Identification of Cardiac Arrhythmias using Natural Resonance Complex Frequencies

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Abstract—An electrocardiogram (ECG) feature extraction system based on the calculation of the complex resonance frequency employing Prony’s method is developed. Prony’s method is applied on five different classes of ECG signals’ arrhythmia as a finite sum of exponentials depending on the signal’s poles and the resonant complex frequencies. Those poles and resonance frequencies of the ECG signals’ arrhythmia are evaluated for a large number of each arrhythmia. The ECG signals of lead II (ML II) were taken from MIT-BIH database for five different types. These are the ventricular couplet (VC), ventricular tachycardia (VT), ventricular bigeminy (VB), and ventricular fibrillation (VF) and the normal (NR). This novel method can be extended to any number of arrhythmias. Different classification techniques were tried using neural networks (NN), K nearest neighbor (KNN), linear discriminant analysis (LDA) and multi-class support vector machine (MC-SVM).

Keywords—Arrhythmias analysis, electrocardiogram, feature extraction, statistical classifiers.

I. INTRODUCTION

CARDIOVASCULAR diseases are the main cause of death globally, where more people die annually from cardiovascular diseases than from any other cause. Approximately 17.5 million people died from cardiovascular diseases in 2005, representing at least 30% of all global deaths according to the world health organization (WHO) report. By 2015, almost 20 million people will die from cardiovascular diseases [1]. Some of the cardiovascular diseases causing death are due to ventricular arrhythmias, valve disease, and coronary artery disease. Since the early detection of these diseases in its initial stages are of great importance to the treatment and life prolonging of the patients.

The electrocardiogram (ECG) signal is one of the most important tools in clinical practice to assess the cardiac status of patients. This signal represents the potential difference between two points on the body surface, versus time [2]. Extracting the features from this signal has been found very helpful in explaining and identifying various cardiac arrhythmias. This could be difficult, when the size of the data of the ECG is huge and the existence of different noise types that may be contained in the ECG signals. Furthermore, manual analysis is considered a very time consuming and is prone to error. Hence arises the importance of automatic recognition and analysis of the ECG signals for extracting the different features available.

Many tools, methods and algorithms from signal processing theory have been proposed, described and implemented over the past few years to extract feature from ECG signals such as, total least squares based Prony modeling algorithm [3], correlation dimension and largest Lyapunov exponent [4], autoregressive model [5], multivariate autoregressive model [6], heartbeat interval combined with the shape and morphological properties of the P, QRS and T waves [7], wavelet transform [8], multiple signal classification (MUSIC) algorithm [9], and efficient formation of morphological wavelet transform features together with the temporal features of the ECG signal [10]. Other researchers reported some techniques to classify ECG signals including the following, linear discriminate analysis technique [7], support vector machines [11], artificial neural network [12, 13], and self organized map [14].

One drawback of these techniques mentioned above is the significant amounts of computation and processing time for extracting the features and the classifications technique employed. Another disadvantage is the small number of arrhythmias that can be classified using a given technique since most techniques can classify two or three arrhythmias. Hence, there is a need for a novel technique to classify a larger number of arrhythmias, and to be amenable for real time implementation especially in intensive care units or from halter devices.

The objective of the present work is to apply Prony’s method to represent the ECG signal as a finite sum of exponentials in order to classify certain cardiac arrhythmias based on the calculation of the complex resonance frequencies and the accompanied poles. Those poles or natural resonance complex frequencies of a signal can be used as a signature and a useful feature for signal discrimination and identification [15]. This technique is employed in many applications, for example, in radar target identification [16], speech recognition [17], frequency perturbation of antenna radiation pattern response [18], and in classifying the underwater source for passive sonar [19]. Prony’s method [20] and Pade’s approximation technique [21] are used to extract the poles of
the reflected radar signal and hence is used as a tool for target identification.

