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PID Controller Design for Following Control of Hard Disk Drive by Characteristic Ratio Assignment Method

Chaoraingern J., Trisuwannawat T. and Numsomran A.

***Abstract*—** The author present PID controller design for following control of hard disk drive by characteristic ratio assignment method. The study in this paper concerns design of a PID controller which sufficiently robust to the disturbances and plant perturbations on following control of hard disk drive. Characteristic Ratio Assignment (CRA) is shown to be an efficient control technique to serve this requirement. The controller design by CRA is based on the choice of the coefficients of the characteristic polynomial of the closed loop system according to the convenient performance criteria such as equivalent time constant and ration of characteristic coefficient. Hence, in this study, CRA method is applied in PID controller design for following control of hard disk drive. Matlab simulation results shown that CRA design is fairly stable and robust whilst giving the convenience in controller's parameters adjustment.

***Keywords*—**Following Control, Hard Disk Drive, PID, CRA

I. INTRODUCTION

Design objectives of the servo controller in following control of hard disk drive are minimum variance regulation of the position of the head. Various sources of disturbances, noise and perturbations have influences on the performance of the servo control system. Especially plant perturbation may arise from external sources or internal load variations. Thus robust controller is needed to stabilize this type of systems for the entire range of disturbance and variations in the plant parameters.

Recently, several robust controllers are developed for uncertain systems. Another robust controller known as Characteristic ratio assignment method is developed and introduced in [1]. The structure of this controller is based on certain relations between characteristic polynomial coefficient. Refer to [2-3] for historical development. By the strength of CRA lies in that the simplest and robust controller, many control systems have been designed successfully using CRA [4-5].

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This paper proposes PID controller design for following control of hard disk drive by characteristic ratio assignment method which satisfies specification of performance of control system and has capability on robustness to disturbances and plant perturbations. Furthermore, it is very convenient in controller's parameters adjustment.

The paper introduces summary details of hard disk drive servo system in section II. Section III gives the CRA design procedure. PID Controller design for the track following control of hard disk drive is explained in section IV. Section V describes the experimental simulations by Matlab program and theirs obtained results. Finally, conclusions are given in section VI.

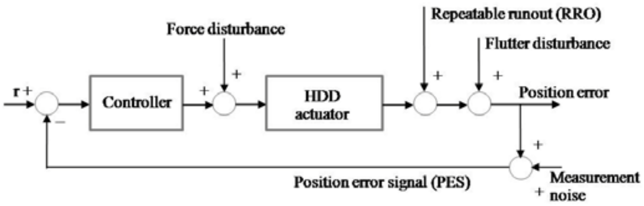

Fig. 1. HDD servo system

II. HARD DISK DRIVE SERVO SYSTEM

A. Plant Model

In this section, we present the modeling of the HDD actuator that has a characteristic of a second order model with poles on the left hand side of the complex plane cascaded with some high-frequency resonance.

Referring to HDD actuator plant model from [7], The dynamics of an HDD actuator can be formulated as a transfer function as follows:

$$\frac{y(s)}{u(s)} = P_f(s) = \frac{K_f}{mT_p} P_{mech}(s) e^{-sT_d} \tag{1}$$

where $u(s)$ is the actuator input (in volts), $y(s)$ are the position (in tracks) of the R/W head, K_f is torque constant, m is mass (kg), T_p is width of track (m : 100kTPI), e^{-sT_d} is input delay consisting of all delay component of the plant, P_{mech} is a transfer function of actuator resonance which is shown as;

$$P_{mech}\left(s\right)=\sum_{i=1}^{n}\frac{A_i}{s^2+2\zeta\omega_i s+\omega_i^2};\quad \omega_i=2\pi f_i \tag{2}$$

where n is the number of significant resonance modes in the frequency range of interest.

This include a nominal model whose natural frequency is 90 Hz and mechanical resonance whose frequency are 3 kHz, 4.1 kHz, 5 kHz, 8.2 kHz, 12.3kHz, and 16.4 kHz. These frequency can be fluctuated in the range of $\pm 5\sim\pm 10\%$, and one can evaluate robust performance of a controller. Fig. 2 and Fig. 3 show Bode diagram of the plant.

