A CTL Specification of Serializability for Transactions Accessing Uniform Data

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Abstract—Existing work in temporal logic on representing the execution of infinitely many transactions, uses linear-time temporal logic (LTL) and only models two-step transactions. In this paper, we use the comparatively efficient branching-time computational tree logic CTL and extend the transaction model to a class of multi-step transactions, by introducing distinguished propositional variables to represent the read and write steps of \( n \) multi-step transactions accessing \( m \) data items infinitely many times. We prove that the well known correspondence between acyclicity of conflict graphs and serializability for finite schedules, extends to infinite schedules. Furthermore, in the case of transactions accessing the same set of data items in (possibly) different orders, serializability corresponds to the absence of cycles of length two. This result is used to give an efficient encoding of the serializability condition into CTL.

Keywords—computational tree logic, serializability, multi-step transactions.

I. INTRODUCTION

As concurrent users access and update databases in terms of transactions, a reliable condition of correctness is needed for the execution of these transactions. The established correctness condition is that of serializability, where an interleaved schedule of concurrent transactions is equivalent to a serial schedule of the transactions. Most work on serializability has modelled histories to be finite with a known fixed bound \([8],[9]\). Recently, with the emergence of new techniques such as web transactions and mobile databases, where an unlimited number of transactions may be incoming and outgoing to the databases in continuous streams, the importance of representing infinite histories has been recognised \([5],[6],[7]\).

One way of representing infinite histories is as models of temporal logic formulae. A benefit of using temporal logic is the availability of powerful model checkers such as NuSMV \([2]\). Model checkers can carry out exhaustive checks of a correctness criterion such as serializability, and are fully automatic and therefore require no special expertise to carry out the verification. The drawback with model checking is that even the most powerful model checkers cannot overcome the theoretical worst-case complexity of model checking inherent from the temporal logic being used. The most benign temporal logic in this respect is CTL which can check whether executions represented by a finite-state machine satisfy a specification with time complexity \( O((|S| + |R|).2^{|f|}) \) \([3]\), where \( |S| \) is the number of states in the finite state machine, \( |R| \) the number of transitions, and \( |f| \) is the length of the specification formula. This is marginally better than for LTL which has a corresponding time complexity of \( O((|S| + |R|).2^{|f|}) \) \([3]\). However, the temporal logics that have been used to specify transactional concurrency include the partial-order temporal logic ISTL in \([10]\), quantified propositional temporal logic QPTL in \([5]\,\ LT\) in \([6]\,\ a\ first-order temporal logic in the first part of \([11]\ \) and a monodic fragment of first-order temporal logic in \([7]\). With the exception of LTL these are, at best, of exponential space complexity, and, at worst, undecidable.

In this paper, we give a computationally efficient specification of serializability in CTL. The serializability condition expressed in CTL is based on acyclicity of conflict graphs. To be able to use such a condition, we prove that acyclicity of conflict graphs corresponds to serializability for infinite schedules. We then assume the further property for our transactions, that they access the same set of data items in different orders. We show that serializability then corresponds to the efficient condition where only cycles of length two need be checked, and this condition is used for the CTL specification. This work advances that of \([5],[6]\,\) which both deal with two-step transactions, to the more normal case of multi-step transactions. We also produces the specification in the slightly more efficient CTL rather than LTL. The paper is organized as follows. In Section II, we give a mathematical model of concurrent multi-step transactions. In Section III, the results on acyclicity of conflict graphs and serializability for infinite schedules are given. From these, serializability is characterized mathematically in a way to be encoded into CTL. The CTL specification is given in Section IV, and conclusions are drawn in Section V.

II. A MODEL OF CONCURRENT MULTI-STEP TRANSACTIONS

A. Steps and histories

The model of concurrent two-step transactions in \([5]\) comprises \( n \) transactions \( \{T_1,\ldots,T_n\} \) occurring infinitely many times, with each transaction containing a read step and a write step each accessing a finite number of data items. In this paper, we define transactions as containing multiple alternate read and write steps, each accessing a single data item. We shall denote a read step and the corresponding write step on the data item \( x_j \) by transaction \( T_i \), as \( r_i(x_j) \) and \( w_i(x_j) \), respectively, and the set of data items accessed by all transactions as \( D \). We say that two steps are conflicting if they belong to different transactions, they access the same data item and at least one of them is a write step. Later in this paper, we shall assume that, given transactions \( T_i \) and \( T_j \), the data items accessed by both are the same, but that the order of access of data
items by transaction $T_i$ is not necessarily the same as that by $T_j$. Precisely, we will assume a finite set of data items $D = \{x_1, \ldots, x_n\}$, an infinite set of (multi-step) transactions $T = \{T_i : i \in \mathbb{N}_1\}$, where $\mathbb{N}_1$ is the set of positive integers, such that all $T_i \in T$ are of the form,

$$T_i = r_1(x_{i_1})w_1(x_{i_1}) \ldots r_k(x_{i_m})w_k(x_{i_m})$$

where $\{x_{i_1}, \ldots, x_{i_m}\} = D$.

