Unsupervised Texture Segmentation via Applying Geodesic Active Regions to Gaborian Feature Space

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Abstract—In this paper, we propose a novel variational method for unsupervised texture segmentation. We use a Gabor filter bank to extract texture features. Some of the filtered channels form a multidimensional Gaborian feature space. To avoid deforming contours directly in a vector-valued space we use a Gaussian mixture model to describe the statistical distribution of this space and get the boundary and region probabilities. Then a framework of geodesic active regions is applied based on them. In the end, experimental results are presented, and show that this method can obtain satisfied boundaries between different texture regions.

Keywords—Texture segmentation, Gabor filter, snakes, Geodesic active regions

I. INTRODUCTION

TEXTURE is an essential feature of object in human vision. Texture analysis and segmentation is an important cue of pattern recognition and image processing. In the recent decades researchers have developed various kinds of feature extractors to measure texture for classification and segmentation[1]. Statistical features based on second-order gray level statistics, Markov random field (MRF), Gibbs random field (GRF), and Gabor filters, Gabor wavelets[2][3] are adopted. Bigün proposed a structure tensor to discriminate texture orientations[4], which is regarded to be able to overcome the redundant drawback of Gabor filters[5][6].

In this paper a filter bank composed of several Gabor filters in different scales and orientations are used to extract texture features. A multidimensional Gaborian feature space is obtained by selecting some most effective channels from them. Then the main problem is object segmentation in this vector-valued space. If we use curves to denote boundaries separating homogeneous regions, object segmentation can be associated with the problem of boundary detection and integration. Since a method named “snakes” or “active contours” is investigated by Kass et al. to approach this problem, it was widely implemented and improved because of its good boundary detecting and noise resisting performance. In this variational framework the contours or surfaces deform iteratively by minimizing a predefined energy function which achieves a minimum value at the boundary of object. Caselles et al. and Malladi et al. proposed the geometric model of deformable contours and surfaces. This modal is given by a geometric flow associated to mean curvature, but not by an energy function. In [7] we can see the relation between snakes and geometric models. By integrating these two approaches geodesic active contours model[8] is presented to find object boundary. It extends the classical snakes method to find a geodesic curve in a Riemannian space with a metric derived from the gray-level information of images. Level-set method is applied to represent the deformation of contours, and it overcomes the topological restriction of snakes[9]. Geodesic active contours model was implemented to texture segmentation[10]. If region information is added to it we will get geodesic active regions model.

Initially, these models are restricted to gray-level images, because the gradient flow is not easy to obtain in vector-valued images such as color (RGB) or medical (MRI). The multidimensional Gaborian feature space can be regarded as a vector-valued image. There are two approaches to do it. One is to process each plane separately and then to integrate the results. The other is to get a unique gradient flow by integrating the vector information[7]. But both these two approaches are enormously more complex than the gray-level one. Paragios[11] and Muñoz[12] used Geodesic active regions model for texture segmentation in another way. In Paragios’s method he did not construct a Riemannian space directly upon the vector-valued space, but developed a supervised method to get the boundary and region information from some predefined texture patterns. And then he could use the geodesic model to deform contours as well as in a gray-level image. But normally these patterns are not easy to get.

So we present an unsupervised approach to apply the geodesic model. After getting the feature space we use a Gaussian mixture model (GMM) to describe the statistical distribution of it. Then we can obtain the probabilities of each pixel belonging to each region and being on the boundaries. By using this information we can construct the needed Riemannian space and use geodesic active regions model to attract an initial contour to the object boundary.

Paper organization. Gabor filter and geodesic active regions model are reviewed in Section II. In Section III our method is introduced in detail. Experimental results are shown in Section IV and conclusions in the last section.

II. METHOD REVIEW

A. Gabor filter

Gabor schemes and Gabor filters have been studied and
implemented in image representation, texture segmentation, and image retrieval in decades. Biological researchers pointed out that Gabor filters are closely related to the function of simple cells in the primary visual cortex, so the Gabor scheme is able to provide a suitable representation for visual information in the combined frequency-space domain [13]. It can achieve the minimum restriction for simultaneous localization in both spatial and frequency domains [14].