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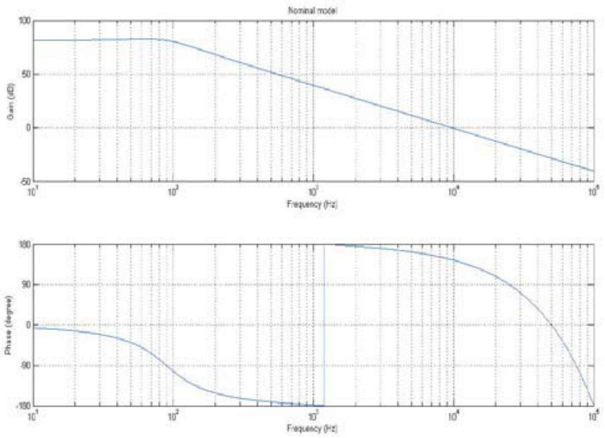

Fig. 2. Bode Diagram of Nominal HDD actuator Model

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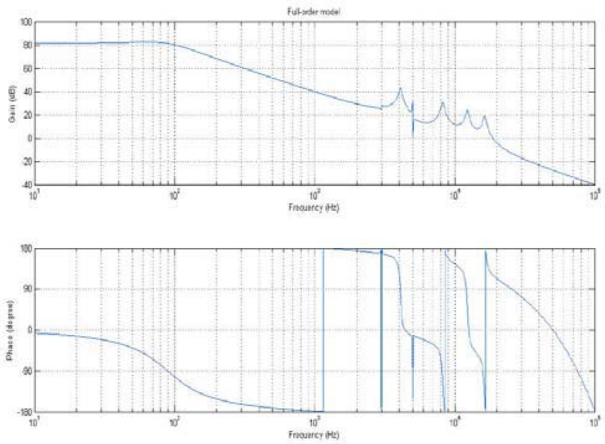

Fig. 3. Bode Diagram of the actual and identified HDD actuator models

B. Disturbance Model

Classified the sources of disturbances [7], which are the error sources contributing to $3\sigma_{pes}$, into following four categories.

Torque noise : AmpTorqueNoise = 1.0e-4 [A] at control input.

Sensor noise : AmpSensorNoise = 1.5e-2 [Track]

Flutter&Suspension :

$$G_{fu}\left(s\right)=\sum_{i=1}^{Nflu}\frac{\alpha_{flu}^{i}\cdot 2\cdot\zeta_{flu}^{i}\cdot{\omega_{flu}^{i}}^{2}}{s^2+2\cdot\zeta_{flu}^{i}\cdot\omega_{flu}^{i}\cdot s+{\omega_{flu}^{i}}^{2}} \tag{3}$$

Repeatable Runout (RRO) is the result of 1st order effects such as error in the servo track writer or disk shift (eccentricity). To be a little more specific, assuming that the RRO,*d(t)*, is a time-varying unknown disturbance consisting of a sum of sinusoids of known frequencies,

FreqRRO = [1, 2, 3]*(rpm/60) [Hz]

AmpRRO = [0, 15, 0.03, 0.006] [Track] at PES

$$d_{rro}\left[k\right]=\sum_{i=1}^{Nrro}\alpha_{rro}^{'}\cdot\sin\left(2\pi\cdot f_{rro}^{'}\cdot k\cdot Ts\right)+\hat{d}_{rro}\left[k\right] \tag{4}$$

Frequency characteristic of the disturbance is shown in Fig. 4~6.