A history $h$ of $T$ is an interleaved sequence of all the read and write steps, of all the transactions in $T$, such that, for each $i \geq 1$, the subsequence of $h$ compromising the steps of $T_i$ is exactly the sequence of steps of $T_i$ occurring in the order that they do in $T_i$. For a history $h$, $h_i$ will denote the (irreflexive) total order between all the read and write steps of $h$. If $T' \subseteq T$, then the projection of $h$ to $T'$, denoted $h_{T'}$, is the history of $T'$, obtained from $h$, by deleting all steps of transactions not in $T'$.

**B. Serializability**

The required correctness condition of ‘serializability’ is that concurrent multi-step transactions should execute in a history whose effect is ‘equivalent’ to a serial execution of all the $T_i \in T$. Our definitions of equivalence and serializability are based on those in [9].

**Definition 1.** Histories $h_1$ and $h_2$ of $T = \{T_i : i \in \mathbb{N}_1\}$ are equivalent, written as $h_1 \sim h_2$, iff for all $i, i' \geq 1, i \neq i'$, and for all $x \in D$,

1. if $r_i(x) < h_i w_i(x)$, then $r_i(x) < h_{i'} w_i(x)$,
2. if $w_i(x) < h_i w_i(x)$, then $w_i(x) < h_{i'} w_i(x)$ and
3. if $w_i(x) < h_i r_i(x)$, then $w_i(x) < h_{i'} r_i(x)$

**Definition 2.** A history $h$ of $T = \{T_i : i \in \mathbb{N}_1\}$ is serializable iff there is a serial history $h_S$ of the form, for each $i \in \mathbb{N}_1$,

$$h_S = \ldots \ldots r_i(x) \ldots w_i(y) \ldots \ldots$$

such that $h \sim h_S$.

**III. A CONDITION FOR SERIALIZABILITY OF MULTI-STEP TRANSACTIONS**

In [5], serializability of infinite histories is characterized in terms of ‘detachable’ steps for certain finite subsequences of steps. We shall determine serializability in terms of acyclicity of ‘conflict graphs’ - a technique widely used for finite histories [9]. We define conflict graphs in Definition 3 and in Theorem 4 give conditions for which acyclicity of conflict graphs correspond to serializability in the case of an infinite number of transactions. In Lemma 5, we give a simpler correspondence in the case where transactions access the same set of data items. This result is used to prove the main result, Theorem 7, which gives the conditions for serializability that will form the basis of the specification in CTL in Section IV.

**Definition 3.** A directed graph is a pair $G = (V, A)$, where $V$ is a set of elements called nodes, denoted nodes($G$), and $A \subseteq V \times V$ is a set of elements called arcs, denoted arcs($G$). A walk in a directed graph $G = (V, A)$ is a sequence of nodes $(v_1, v_2, \ldots, v_n)$ such that $(v_i, v_{i+1}) \in A$ for $i = 1, \ldots, n-1$. A walk with no nodes repeated is called a path; it is a cycle when only the first and last node coincide. For each history $h$, there is a directed graph $G(h)$ called the precedence graph or conflict graph of $h$. This graph has the transactions of $h$ as its nodes, and contains an arc $(T_i, T_j)$, where $T_i$ and $T_j$ are distinct transactions of $h$, whenever there is a step of $T_i$ which conflicts with a subsequence (in $h$) of step of $T_j$.

**Theorem 4.** A history $h$ of an infinite number of multi-step transactions $T = \{T_i : i \in \mathbb{N}_1\}$, accessing data items in some finite set $D$ (though not necessarily accessing the same data items), is serializable iff the conflict graph $G(h)$ is acyclic.