The advantage of Prony’s method is its simplicity and applicability in real-time scenarios where a large amount of data may exist. The poles computed were classified using different classifier models to determine the different arrhythmias including normal rhythm (NR), ventricular couplet (VC), ventricular tachycardia (VT), ventricular bigeminy (VB), and ventricular fibrillation (VF). The verification and the validation of the present method are accomplished using arrhythmias chosen from MIT-BIH database [22].

The paper is organized as follows. Section II introduces the methodology of the proposed algorithm for heartbeat classification schema. Section III presents ECG signals preprocessing procedure employed. Section IV gives a review on the feature extraction method of the proposed algorithm. Section V introduces the classifier models. The simulation results, discussion of results and the conclusion are presented in Section VI, VII and VIII respectively.

II. METHODOLOGY

Fig. 1 depicts the stages of the proposed algorithm for heartbeat classification schema. It consists of three stages: preprocessing stage, feature extraction stage, and classification stage. The ECG signals are first preprocessed to remove baseline wander, power line interference, and high frequency noise. Then, the ECG signal undergoes different processes to enhance signal quality, and to omit equipment and environmental effects. Next, Prony’s method is applied to model ECG signals. The natural resonance complex frequencies are calculated and are used as feature. Finally, various classifier models are employed to test those features and the diagnosis is reached.

![ECG Signal Diagram](https://example.com/ECG_Signaling_Diagram.png)

**Fig. 1 Heartbeat Classification Diagnostic Diagram**

![Preprocessing Diagram](https://example.com/Preprocessing_Diagram.png)

**III. PREPROCESSING**

All ECG data from the MIT-BIH have been filtered to remove the noise that may influence the ECG signal including, baseline wander, artifact, and power line interference. The presence of these noise sources in the signal may mislead the feature extraction and classification. Butterworth band pass filter is designed to remove these low and high unwanted bands of frequencies [23]. The cutoff frequencies of the band pass filter are selected to lie in the range 0.5 to 40 Hz.

IV. REVIEW ON PRONY’S METHOD

Prony method [24]-[27] is a technique for modelling functions or sampled data as a linear combination of damped exponentials.

It is desired to determine an approximation for the ECG to take the form,

\[ f(t_i) = \sum_{\alpha=1}^{P} R_\alpha \exp(s_\alpha t_i), \quad \alpha = 1, 2, 3, ..., P, \quad i = 0, 1, 2, 3, ..., D - 1. \]

Where \( f(t_i) \) is the ECG signal defined at D sampling points \( t_0, t_1, t_2, ..., t_{D-1} \), \( s_\alpha \) is the \( \alpha \)th pole, \( R_\alpha \) is the \( \alpha \)th pole’s amplitude. It is useful to express the equation (1) in discrete sampled data form as it may normally been found in practice, thus,

\[ f(t_i) = \sum_{\alpha=1}^{P} R_\alpha \exp(s_\alpha i \delta t) = \sum_{\alpha=1}^{P} R_\alpha (X_\alpha^i), \quad \alpha = 1, 2, 3, ..., P, \quad i = 0, 1, 2, 3, ..., D - 1. \]

Where \( X_\alpha^i = \exp(s_\alpha i \delta t) \), and the size of the sampling interval is defined as \( \delta t \).

The above set of nonlinear equations (2) have both two sets of unknowns \( X_\alpha^i \)'s and \( R_\alpha^i \)'s. If the constants \( X_\alpha^i \)'s were known, this set would comprise \( D \) linear equations in the \( P \) unknowns \( R_\alpha \)'s and could be solved exactly if \( D = P \) or approximately, by using least square method if \( D > P \). However, if the \( X_\alpha^i \)'s are to be determined, at least \( 2P \) equations are needed.