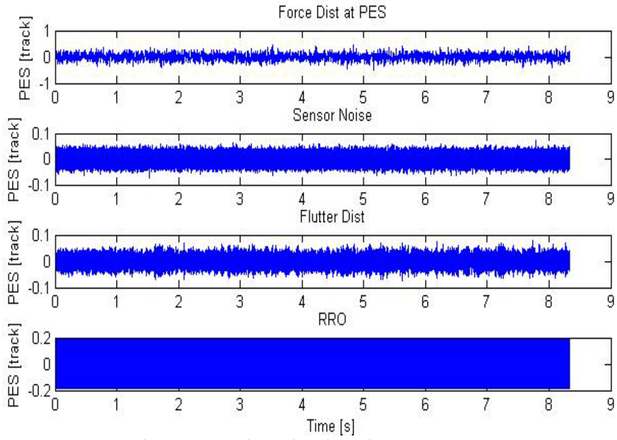

Fig 4.PES of Each Disturbance Source

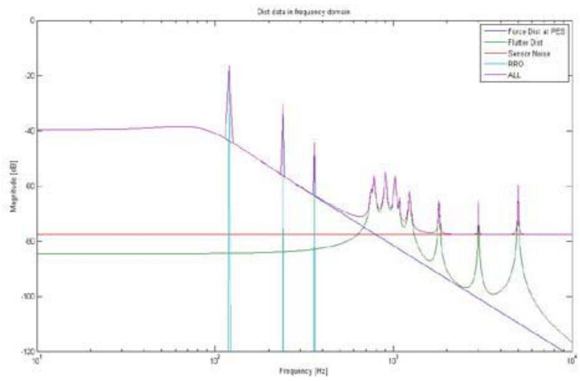

Fig. 5. Disturbance Data in Frequency Domain

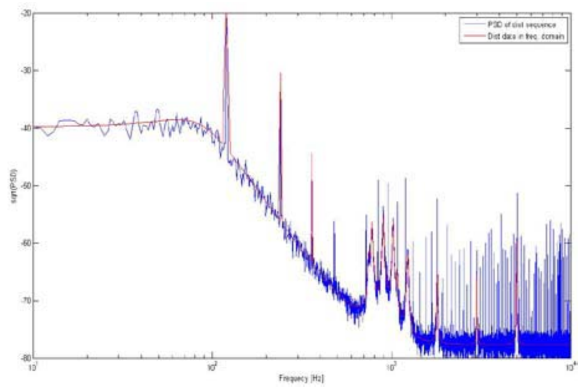

Fig. 6. Frequency Response of Plant with all Disturbance

III. CHARACTERISTIC RATIO ASSIGNMENT

Naslin [1] has studied the problems to optimize a damping ratio of control systems, and he found that the damping ratio relates with characteristic ratio The relation of characteristic equation can be illustrated as follow.

$$p(s) = a_n s^n + a_{n-1}s^{n-1} + \ldots + a_1 s + a_0,\ \forall a_i > 0 \quad (5)$$

Where, the characteristic ratio is given by,

$$\alpha_1 = \frac{a_1^2}{a_0 a_2},\ \alpha_2 = \frac{a_2^2}{a_1 a_3},\ \ldots,\ \alpha_{n-1} = \frac{a_{n-1}^2}{a_{n-2}a_n} \quad (6)$$

and the inverse of characteristic equation is given as

$$b_0 = \frac{a_0}{a_1},\ b_1 = \frac{a_1}{a_2},\ \ldots,\ b_{n-1} = \frac{a_{n-1}}{a_n} \quad (7)$$

Otherwise, it is able to depict the relation of coefficient ration and characteristic pulsatances as,

$$\alpha_1 = \frac{b_1}{b_0},\ \alpha_2 = \frac{b_2}{b_1},\ldots,\ \alpha_{n-1} = \frac{b_{n-1}}{b_{n-2}} \quad (8)$$

Thus the time constant is given by

$$\tau = \frac{a_1}{a_0} \quad (9)$$

Given equation (6)-(7), it is able to describe in another form by the coefficient of characteristic equation

$$A = \begin{bmatrix} a_n & a_{n-1} & \cdots & a_1 & a_0 \end{bmatrix} \quad (10)$$

$$B = \begin{bmatrix} b_{n-1} & b_{n-2} & \cdots & b_1 & b_0 \end{bmatrix} \quad (11)$$

$$C = \begin{bmatrix} c_{n-2} & c_{n-3} & \cdots & c_1 & c_0 \end{bmatrix} \quad (12)$$

Where

$$b_i = \frac{a_i}{a_{i+1}}\ \ i = 0,1,2,\ldots,n-1 \quad (13)$$

$$c_i = \frac{b_{i+1}}{b_i}\ \ i = 0,1,2,\ldots,n-2 \quad (14)$$

Design controller by CRA method, assigns value of time constant (τ) and characteristic pulsatances (α_i) up to system requirement, To define the value of α_i, it is under the rule of Lipatov and Sokolov [2] in order to retain the system stability which is given by,

$$\sqrt{\alpha_i \alpha_{i+1}} > 1.4656, i = 1,2,\ldots,n-2 \quad (15)$$

$$\alpha_i \geq 1.12374\alpha_i^*, i = 2,3,\ldots,n-2 \quad (16)$$

$$\alpha_i^* = \frac{1}{\alpha_{i+1}} + \frac{1}{\alpha_i - 1}, \alpha_n = \alpha_0 = \infty \quad (17)$$

A. Adjustment speed response of control system

To adjust a speed response of control system, the CRA method can be tuned by changing a value of time constant as shown in the next equation. Assume, the transfer function is given as

$$G(s) = \frac{a_0}{a_n s^n + a_{n-1}s + \ldots + a_1 s + a_0} \quad (18)$$

Then it is arranged in new format

$$G(s) = \frac{a_0/a_n}{s^n + \frac{a_{n-1}}{a_b}s + \ldots + \frac{a_1}{a_n}s + \frac{a_0}{a_n}} \quad (19)$$

From equation (19) is able to construct the coefficient as inverse form of characteristic equation

$$A = \begin{bmatrix} 1 & \prod_{i=n-1}^{n-1} b_i & \cdots & \prod_{i=2}^{n-1} b_i & \prod_{i=1}^{n-1} b_i \end{bmatrix} \quad (20)$$

When increasing a gain with equivalent ratio by k, it is obtained by

$$A = \begin{bmatrix} 1 & k\prod_{i=n-1}^{n-1} b_i & \cdots & k^{n-1}\prod_{i=2}^{n-1} b_i & k^n\prod_{i=1}^{n-1} b_i \end{bmatrix} \quad (21)$$