**Proof:**

If Let $h$ be a history of $T$ such that $G(h)$ is acyclic. Assume that, for some $T_i \in T$, we have the following infinite regression of arcs:

$$\ldots, (T_{k=0+1}, T_{k=0}), \ldots, (T_{k=1}, T_{k=0}), (T_{k=1}, T_1)$$

where $\{T_{k=0}, \ldots, T_{k=2}, \ldots\} \subseteq T$. Then, as only finitely many data items are accessed by the transactions, and as each step may be preceded by only finitely many steps in $h$, there exist $l, j \geq 0$ such that we have the following order of steps in $h$ (assuming, without loss of generality, that the arcs in (1) are as the result of write-read conflicts):

$$w_{k+1}(x) < h r_k(x) < \ldots < h w_{k+1}(x) < h r_k(x)$$

and $w_k(x)$ does not precede $r_k(x)$ in $h$, i.e.

$$r_k(x) < h w_k(x)$$

From (2) and (3), we produce the cycle

$$(T_{k=1}, T_{k-0+1}), \ldots, (T_{k=1}, T_{k=0}), (T_{k=1}, T_{k=0})$$

This contradiction shows that (1) cannot occur. It follows that we can define, inductively, the sequence $i^1, i^2, \ldots$ thus:

$$i^1 = \min\{k \in \mathbb{N}_1 : \text{ for all } i \neq k, (T_i, T_k) \notin \text{ arcs}(G(h))\}$$

$$\ldots$$

$$i^n = \min\{k \in \mathbb{N}_1 : \text{ for all } i \neq k, i^1, \ldots, i^{n-1}, (T_i, T_k) \notin \text{ arcs}(G(h))\}$$

$$\ldots$$

Firstly, we show that $\{i^1, \ldots, i^n, \ldots\} = \mathbb{N}_1$. Suppose, on the contrary, that there is some $i^t \in \mathbb{N}_1$ such that $i^t \neq i^{n'}$ for any (all) $n \in \mathbb{N}_1$. As the situation (1) cannot occur, we can choose $i^t$ to be such that

$$(T_1, T_{i_t}) \text{ implies } T_i = i^t \text{ for some } n \in \mathbb{N}_1$$

Intuitively, $i_t$ is the ‘earliest’ transaction for which $T_{i_t} \neq T_i$ for any $n \in \mathbb{N}_1$. Now choose $n' \in \mathbb{N}_1$ to be such that:

(a) $i^{n'+1} > i^t$

(b) all the steps of any $T_{i_{n'}}$, where $n \geq n'$, come after the steps of $T_{i_t}$ in $h$. 

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Assume that we have that
\[(T_i, T_i') ∈ arcs(G(h)) \text{ for some } i ∈ N_1, i ≠ i^1, \ldots, i^{n'}, (5)\]
This implies, by (4), that
\[T_i = T_i' = T_i'' \text{ for some } n ∈ N_1\]
which implies, by the assumption at (5) that \(i ≠ i^1, \ldots, i^{n'}\),
\[T_i = T_i'' \text{ for some } n > n'\]
This, in turn, implies, by (b), that all the steps of \(T_i = T_i''\) come after the steps of \(T_i'\) in \(h\) and so \((T_i, T_i') ∉ arcs(G(h))\). This contradicts (5) and so the assumption at (5) cannot hold. But, then,
\[i^0 > i^{n'+1} ≥ i'\]
This last contradiction means that the assumption that \(i' ≠ i^n\) for all \(n ∈ N_1\) is false. It follows that \(\{i^1, \ldots, i^{n-1}\} = N_1\).
We construct the serial history \(h_S\) of \(T\) given by \(h_S = z\) all steps of \(T_1\) \(w_x(y)\) \(z\) all steps of \(T_n\)
and show that \(h ∼ h_S\) by showing that Definition 1(1), (2) and (3) hold. For Definition 1(1), suppose that \(r_1(x) <_h w_x(y)\) for transactions \(T_i, T_i' \in T\). Then, \((T_i, T_i') ∉ arcs(G(h))\). In the sequence \(i^1, i^2, \ldots, i^n\), above, we cannot have \(i = i^1\) and \(i' = i^{n'}\) for some \(n' < n\), since, from the definition of \(n'\), that would imply that \((T_i, T_i') ∉ arcs(G(h))\). Thus, \(h_S\) is of the form
\[\ldots r_i(c) \ldots w_x(y) \ldots \ldots r_i(x) \ldots w_x(y) \ldots \ldots\]
and Definition 1(1) holds as required. The proof of Definition 1(2) and (3) are similar.