Gabor function is a complex sinusoid modulated by a Gaussian envelope. Because of this envelope it can extract localized frequency features such as texture among a given local area. A typical 2-D Gabor function  and its Fourier transform have the following form:

\[
g(x, y) = \frac{1}{2\pi \sigma_x \sigma_y} \exp\left( -\frac{x^2}{2\sigma_x^2} - \frac{y^2}{2\sigma_y^2}\right)
\]

\[
h(x, y) = g(x, y) \cdot \exp[2\pi j W x]
\]

\[
H(u, v) = \exp\left( -\frac{1}{2} \left( \frac{(u-W)^2}{\sigma_u^2} + \frac{v^2}{\sigma_v^2}\right)\right)
\]

where \(\sigma_u\) and \(\sigma_v\) are the standard deviations in two axes of the Gaussian functions. They determine the size of the receptive field. \(W\) is the frequency of the sinusoidal plane wave along the horizontal axis. The complex magnitude of the filtered image will be maximized over the regions which have a close spatial and frequency attributes to the Gabor function. Because textures are various widely, a bank composed of several Gabor filters in different scales and orientations is commanded.

**B. Geodesic Active Regions**

Geodesic active contours and geodesic active regions models have been successively used in the context of gray-level images. Geodesic active contours is derived from both the energy based active contours and geometric curve evolution. It can derive the relation between them, and is regarded as a geometric alternative for snakes.

A given image is denoted as \(I : [0, w] \times [0, h] \to \mathbb{R}\). We use \(C(p) : [0, 1] \to \mathbb{R}^2\) to denote a parameterized curve. The classical snakes model associates the curve \(C\) with an energy as

\[
E(C) = \alpha \int \nabla I(C) \, dp + \beta \int \nabla^2 I(C) \, dp + \gamma \int \nabla I(C) \, dp
\]

(2)

where the first two terms basically control the smoothness and the third one attracts the contour to the object boundary.

In novel geometric model if we use a level-set \(u : \mathbb{R}^2 \to \mathbb{R}\) to denote the curve \(C\), the proposed 2D deformation is given by

\[
u_I = (\nu + \kappa) |\nabla u|
\]

(3)

where \(\nu\) is a positive constant and \(\kappa\) is the mean curvature.

When the flexibility parameter \(\beta\) in (2) is set to be zero minimizing the energy is equivalent to minimizing the length of a curve in a Riemannian space with some predefined metric.

\[
L = \int g(\nabla I(C)) \, ds
\]

(4)

where \(L\) denotes the Euclidean length of the curve, and \(g(r) : [0, \infty] \to \mathbb{R}^2\) is an inverse edge detector. It is called geodesic active contours model. In this model finding the object boundary is to find a geodesic curve in a Riemannian space by minimizing (4).

The snakes model and geodesic active contours model are based on the boundary information and are sensitive to local minima. An edge and region based information within a deformable boundary finding framework was applied in medical image segmentation [15]. It was called geodesic active regions. In Paragios’s Ph.D thesis [16] he defined the energy function in geodesic active regions model as:

\[
E(\partial R) = \alpha \int \left( g(p_u, I(\partial R)) - \bar{c}(R) \right)^2 \, ds - \beta \int \left( \log p_x(I(\partial R)) \right)^2 \, ds + \int \log p_x(I(\partial R)) \, ds
\]

(5)

where the image is composed of two non-overlapping region \(R_a\) and \(R_b\), and \(\partial R\) is the boundaries. \(p_a()\) and \(p_b()\) are the conditional region probability density functions of a pixel belonging to \(R_a\) and \(R_b\). \(p_c()\) is the probability of pixel on the boundaries. Then finding object boundaries is equivalent to minimizing (5).

**III. Application of Geodesic active regions to the Gaborian feature space**

**A. Get the Gaborian feature space**

Firstly we must choose an appropriate Gabor filter bank to cover all the wanted frequencies and orientations. The Gabor functions form a complete but non-orthogonal basic set. If we set the frequency range to be \([U_a, U_b]\), a filter bank parameters setting method is proposed by

\[
a = \frac{U_k}{U_j}\n\]

(5)

\[
U(i) = a^i U_j
\]

\[
\sigma_{a(i)} = \frac{(a-1) U_{a(i)}}{(a+1) \sqrt{2\ln 2}}
\]

where the bank is composed of \(S \times K\) Gabor filters, and \(n = 0, 1, ..., S - 1\) and \(m = 0, 1, ..., K - 1\) are the frequency and orientation indexes of each filter. After filtering several most effective channels are selected to form a Gaborian feature space.