Equation (20) and (21), coefficient ratio is unchanged, but the time constant is able to change as shown in equation (22) (23)

$$G_k(s) = \frac{k^n a_0}{a_n s^n + k a_{n-1}s^{n-1} + \ldots + k^{n-1}a_1 s + k^n a_0} \quad (22)$$

$$\tau = \frac{1}{k}\left(\frac{a_1}{a_0}\right) \quad (23)$$

Where $0 < k < 1$

B. Adjustment damping ratio of control system

To adjust the damping ratio, if a system is high order then there are many parameters (α_i) to be adjusted. Thus the CRA method is designed for tuning only one parameter that follows as,

$$G(s) = \frac{1}{\frac{a_n}{a_0}s^n + \frac{a_{n-1}}{a_0}s^{n-1} + \ldots + \frac{a_1}{a_0}s + 1} \quad (24)$$

$$A = \begin{bmatrix} \frac{1}{\prod_{i=0}^{n-1} b_i} & \frac{1}{\prod_{i=0}^{n-2} b_i} & \cdots & \frac{1}{\prod_{i=0}^{1} b_i} & \frac{1}{\prod_{i=0}^{0} b_i} \end{bmatrix} \quad (25)$$

$$A = \begin{bmatrix} \frac{1}{\left[\prod_{i=0}^{n-2} c_i \prod_{i=0}^{n-3} c_i \cdots \prod_{i=0}^{0} c_i\right] b_0^n} & \frac{1}{\left[\prod_{i=0}^{n-3} c_i \prod_{i=0}^{n-4} c_i \cdots \prod_{i=0}^{0} c_i\right] b_0^{n-1}} & \cdots & \frac{1}{b_0} & 1 \end{bmatrix} \quad (26)$$

When the coefficient ratio is increased by k then the new characteristic equation is given by

$$A = \begin{bmatrix} \frac{1}{k^{\frac{1}{2}n^2 - \frac{1}{2}}\left[\prod_{i=0}^{n-2} c_i \prod_{i=0}^{n-3} c_i \cdots \prod_{i=0}^{0} c_i\right] b_0^n} \\ \frac{1}{k^{\frac{1}{2}n^2 - \frac{1}{2} - 1}\left[\prod_{i=0}^{n-3} c_i \prod_{i=0}^{n-4} c_i \cdots \prod_{i=0}^{0} c_i\right] b_0^{n-1}} & \cdots & \frac{1}{b_0} & 1 \end{bmatrix} \quad (27)$$

Finally, equation (26) and (27) are formulated to the equation (28) when $k > 1$; damping ration will be increased then $0 < k < 1$; damping ratio will be decreased.

$$G_k(s) = \frac{k^{\frac{1}{2}n^2 - \frac{1}{2}n} a_0}{a_n s^n + k^{n-1}a_{n-1}s^{n-1} + \ldots + k^{\frac{1}{2}n^2 - \frac{1}{2}n - 1}a_1 s + k^{\frac{1}{2}n^2 - \frac{1}{2}n}a_0} \quad (28)$$

IV. PID Controller Design by CRA

In this section, the design procedures of PID Controller via CRA which are verified through the MATLAB.

In this paper, we only consider a second-order model for the HDD actuator at this stage. We will then put the resonance modes back when we are to evaluate the performance of the

overall design. Thus, in our design, we first use the following simplified model of the HDD actuator:
Plant transfer function

$$G_p(s) = \frac{p_1 s + p_0}{q_2 s^2 + q_1 s + q_0} \tag{29}$$

PID transfer function

$$G_c(s) = \frac{\left[\left(K_p + K_d A\right)s^2 + \left(K_i + K_p A\right)s + K_i A\right]}{s(s+A)} \tag{30}$$

Closed Loop transfer function

$$G_{cl}(s) = \begin{bmatrix} \left(K_d A p_1 + K_p p_1\right)s^3 + \\ \left(K_p A p_1 + K_i p_1 + K_d A p_0 + K_p p_0\right)s^2 + \\ \left(K_i A p_1 + K_p A p_0 + K_i p_0\right)s + K_i A p_0 \end{bmatrix} / \begin{bmatrix} q_2 s^4 + \left(K_p p_1 + K_d A p_1 + q_1 + A q_2\right)s^3 + \\ \left(A q_1 + K_i p_1 + K_p p_0 + K_p A p_1 + K_d A p_0 + q_0\right)s^2 \\ + \left(K_p A p_0 + K_i A p_1 + K_i p_0 + A q_0\right)s + K_i A p_0 \end{bmatrix} \tag{31}$$