Only if
Let \(h\) be a serializable history. This means that there is a serial history \(h_S\) such that \(h ∼ h_S\). This implies, by Definitions 1 and 3, that \(G(h) = G(h_S)\). As \(G(h_S)\) is necessarily acyclic, since it must be a subgraph of the total order under which the transactions occur in \(h_S\), we conclude that \(G(h)\) is acyclic.

In the case where all transactions access the same set of data items, serializability is guaranteed if \(G(h)\) has no cycle of length 2.

**Lemma 5.** Let \(h\) be a history of multi-step transactions \(T = \{T_i : i ∈ N_1\}\) accessing the same set of data items \(D\) (in possibly different orders). Then, if \(G(h)\) has a cycle, there are transactions \(T_i, T_i'\) such that \(G(h)\) has the cycle \((T_i, T_i'), (T_i', T_i)\).

**Proof:**
Assume that \(G(h)\) has a cycle
\[(T_i, T_{i+1}), \ldots, (T_{i(n-1)}, T_{i+n}), (T_{i+n}, T_i)\]
where \(n > 2\), but no such cycle for \(n = 2\). We will derive a contradiction. Choose any \(x ∈ D\). Then, for \(1 ≤ j ≤ n - 1\),
\[w_{i+j}(x) <_h r_{i+j}(x)\]
otherwise \((T_{i+j+1}, T_{i+j})\) is an arc in \(G(h)\) and, from (7), \((T_{i+j}, T_{i+j+1})\) is also an arc in \(G(h)\) giving a cycle between \(T_i\) and \(T_{i+n}\), and, contrary to our assumption that there are no cycles of length 2. From (8) we have that
\[r_i(x) <_h w_i(x) <_h \ldots <_h r_i(x) <_h w_i(x) <_h r_i(x)\]
(9)
The contradiction, from (9), that \(r_i(x) <_h r_i(x)\), means that our assumption that there is no cycle between two transactions is incorrect.

**Definition 6.** We say that \(T_i\) comes before \(T_i'\) in \(h\) iff \(w_i(x) <_h r_i(y)\), where \(x\) and \(y\) are the first data items accessed by \(T_i\) and \(T_i'\), respectively.

**Theorem 7.** A history \(h\) of multi-step transactions \(T = \{T_i : i ∈ N_1\}\) is serializable iff for any two distinct transactions \(T_i\) and \(T_i'\), one of them, \(T_i\), say, is such that
(i) \(T_i\) comes before \(T_i'\) in \(h\), and
(ii) for all \(x ∈ D\), \(w_i(x) <_h r_i(y)\)

**Proof:**
Let \(h\) be not serializable. We show that there are \(T_i\) and \(T_i'\) such that the conditions (i) and (ii) do not both hold. To have \(h\) not serializable, by Theorem 4 and Lemma 5, that there is a cycle in the precedence graph \(G(h)\), \((T_i, T_i'), (T_i', T_i)\). Assume that (i) holds for \(T_i\) and \(T_i'\), i.e. \(T_i\) comes before \(T_i'\). Here, letting \(x\) and \(y\) denote the first data items accessed by \(T_i\) and \(T_i'\), respectively, there are a limited number of cases causing the cycle:
\[\ldots w_i(x) \ldots r_i(y) \ldots w_i(y) \ldots r_i(z) \ldots w_i(z) \ldots \]
\[\ldots w_i(z) \ldots \]
\[\ldots w_i(x) \ldots r_i(y) \ldots w_i(y) \ldots r_i(z) \ldots w_i(z) \ldots \]
\[\ldots w_i(z) \ldots \]
\[\ldots w_i(x) \ldots r_i(y) \ldots w_i(y) \ldots r_i(z) \ldots w_i(z) \ldots \]
\[\ldots w_i(z) \ldots \]
\[\ldots w_i(x) \ldots r_i(y) \ldots w_i(y) \ldots r_i(z) \ldots w_i(z) \ldots \]
\[\ldots w_i(z) \ldots \]
In the all cases (10)-(13) condition (ii) is breached because \(r_i(z) <_h w_i(z)\) (underlined).