**B. Estimate the Gaussian mixture model**

Paragios filtered the given image with a serial of texture patterns by which the image is composed to get the boundary
and region information needed in geodesic active regions model. In normal cases we do not have these patterns, so we are inclined to develop an unsupervised approach. Gaussian mixture model is applied to describe the statistical distribution of the feature space. From it we are able to get the probabilities of each pixel belonging to each texture component and being on the boundaries. Also we can avoid getting the geometric flow directly in a vector-valued space.

Successive estimation algorithms such as the expectation maximization (EM) algorithm[17] can be used to estimate the free parameters of the mixture model from the whole vector set. Before doing that we roughly estimate the center vector of each component by using mean shift method to cluster some sampled vectors in the feature space into several classes. If we get K classes we use a K-component Gaussian mixture model to describe the statistical distribution of these vectors.

\[ P(x | \theta) = \sum_{k=1}^{K} P(x | k) P(k) \]  

where \( \theta \) contains three quantities for each Gaussian: the centroid \( \mu_k \), the covariance matrix \( \Sigma_k \) and the prior \( P(k) \).

C. Adopt the Geodesic active regions model

Assume the image is composed by K different texture components. A region \( R_i \) consisting of the \( i^{th} \) component is denoted by a parameterized curve \( C_i(p):[0,1] \to R^2 \). We define an energy function among it as:

\[ E(R_i) = \alpha \int g(\{\nabla p_i(C_i(p))\}) dp - \beta \int \log p_i(L_{xy}) dxdy \]  

where \( p_i() \) is the probability of pixel on the boundaries of the \( i^{th} \) component and \( p_i() \) is the probability of pixel belonging to the \( i^{th} \) component. The first term attracts the curves to boundaries and the second term retains the homogeneity of the region.

To minimize this energy is equivalent to solve a partial differential equation (PDE). If we define a level function \( \Phi \in R^2 \to R \) so that the initial curve is represented as the zero level-set \( C = \{ (x,y) : \Phi(x,y) = 0 \} \), the curve should deform according to the following equation by using Euler-Lagrange equations:

\[ u_i = \alpha \left[ g(\{\nabla p_i(u)\}) \kappa - \nabla g(\{\nabla p_i(u)\}) \cdot \bar{N} \right] - \beta \log(p_i(u)) \]  

where \( g() \) is an inverse edge detector and \( \kappa \) is the mean curvature. Function \( p_i() \) can be got directly from the GMM. To compute \( p_i() \) we assume all the edges are in only four directions shown in Fig. 1. We compute the average probability of each side neighborhood belonging to the \( i^{th} \) component. Then select the direction which has a maximum contrast between the two sides. If in this direction the probabilities of the two sides are \( p_i \) and \( p_j \) \( (p_i > p_j) \), we set the probability of pixel \( (x,y) \) being on the boundary of region \( R_i \) as:

\[ P_i(x,y) = \frac{(p_i - p_j)}{(p_i + p_j)} \]  

IV. EXPERIMENTAL RESULTS AND DISCUSSION

A fixed Gabor filter bank \( (S = 3, K = 4, U_1 = 0.044, U_2 = 0.35) \) is used to test the performance of our method with synthetic images composed of textures from the Brodatz texture database. The filter bank includes twelve Gabor filters in three scales and four orientations. Narrow band level-set and coupled level-set methods are used to deform the initial curves according to geodesic active regions model.

Firstly we show two samples composed of two homogeneous textured components. Fig. 2a is an image composed of D21 and D51, and coupled level-set method is used. After filtering and channel selection we get a 3-dimensional feature space. Fig. 2b and Fig. 2c are the interim result. Finally we get Fig. 2d for the segmentation result. It shows that in this case it can achieve accurate boundaries.
In this paper, an unsupervised texture segmentation method is proposed. Texture features are extracted by using a Gabor filter bank. A geodesic active regions model is presented and applied to this multidimensional Gaborian feature space. We use Gaussian mixture model to describe the statistical distribution of this vector-valued space to avoid directly deforming curves in it. The needed boundary and region information is obtained from this GMM. In the end experimental results show that by minimizing the defined energy we can get satisfied boundaries between different texture regions.

REFERENCES