As CRA method, we assign the value of equivalent time constant τ and characteristic ratio $\alpha_1, \alpha_2, \alpha_3$, hence the $P(s)$ can be expressed as

$$P(s) = a_4 s^4 + a_3 s^3 + a_2 s^2 + a_1 s + a_0, \tag{32}$$

where

$$a_i = \frac{a_o \tau^i}{\left(\alpha_{i-1}\alpha_{i-2}^2 \cdots \alpha_2^{i-2}\alpha_1^{i-1}\right)}$$

and characteristic equation of closed loop transfer function is

$$P_{cl}(s) = q_2 s^4 + \left(K_p p_1 + K_d A p_1 + q_1 + A q_2\right)s^3 + \left(A q_1 + K_i p_1 + K_p p_0 + K_p A p_1 + K_d A p_0 + q_0\right)s^2 + \left(K_p A p_0 + K_i A p_1 + K_i p_0 + A q_0\right)s + K_i A p_0 \tag{33}$$

Equated the above $P(s)$ to the $P_{cl}(s)$ of the plant in equation (33), the parameters of PID controller are received.

V. Simulation & Results

We now present track following control system design for control amount of NRPE (nonrepeatable position error), RPE(repeatable position error), and PES (position error signal) by CRA methods.

The overall control system that proceeds by Matlab simulink is depicted as in Fig. 7.

As HDD VCM plant parameters in table I, we obtained a simplified model of the HDD actuator:

$$G_p(s) = \frac{1.137e-13s + 3.745e9}{s^2 + 565.5s + 3.198e5} \tag{34}$$

From this data formed by equation (34) then it is obtained the characteristic equation of closed loop transfer function as

$$P_{cl}(s) = s^4 + \left(K_p \cdot (1.137e-13) + K_d A \cdot (1.137e-13) + 565.5 + A\right)s^3 + \begin{pmatrix} A \cdot 565.5 + K_i \cdot (1.137e-13) + K_p \cdot (3.745e9) + \\ K_p A \cdot (1.137e-13) + K_d A \cdot (3.745e9) + 3.198e5 \end{pmatrix} s^2 + \begin{pmatrix} K_p A \cdot (3.745e9) + K_i A \cdot (1.137e-13) + \\ K_i \cdot (3.745e9) + A \cdot (3.198e5) \end{pmatrix} s + K_i A \cdot (3.745e9) \tag{35}$$

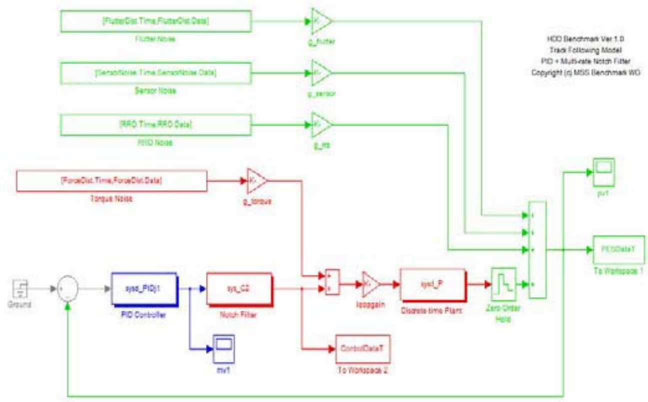

Fig. 7. Sumilation of Track-following Control system configuration for the HDD actuator

TABLE I
HDD VCM Plant Parameters

Symbol	Description	Quantity
Plant Parameters		
m	Mass of head actuator [kg]	1e-3
T_p	Width of track [m] 100kTPI	2.54
K_f	Equivalent Force constant [N/A]	0.9512
T_d	delay [s]	10.0e-6
Rotation speed of disk and sampling time		
Rpm	rotational speed	7200
num_servo	Number of sector per track	220
T_s	Sampling time of Position Error Signal (PES)	60/rpm/num_servo
T_c	Sampling time of control input	Ts/2

According to equation (32), design of PID controller based on CRA, is assigned parameter follow as time constant τ=0.0016, characteristic ratio $\alpha_1 = 1.2,\ \alpha_2 = 6,\ \alpha_3 = 6$ then the characteristic equation is,

$$P(s) = s^4 + (2.70e+4)s^3 + (1.21e+8)s^2 + (9.11e+10)s + (5.69e+13) \tag{36}$$

Equated the above $P_{cl}(s)$ in equation (35) to the $P(s)$ of the plant with the PID controller in equation (36), the parameters of PID controller are

$K_p = 8.1332e-004, K_i = 0.5753,$

$K_d = 1.0423e-006, A = 2.6435e+004$

We then discretize it using a Tustin transformation with a sampling frequency of $3.7879e-005$ kHz. The discretized controller is given by

$$G_c(z) = \frac{0.01918z^2 - 0.0378z + 0.01863}{z^2 - 1.333z + 0.3328}, \tag{37}$$

$sampling\ time: 3.7879e-005$

Testing the result of the process response by using MATLAB Simulink in fig 7, the results are shown as follows Fig. 8~11.