**Only if**
Assume that we have that \(h\) does not satisfy conditions (i) and (ii) for all \(T_i, T_i'\). We show that \(h\) is not serializable. Firstly, suppose that condition (i) holds, but that condition (ii) does not hold for some \(T_i\) and \(T_i'\). Then, if \(x\) and \(y\) are the first data items accessed by \(T_i\) and \(T_i'\), respectively,
\[w_i(x) <_h r_i(y)\]
(14)
giving the arc \((T_i, T_j)\) in \(G(h)\). As condition (ii) does not hold, there is \(z \in D\) such that
\[
r_i(z) <_h w_i(z)
\]
This gives the arc \((T_i, T_j)\) and hence a cycle in \(G(h)\). By Theorem 4, \(h\) is not serializable. Secondly, suppose condition (i) does not hold for some \(T_i, T_j\). Then, by Definition 6, if \(x\) and \(y\) are the first data items accessed as above,
\[
r_i(x) <_h w_i(y) \quad (15)
\]
and
\[
r_i(y) <_h w_i(x) \quad (16)
\]
From (15), if \(x = y\), \((T_i, T_j)\) is an arc in \(G(h)\), and from (16) \((T_i, T_j)\) is an arc in \(G(h)\). This gives a cycle and shows, by Theorem 4, that \(h\) is not serializable. But, if \(x \neq y\), \(T_i\) accesses \(y\) later, and \(T_j\) accesses \(x\) later. Thus, by (16),
\[
r_i(y) <_h w_i(x) <_h r_i(y) <_h w_i(y) \quad (17)
\]
and, by (15),
\[
r_i(x) <_h w_i(y) <_h r_i(x) <_h w_i(x) \quad (18)
\]
From (17), \((T_i, T_j)\) is an arc in \(G(h)\) and, from (18), \((T_j, T_i)\) is an arc in \(G(h)\) giving a cycle.

IV. SPECIFICATION OF SERIALIZABILITY IN CTL

We present a CTL specification of infinite histories composed of \(n\) transactions each accessing all of \(m\) data items, and repeating infinitely often. The aggregate of all the repetitions of the \(n\) transactions will constitute the infinite number of transactions \(\{T_i : i \in \mathbb{Z}\_+\}\) of the previous section. Such concurrent repeating or ‘iterating’ transactions were originally investigated in [4] and temporal logic models have been given in [5] and [6]. In [5] and [6] each iteration of a transaction is called an occurrence, and every occurrence of a particular transaction comprises the same two (read and write) steps. We improve this to a case of multi-step transactions where, different occurrences of particular transactions access the same data items, but in possibly different orders. So, the order of access of data items may be different between different transactions and between different occurrences of the ‘same’ transaction. Actually, in our model here, different occurrences of the ‘same’ transaction bear no relation to each other. As such, we model, not so much the same \(n\) transactions iterating, but a more general case of an infinite number of (possibly totally unrelated) transactions where there is a limit of \(n\) on how many are active at any given time.

The syntax for CTL is given in Section IV.A and the semantics in Section IV.B. The specification of the multi-step transactions model is in Section IV.C and serializability is specified in IV.D.

A. Syntax

The alphabet of CTL consists of a set of propositions symbols \(p_0, p_1, \ldots\) distinguished read/write step propositional symbols \(r_i(x_j), w_i(x_j)\) \((1 \leq i \leq n, 1 \leq j \leq m)\), booleans \(\neg, \lor, \land, \top, \bot\), quantifiers \(\exists\), \(\forall\), and temporal operators \(X, F, G\) and \(U\). Formule in CTL are generated by:
\[
\phi ::= p_i | r_i(x_j) | w_i(x_j) | \neg \phi | \phi_1 \lor \phi_2 | \phi_1 \land \phi_2 | AX\phi | EX\phi | AF\phi | EF\phi | AG\phi | EG\phi | A[\phi_1 U \phi_2] | E[\phi_1 U \phi_2]
\]
Note that, despite their appearance, \(r_i(x_j)\) and \(w_i(x_j)\) are propositions, and not predicates, in the logic. The symbols \(\land\) and \(\top\) will also be used to denote the truth values false and true respectively and the abbreviations \(\Rightarrow\) and \(\Leftrightarrow\) will have their usual logical meaning.