Using Prony’s method procedure, one can define a polynomial \( A(M) \) of order \( P \) in the variable \( M \), having the same \( \alpha \) roots appearing in equations (1) to (2), thus,

\[ A(M) = \sum_{\alpha=0}^{P} a_\alpha M^\alpha \quad (3) \]

Equation (3) can be written in terms of its roots as,

\[ A(M) = \prod_{\alpha=0}^{P} (M - X_\alpha) = 0 \quad (4) \]

Where \( X_\alpha \)'s are the roots of the above equation.

In order to determine the coefficients \( a_0, a_1, a_2, ..., a_P \) in equation (3), the first equation in (2) will be multiplied by \( a_0 \), the second equation by \( a_1 \), and this process is repeated until the \( Pth \) equation where it is to be multiplied by \( a_P \). Thus the
following set of equations is obtained and can be written in the following matrix form,

\[
\begin{bmatrix}
    a_0f_0 \\
    a_1f_1 \\
    \ldots \\
    a_Pf_P
\end{bmatrix} =
\begin{bmatrix}
    a_0 & a_0 & \ldots & a_0 \\
    a_1 & a_X_1 & \ldots & a_X_1 \\
    \ldots & \ldots & \ldots & \ldots \\
    a_P & a_X_P & \ldots & a_X_P
\end{bmatrix}
\begin{bmatrix}
    R_1 \\
    R_2 \\
    \ldots \\
    R_P
\end{bmatrix}
\]

Where, \( j = 0, 1, 2, \ldots, P \), \( \alpha = 1, 2, 3, \ldots, P \)

Adding the above set of equations (5) gives,

\[
\sum_{a=1}^{P} A(X_a) = \sum_{j=0}^{P} a_j f_j
\]

Where \( A(X_a) \) is defined in equation (4), \( X_a \) is the roots of \( A \), thus, equation (6) yields,

\[
\sum_{j=0}^{P} a_j f_j = 0
\]

A set of \( D - P - 1 \) additional equation can be obtained similar in a way by repeating the steps explained above, starting from \( f_i \) to \( f_{(D-P-1)} \), giving the following set of equations,

\[
\begin{bmatrix}
    f_0 \\
    f_1 \\
    \ldots \\
    f_P \\
    \ldots \\
    f_{D-P-1}
\end{bmatrix} =
\begin{bmatrix}
    a_0 \\
    a_1 \\
    \ldots \\
    a_P
\end{bmatrix}
\]

Since the ordinates \( f_i \) are known, and by taking \( a_P = 1 \) (linear predictor constraint), equation (8) generally can be solved directly for the \( a_i \)'s if \( D = 2P \), or solved approximately by using the least square method if \( D > 2P \). After computing the \( a_i \)'s coefficients, the \( X_a \)'s can be calculated as the root of equation (3). Equation (2) becomes a set of linear equations in \( R \). Thus, \( R \) can be found from the first \( P \) equations (2) or by applying the least square method to the entire set.

The poles and the accompanied natural resonance complex frequencies, \( \omega_a \) of the ECG signal can be directly calculated as,

\[
s_a = \frac{1}{\delta} \log (X_a)
\]

V. CLASSIFIER MODELS

A. Artificial Neural Network

Neural network are analytical techniques modeled analogous to the process of learning in the cognitive system and the neurological functions of the brain [28]. They are capable of predicting new observations from previous observations after executing the learning process using the past existing data. Neural networks or artificial neural networks (ANNs) can be defined as a computational system consisting of a set of highly interconnected processing elements, called neurons, which process information as a response to external stimuli [29]. Stimuli are transmitted from one processing element to another via synapses or interconnection, which can be excitatory or inhibitory. ANNs are useful in application areas such as pattern recognition, classification, etc. [32].

A multilayer feed forward network named the multilayer perceptrons (MLPs) is employed as a class of neural networks. Usually, MLP is made up of several layers of neurons. Each layer is fully connected to the next one. MLP consists of two phases, the training phase and the testing phase. During the training phase, the features are applied at the input and the corresponding desired classes are at the output of MLP classifier. A training algorithm is executed to adjust the weights and the bias until the actual output of the MLP matches the desired output and performance satisfaction is reached. In the test phase, a set of test features, which are not part of training features, are applied to the trained MLP classifier to test the classification of the unknown features.