Fig. 8 show the response of track-following control as NRPE 3sigma = 9.954925%TP, RPEpp = 16.328410%TP , (elapsed time = 5.47222 (s)).

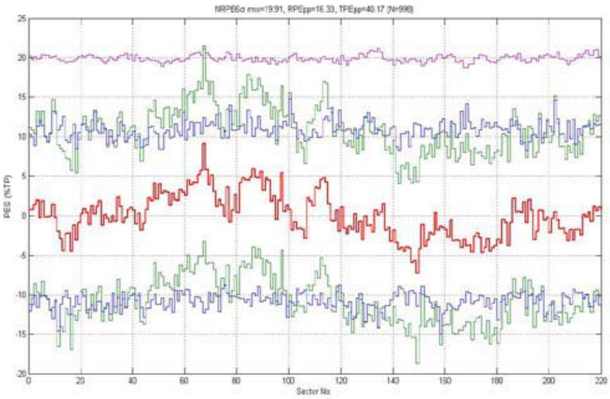

Fig. 8. PES (%TP)Response of track following control system

The power spectral density (PSD) response of track following control system is given in Fig. 9. The overall response of the plant together with the PID control input are given in Fig. 10.

Fig. 11 shows the histograms of the tracking errors of the respective control systems under the disturbance of the runouts. The values of $3\sigma_{NRPE}, 3\sigma_{PES}$ show in (38).

$$3\sigma_{NRPE} = 9.95(\%TP) \quad and \quad 3\sigma_{PES} = 12.92(\%TP) \tag{38}$$

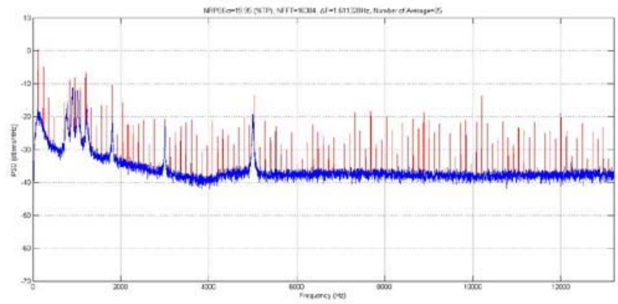

Fig. 9. PSD of control system

The result of the simulation shows that the amount of NRPE 3 sigma is satisfied the specification but not for RPEpp and PES 3 sigma so the PID controller parameters should be adjusted

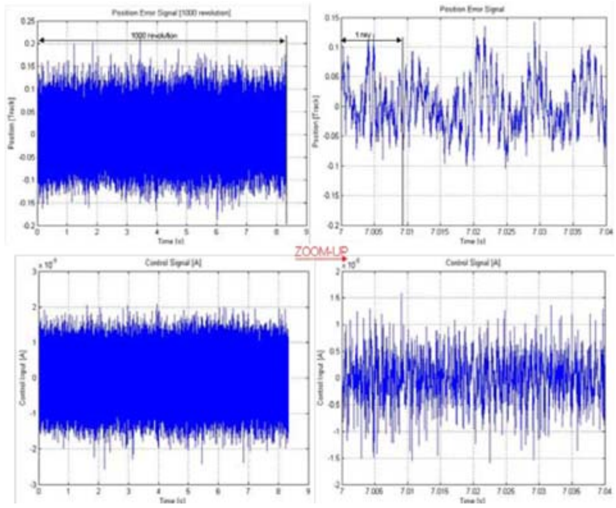

Fig. 10. PES and Control Signal

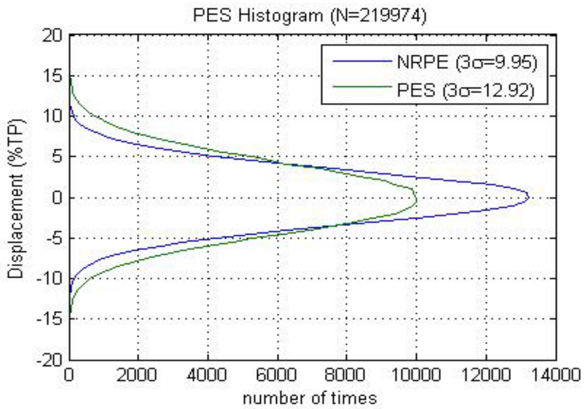

Fig. 11. $3\sigma_{NRPE}, 3\sigma_{PES}$ of Response of Track-following Control System

Adjustment response of control system by using CRA method for optimization of PID controller's parameters are described as follow details

The characteristic equation is,

$$P(s) = s^4 + (2.70e+4)k^3s^3 + (1.21e+8)k^5s^2 + (9.11e+10)k^6s + (5.69e+13)k^6 \tag{39}$$

where k are 1.25.