B. Semantics of CTL

An interpretation for CTL, \(I(s_a)\), at a given state \(s_a \in S\), where \(S\) is a set of states, assigns truth values \(p_i I(s_a)\), \(r_i(x_j) I(s_a)\) and \(w_i(x_j) I(s_a)\) \((\in \{\bot, \top\})\) to propositional symbols \(p_i\), \(r_i(x_j)\), and \(w_i(x_j)\), respectively. A \(\mathit{interpretation}\) \(I\) over \(S\) is a set of interpretations \(I = \{I(s_a) : s_a \in S\}\). A \(\mathit{Kripke structure}\) \(M\) is a triple \(<S, R, I>\), where \(S\) is a set of states, \(R \subseteq S \times S\) a transition relation such that, for all \(s \in S\), there exists \(s' \in S\) with \((s, s') \in R\), and \(I\) is an interpretation over \(S\). A \(\mathit{path}\) in \(M\) is an infinite sequence of states, \(\pi = s_a, s_{a+1}, \ldots\), such that, for every \(b \geq a, (s_b, s_{b+1}) \in R\). The set of paths that start in state \(s_a\) is denoted \(\mathit{Paths}(s_a)\). As each state in a Kripke structure is required to have at least one successor, it follows that \(\mathit{Paths}(s_a) \neq \{\}\) for any state \(s_a\). The \(\mathit{semantics}\) of a CTL formula \(\phi\) is given by the truth relation \(M, s_a \models \phi\) which means that \(\phi\) holds at state \(s_a\) in the Kripke structure \(M\). The relation \(\models\) is defined inductively as follows
\[
M, s_a \models p_i \text{ iff } p_i I(s_a) = \top
\]
\[
M, s_a \models r_i(x_j) \text{ iff } r_i(x_j) I(s_a) = \top
\]
\[
M, s_a \models w_i(x_j) \text{ iff } w_i(x_j) I(s_a) = \top
\]
\[
M, s_a \models \neg \phi \text{ iff } M, s_a \not\models \phi
\]
\[
M, s_a \models \phi_1 \lor \phi_2 \text{ iff } M, s_a \models \phi_1 \text{ or } M, s_a \models \phi_2
\]
\[
M, s_a \models \phi_1 \land \phi_2 \text{ iff } M, s_a \models \phi_1 \text{ and } M, s_a \models \phi_2
\]
\[
M, s_a \models AX\phi \text{ iff, for all } \pi \in \mathit{Paths}(s_a), M, s_{a+1} \models \phi
\]
\[
M, s_a \models EX\phi \text{ iff there exists } \pi \in \mathit{Paths}(s_a) \text{ such that } M, s_{a+1} \models \phi
\]
\[
M, s_a \models AF\phi \text{ iff, for all } \pi \in \mathit{Paths}(s_a), \text{ there exists } b \geq a \text{ such that } M, s_b \models \phi
\]
\[
M, s_a \models EF\phi \text{ iff there exists } \pi \in \mathit{Paths}(s_a) \text{ and } b \geq a \text{ such that } M, s_b \models \phi
\]
\[
M, s_a \models AG\phi \text{ iff, for all } \pi \in \mathit{Paths}(s_a), \text{ and, for all } b \geq a, M, s_b \models \phi
\]
\[
M, s_a \models EG\phi \text{ iff there exists } \pi \in \mathit{Paths}(s_a) \text{ such that, for all } b \geq a, M, s_b \models \phi
\]
\[
M, s_a \models A[\phi_1 U \phi_2] \text{ iff, for all } \pi \in \mathit{Paths}(s_a), \text{ there is some } c \geq a \text{ such that } M, s_c \models \phi_2 \text{ and, for all } a \leq b < c, M, s_b \models \phi_1
\]
\[
M, s_a \models E[\phi_1 U \phi_2] \text{ iff there exists } \pi \in \mathit{Paths}(s_a) \text{ such that, for some } c \geq a, M, s_c \models \phi_2 \text{ and, for all } a \leq b < c, M, s_b \models \phi_1
$r_i(x_j)$ $\sim$ active transaction $T_i$ has read data item $x_j$

$w_i(x_j)$ $\sim$ active transaction $T_i$ has written to data item $x_j$

The multi-step transactions model is characterized by the following properties:

(C1) **Read/write alternation**

A transaction $T_i$ cannot have read two distinct data items without having written to one of them, i.e. $r_i(x_j)$ and $r_i(x_j')$ cannot both be true if $w_i(x_j)$ and $w_i(x_j')$ are both false.

(C2) **Write implies read**

A transaction $T$, can only have written to $x_j$ if it has read $x_j$, i.e. if $w_i(x_j)$ is true, then $r_i(x_j)$ must be true.

(C3) **Read/write steps remain true to transaction end**

If a read/write step has taken place, the corresponding propositions remain true until the transaction ends, i.e. $r_i(x_j)/w_i(x_j)$ once true, remain true until all other steps $r_i(x_j')$ and $w_i(x_j')$ ($x_j' \in D$) are true.