B. K- Nearest Neighbor

K-Nearest Neighbor (KNN) is based on the principle that the instances within a dataset will generally exist in close proximity to other instances that have similar properties. If the instances are tagged with a classification label, then the value of the label of an unclassified instance can be determined by observing the class of its nearest neighbors [30]. The KNN locates the K nearest instances to the query instance and determines its class by identifying the single most frequent class label. More details about KNN can be found in [30, 31].

C. Linear Discriminate Analysis

The aim of linear discriminate analysis (LDA) known as Fisher's LDA is to use hyper planes to separate the data of the
different classes [32, 33]. The separating hyper plane is obtained by seeking the projection that maximizes the distance between-classes and minimizes the distance within-classes. To solve an N-class problem (N > 2) several hyper planes are used. This technique has less computational requirements making it suitable for such class of problems. Moreover this classifier is simple to use and generally provides good results in many applications.

D. Support Vector Machines

Support vector machine (SVM) maps the input vectors to a higher dimensional space where a maximal separating hyper plane is constructed. Two parallel hyper planes are constructed on each side of the hyper plane separating the data [34]. The separating hyper plane is the hyper plane that maximizes the distance between the two parallel hyper planes. An assumption is made that the larger the margin between these parallel hyper planes is, the better the impact of the classifier will be.

Although SVM were primarily designed for binary classification problems, it can be used in multi-class classification problems. The most common approaches to create M-class classifiers, are the “one versus the rest” and the “pair wise classification”. In this paper, “one versus the rest” approach is presented. Detailed information on SVM can be found in [34].

VI. SIMULATION RESULTS

The proposed algorithm for heartbeat classification schema was tested on the MIT-BIH Arrhythmia Database. The data set used for this work was comprises five different types including normal (NR), ventricular couplet (VC), ventricular tachycardia (VT), ventricular bigeminy (VB), and ventricular fibrillation (VF). Each type was represented by 64 different patients signals having duration of three seconds long, they were taken from lead number two (MLII). The VF signals were sampled at 250 sample/sec, while the others were sampled at 360 sample/sec. Resembling adjustment is employed.

In order to investigate the validity of the proposed method, four classifiers model are employed. Neural networks, K nearest neighbor, linear discriminate analysis and multi-class support vector machine. All classifier models were designed and tested using the poles and the accompanied complex resonant frequencies sets extracted from ECG signals using Prony’s method. All features sets are divided into independent training and testing sets using n-fold cross validation method. This scheme randomly divides the available data into n approximately equal size and mutually exclusive folds. For an n-fold cross validation run, the classifiers are trained with a different n fold used each time as the testing set, while the other n-1 folds are used for the training data. In this study three fold cross validation were employed.

A feed forward multilayer perceptron (MLP) neural network with three layers is implemented, input layer, hidden layer, and output layer. The number of neurons selected at input layer is equal to the number of poles and the accompanied complex frequencies. The neurons at the output layer are selected according to the number of classes. One step secant back propagations training function is used to update the weight. The Tan-Sigmoid function is used as the transfer function in the first and second layers, and pure line function is used in the output layer. An error-correction rule is used to adjust the synaptic weights; where the error is the difference between the target and actual network output.

The distance function applied for K nearest neighbor technique is the Euclidean distance to match the test examples with training examples, and for different values of k, where k is taken to be 1, 3 and 5. One versus the rest MC-SVM technique with linear training algorithm is employed in this work.