Equated the above $P_{cl}(s)$ in equation (35) to the $P(s)$ of the plant with the PID controller in equation (39), the parameters of PID controller are

$K_p = 0.0017, K_i = 1.1120,$

$K_d = 1.7132e-006, A = 5.2169e+004$

We discretized it using a Tustin transformation with a sampling frequency of $3.7879e-005$ kHz. The discretized controller is given by

$$G_c(z) = \frac{0.04665z^2 - 0.09157z + 0.04497}{z^2 - 1.006z + 0.00601}, \tag{40}$$

$$sampling\ time: 3.7879e-005$$

Testing the result of the process response by using MATLAB Simulink in fig 7, the results are shown as follows Fig. 12~17.

Fig. 12 show the response of track-following control as NRPE 3sigma = 8.947490 %TP, RPEpp = 11.944662%TP (elapsed time = 5.37241 (s)).

The power spectral density (PSD) response of track following control system is given in Fig. 13. The overall response of the plant together with the PID control input are given in Fig. 14.

Fig. 15 shows the histograms of the tracking errors of the respective control systems under the disturbance of the runouts. The values of $3\sigma_{NRPE}, 3\sigma_{PES}$ satisfied design specification as following value.

$$3\sigma_{NRPE} = 8.94(\%TP) \quad and \quad 3\sigma_{PES} = 11.02(\%TP) \qquad (41)$$

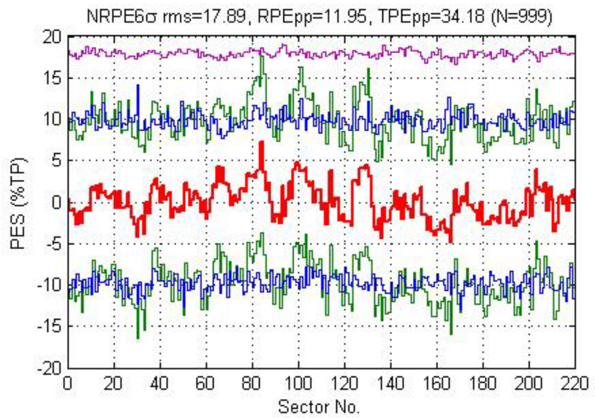

Fig. 12. PES (%TP)Response of track following control system

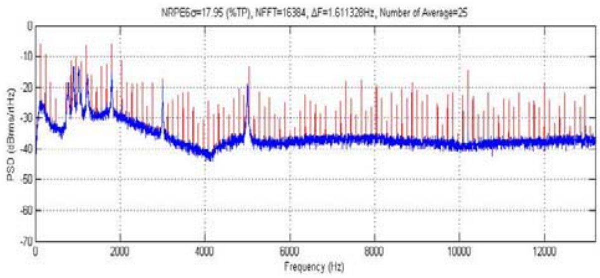

Fig. 13. PSD of control system

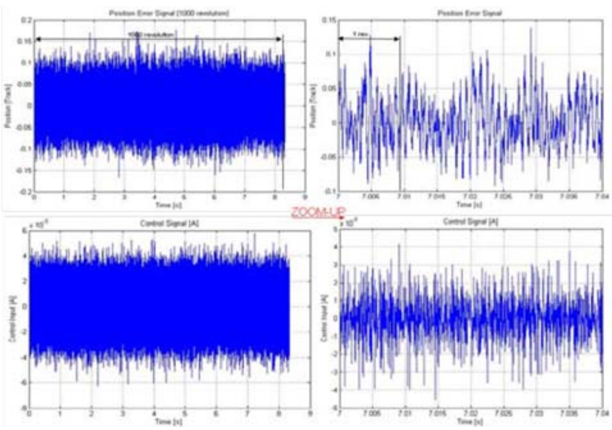

Fig. 14. PES and Control Signal

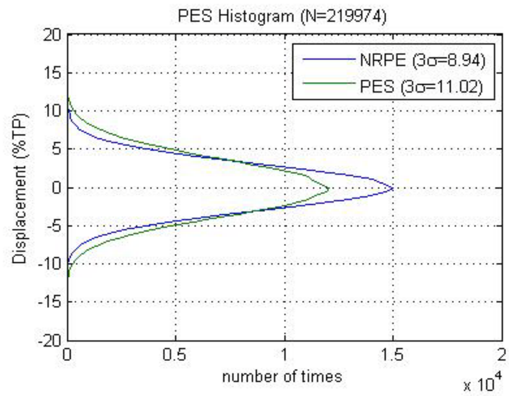

Fig. 15. $3\sigma_{NRPE}, 3\sigma_{PES}$ of Response of Track-following Control System

The effects of ±10% loop gain and plant parameters which classified to 10 models on NRPE 3 sigma and RPEpp are investigated in Fig. 18-19 respectively.