(C4) **End of transaction occurrence**

After a transaction occurrence ends, at most one read step $r_i(x_j)$ and no write steps $w_i(x_j)$ can be true in any next state.

(C5) **At most one step occurs at each successive state**

No two distinct steps can both be false in a state, and then both true in a next state.

Given a state $s_n$, and a path $\pi \in Paths(s_n)$, there corresponds a sequence of read and write step propositions that become true in $s_n, s_{n+1}$, $\ldots$. In this way, $\pi$ yields a history of infinitely many occurrences of the transactions $T_1, \ldots, T_n$. We illustrate this correspondence between paths and histories in Figure 1.

In Figure 1, we have $D = \{x, y, z\}$ and transactions

$$T_1 = r_1(x)w_1(x)r_1(y)w_1(y)r_1(z)w_1(z)$$

and

$$T_2 = r_2(x)w_2(x)r_2(z)w_2(z)r_2(y)w_2(y)$$

Interpretations for read and write step propositions are given for successive states, and the top of each column displays the unique proposition that becomes true in the particular state. The corresponding history $h$ is:

$$h = r_1(x)w_1(x)r_1(y)w_1(y)r_1(z)w_1(z)r_2(x)w_2(x)r_2(z)w_2(z)r_2(y)w_2(y)r_1(x) \ldots$$

We encode the conditions (C1)-(C5) as $\sigma_0, \sigma_2, \sigma_3, \sigma_4$ and $\sigma_5$ respectively, below. We use an extra proposition $\text{end} T_i$ to mark the states at which an occurrence of $T_i$ ends, i.e. the states at which $r_i(x_j)$ and $w_i(x_j)$ are true for all $x_j$. This is defined in $\sigma_0$ as follows:

$$\sigma_0 = \bigwedge_{1 \leq i \leq n} \Box G(\text{end} T_i) \iff \bigwedge_{1 \leq i \leq n} (r_i(x_j) \land w_i(x_j))$$

Conditions (C1)-(C5) are given below:

(C1) **Read/write alternation**

A transaction $T_i$ cannot have read two distinct data items

$$\sigma_1 = \bigwedge_{1 \leq i \leq n} \bigwedge_{1 \leq j \neq j' \leq m} (\neg \text{EF}(r_i(x_j) \land r_i(x_j') \land \neg w_i(x_j) \land \neg w_i(x_j'))$$
(C2) Write implies read
A transaction $T_i$ can only have written to $x_j$ if it has read $x_j$, i.e. if $w_i(x_j)$ is true, then $r_i(x_j)$ must be true.

$$\sigma_2 = \bigwedge_{1 \leq i \leq n} \bigwedge_{1 \leq j \leq m} AG(w_i(x_j) \Rightarrow r_i(x_j))$$

(C3) Readwrite steps remain true to transaction end
If a read/write step has taken place, the corresponding propositions remain true until the transaction ends, i.e. $r_i(x_j)/w_i(x_j)$ once true, remain true until all other steps $r_i(x'_j)$ and $w_i(x'_j)$ ($x'_j \in D$) are true.

$$\sigma_3 = \bigwedge_{1 \leq i \leq n} \bigwedge_{1 \leq j \leq m} \left( (r_i(x_j) \land \neg endT_i) \Rightarrow AXr_i(x_j) \right) \land$$
$$\left( w_i(x_j) \land \neg endT_i \Rightarrow AXw_i(x_j) \right)$$

(C4) End of transaction occurrence
After a transaction occurrence ends, at most one read step $r_i(x_j)$ and no write steps $w_i(x_j)$ can be true in any next state.

$$\sigma_4 = \bigwedge_{1 \leq i \leq n} \bigwedge_{1 \leq j \leq m} AG(\neg endT_i \Rightarrow AX) \bigvee \bigwedge_{1 \leq i \leq n} \bigwedge_{1 \leq j \neq j' \leq m} \left( \neg(r_i(x_j) \land \neg r_i(x_{j'})) \land \neg(w_i(x_j) \land \neg w_i(x_{j'})) \right)$$

(C5) At most one step occurs at each successive state
No two distinct steps can both be false in a state, and then both true in a next state.

$$\sigma_5 = \bigwedge_{1 \leq i \leq n} \bigwedge_{1 \leq j \leq m} \left( \neg\left( (r_i(x_j) \land \neg r_i(x_{j'})) \land EX(r_i(x_j) \land r_i(x_{j'})) \right) \land \neg\left( (r_i(x_j) \land \neg w_i(x_{j'})) \land EX(r_i(x_j) \land w_i(x_{j'})) \right) \land \neg\left( (w_i(x_j) \land \neg w_i(x_{j'})) \land EX(w_i(x_j) \land w_i(x_{j'})) \right) \right)$$