Following the guidelines proposed by the Association for the Advancement of Medical Instrumentation (AAMI) [35], three benchmark parameters were used to assess the algorithm performance: accuracy, specificity and sensitivity. From Table 1, they are defined as,

\[
\text{Accuracy} = \frac{\text{TP} + \text{TN}}{\Sigma} \quad (10)
\]

\[
\text{Sensitivity} = \frac{\text{TP}}{\text{TP} + \text{FN}} \quad (11)
\]

\[
\text{Specificity} = \frac{\text{TN}}{\text{TN} + \text{FP}} \quad (12)
\]

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\[
\text{TP} = \text{VCV} + \text{VVT} + \text{VVB} + \text{VVF}
\]

\[
\text{FP} = \text{NRV} + \text{NRV} + \text{NRV} + \text{NRV}
\]

Where, TP, TN, FP, and FN stand for true positives, true negatives, false positives, and false negatives respectively. True positives represent abnormal beats classified in their respective classes whereas true negative represents normal beats classified as normal. False positives represent normal beats classified as abnormal and false negatives represent abnormal beats classified as normal [7, 35].
VII. IMPLEMENTATION PROCEDURE AND DISCUSSION OF RESULTS

Initially, ECG signals were filtered using a Butterworth band pass filter with cutoff frequencies of 0.5 to 40 Hz to reduce the noise. Figure 2.a., shows a normal ECG signal corrupted with baseline drift and power line interference noise. In Figure 2.b., shows a noise free ECG signal as a result of applying the Butterworth band pass filter.

Fig. 2.a. Unfiltered normal ECG signal

Fig. 2.b. Filtered normal ECG signal

The next step is to extract the poles and the accompanied complex resonance frequencies using Prony’s method for all the filtered ECG signals. A reconstruction of all these ECG signals using the previous calculated poles and the accompanied complex resonance frequencies proves the exactness of the method employed. This is shown in figures 3, 4, 5, 6, and 7. Both the original filtered ECG signal and the constructed ECG signal from Prony’s method coincide on each other.

Application of various classifier models to test those features is the final stage of the proposed schema. Table 2, shows the result of all classifier models used in this paper. The artificial neural network gives the best results, where the accuracy, specificity and sensitivity reached to 100%. The accuracy and the specificity for K nearest neighbor (k = 1) reach to 93.33% and 100% respectively, which is more accurate than MC-SVM and LDA. On other hand, the sensitivity of MC-SVM and LDA is more accurate than KNN which it is equal to 100% for MC-SVM and 98.65% for LDA. This means, MC-SVM and LDA are more accurate in identification of arrhythmia with patients suffering from abnormal beats than KNN.

Fig. 3. A patient ECG and simulated ECG with NR.

Fig. 4. A patient ECG and simulated ECG with VC.

Fig. 5. A patient ECG and simulated ECG with VT.
the author presented method achieves higher specificity and sensitivity than the Owis technique method [3].

<table>
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<th>Sensitivity</th>
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<td>Owis et al [4]</td>
<td>KNN, k=1</td>
<td>34.38%</td>
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<td></td>
<td>KNN, k=3</td>
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<td>KNN, k=5</td>
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<td>KNN, k=1</td>
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<td>97.47%</td>
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<td></td>
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<td></td>
<td>KNN, k=5</td>
<td>76.19%</td>
<td>95.89%</td>
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</table>

VIII. CONCLUSION

A new feature extraction technique of ECG signal based on complex resonance frequencies and the accompanied poles using Prony’s method is presented. The results show the technique’s ability to detect the different arrhythmias. Also multilayer perceptrons neural network has proved its accuracy compared to other techniques as an excellent classifier model. In addition MC-SVM and LDA are more accurate in identification of arrhythmia with patients suffering from abnormal beats than KNN.

REFERENCES


Table 3 shows the comparison between the proposed technique with other method introduced by Owis et al [3] using the same data. They developed a feature extraction technique using the correlation dimension and largest Lyapunov exponent, to model the chaotic nature of five different classes of ECG signals. It can be seen from Table 3,