It is seen that the system is fairly robust under both disturbance and parameter perturbations.

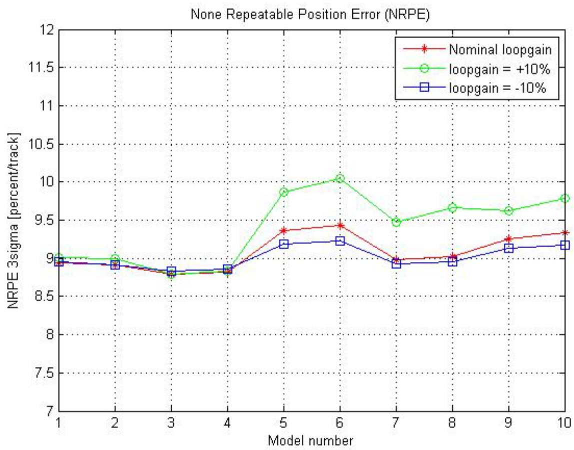

Fig. 16. Non Repeatable Position Error (NRPE) of plant with perturbations

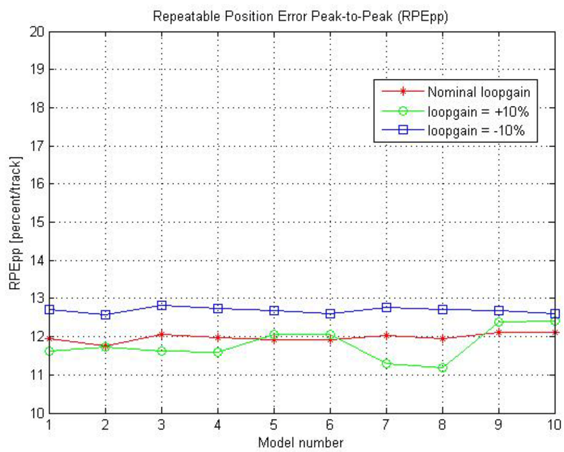

Fig. 17. Repeatable Position Error (RPE) of plant with perturbations

The result of the simulation shows that when k was changed from 1 to 1.25, the adjusted PID parameters could control the response of control system to satisfy the NRPE 3 sigma, RPEpp and PES 3 sigma specification even thought it was under both disturbance and parameter perturbations.

VI. CONCLUSION

The results reveal us that the approach can give an effective scheme for control amount of NRPE (nonrepeatable position error), RPE (repeatable position error) and PES **(**position error signal) by the effective methods. Hence CRA is flexible and can be used efficiently for this purpose to overcome the effects of the disturbances as well as plant perturbations. Furthermore, it is very convenient as a fast adjustment of controller's parameter's for improving the performance of control system by one parameter.

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REFERENCES

[1] P. Naslin 1969. *Essentials of optimal control*, London Boston Technical Publishers Inc, Cambridge, MA, pp. 30-51
[2] A.V. Lipatov and N.I. Sokolov, "Some Sufficient Conditions for Stability and Instability of Continuous Linear Stationary Systems," Translated from Automatika, No.9,1978, pp. 30-37
[3] Y.C. Kim, L.H. Keel and S.P. Bhattacharyya, "Transient Response Control via Characteristic Ratio Assignment" *IEEE Transactions On Automatic Control*, Vol.48, No.12, 2003, pp.2238-2244
[4] Chaorai-ngern, J. Numsomran, A. Suesut, T. Trisuwannawat, T. Tipsuwanporn, V. , "PID Controller Design using Characteristic Ratio Assignment Method for Coupled-Tank Process" Computational Intelligence for Modelling, Control and Automation, 2005 and International Conference on Intelligent Agents, Web Technologies and Internet Commerce, International Conference on Publication, Nov.2005 Volume: 1, pp. 590-594
[5] Numsomran, A. Wongkhum,T. Suksri, T. Nilas, P.Chaoraingern, J., "Design of decoupled controller for TITO system using characteristic ratio assignment", ICCAS '07. International Conference on Publication Date: 17-20 Oct. 2007 , pp. 957-962
[6] Ben M. Chen, Tong H. Lee, Kemao Peng and Venkatakrishnan Venkataramanan,"Hard Disk Drive Servo Systems", 2nd Edition, Springer
[7] M. Hirata, H. Uchida and T. Hara, MSS benchmark working group, Copyright © 2004-2005