We denote by $\sigma_{trans}$ the specification of the transactions model, i.e.

$$\sigma_{trans} = \sigma_0 \land \sigma_1 \land \sigma_2 \land \sigma_3 \land \sigma_4 \land \sigma_5$$

D. Specification of serializability

We encode conditions (i) and (ii) of Theorem 7. We make use of additional propositions $before_{i,i'}$ ($1 \leq i \neq i' \leq n$), each of which is true in a state if the current occurrence of $T_i$ comes before the current occurrence of $T_i'$. We have that $before_{i,i'}$ becomes true either if $T_i$ has performed a write step and $T_{i'}$ has not performed any read steps, or in a state which comes after a state in which the occurrence of $T_{i'}$ ended and $T_i$ had previously performed a write step. This is specified as $\sigma_6$:

$$\sigma_6 = \bigwedge_{1 \leq i \neq i' \leq n} AG(\neg before_{i,i'} \Rightarrow A(\neg before_{i,i'}))$$

$$\left( \bigvee_{1 \leq j \leq m} w_i(x_j) \land \bigvee_{1 \leq j' \leq m} \left( \neg r_i(x_{j'}) \land w_i(x_{j'}) \land before_{i,i'} \right) \right)$$

Also, we need to ensure that $before_{i,i'}$, once true, remains true until the end of the occurrence of $T_i$, and then becomes false. This is given by $\sigma_7$:

$$\sigma_7 = \bigwedge_{1 \leq i \leq n} \bigwedge_{1 \leq i',i'' \leq n} \left( (before_{i,i'} \land \neg endT_i) \Rightarrow AXbefore_{i,i'} \right) \land$$

$$\left( endT_i \Rightarrow AX\neg before_{i,i'} \right)$$

Theorem 7 condition (i) can then be encoded as $\sigma_8$ which states that, if $T_i$ and $T_{i'}$ are active, one of them must come before the other:

$$\sigma_8 = \bigwedge_{1 \leq i \leq n} \bigwedge_{1 \leq i',i'' \leq n} \left( (before_{i,i'} \lor before_{i,i''}) \right)$$

Theorem 7 condition (ii) is encoded as $\sigma_9$:

$$\sigma_9 = \bigwedge_{1 \leq i \leq n} \bigwedge_{1 \leq j \leq m} AG(before_{i,i'} \Rightarrow \neg(r_i(x_j) \land \neg w_i(x_j)))$$

We denote by $\sigma_{sz}$ the specification of the serializability condition, i.e.

$$\sigma_{sz} = \sigma_6 \land \sigma_7 \land \sigma_8 \land \sigma_9$$

V. CONCLUSIONS

We have given a method using CTL for specifying and verifying the correctness of concurrent executions of multi-step transactions produced by schedulers. For example, a scheduler might be specified as a finite-state machine in NuSMV, corresponding to a structure Sched for CTL. The specification would then be checked to see that the transactions model had been specified in the correct way. This would mean running the NuSMV model checker to show that

$$Sched, s_n \models \sigma_{trans}$$

Serializability could then be verified by using the NuSMV model checker to show that

$$Sched, s_n \models \sigma_{sz}$$

A preliminary case study of the use of CTL to verify serializability of mobile transactions in this way has been conducted in [1]. The CTL method here improves on previous work in two respects - there is a slight efficiency gain in using CTL as opposed to LTL and the more usual case of multi-step transactions can be modelled.

We have defined a serializability condition that scales well with increasing numbers of transactions and data items. However, this has come at a price as we have added the assumption that transactions access the same set of data items, albeit in different orders. In fact, there are many applications where this assumption is realistic. For example, people booking meals at restaurants over mobile phones. Some may book the main course first, then maybe dessert, then starters, and finally tea or coffee. Others may choose to book in a different order.
The availability of one course may influence the choice of another course and serializability of the booking transactions for the whole meals would be the appropriate correctness condition. Furthermore, we have investigated other different assumptions on the transactions model, where there is an order on the accessed set of data items and transactions may access different subsets of data items, that result in a similar serializability condition that only needs to check for cyclicity between pairs of transactions and would have a similar efficient encoding into $\text{CTL}$.

**REFERENCES**


