Accurate Crosstalk Analysis for RLC On-Chip VLSI Interconnect

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Abstract—This work proposes an accurate crosstalk noise estimation method in the presence of multiple RLC lines for the use in design automation tools. This method correctly models the loading effects of non-switching aggressors and aggressor tree branches using resistive shielding effect and realistic exponential input waveforms. Noise peak and width expressions have been derived. The results obtained are at good agreement with SPICE results. Results show that average error for noise peak is 4.7% and for the width is 6.15% while allowing very fast analysis.

Keywords—Crosstalk, Distributed RLC segments, On-Chip Interconnect, Output response, VLSI, Noise Peak, Noise Width.

I. INTRODUCTION

THE minimum feature size in VLSI circuits is shrinking; signal integrity issues gain importance due to increased coupling between nets in VLSI circuits, which results in crosstalk noise. Decreasing feature size affects the crosstalk noise problem and also affects the design’s timing and functionality goals [1-2]. If the crosstalk effects on the victim net are large, they can propagate into storage elements that connect to victim line and can cause permanent errors. Several proposals have been made which model the crosstalk effects using simple lumped and/or distributed RC circuit models. Vittal [3] modeled each aggressor and victim net by a simple L-type lumped RC circuit and obtained a bound for crosstalk noise using a step input. Later extensions to this model are made in [4] and [5]; where a saturated ramp input or a noise using a step input. Later modifications are made to this model and an improved π-type lumped RC circuit has been considered. Cong et al. later proposed a 2-π model [6] that offered more accuracy than previous models. In this model, the victim line is modeled using a saturated ramp while the aggressor line is simplified as a saturated ramp at the coupling node. Later modifications are made to this model and an improved 4-π model has been proposed [7]. In [7], the model [6] is extended to include the aggressor distributed line characteristics. However, the approach uses decoupling, and during the decoupling it ignores the victim loading effect on aggressor coupling node. Then a new approach in [8] was proposed based on 4-π model which was extension of [7]. It introduces a new multi-line model that considers non-switching aggressors as well as switching aggressors. With faster rise times and lower resistance, long wide wires in the upper metal layers exhibit significant inductive effects. An efficient resistance-inductance-capacitance (RLC) model of the on-chip interconnect is, therefore, critical in high level design, logic synthesis and physical design. A closed form expression for the cross-talk noise between two identical RLC lines is developed in [9], assuming that the two interconnects are loosely coupled. In [10], a technique to decouple coupled RLC interconnects into independent interconnects is developed based on a modal analysis. This decoupling method, however, assumes a TEM mode approximation, which is only valid in a two-dimensional structure with a perfect current return path in the ground plane directly beneath the conductors [11]. An estimate of crosstalk noise among multiple RLC interconnects is required to efficiently implement shielding techniques. Inserting shield lines can greatly reduce both capacitive coupling [12] and mutual inductive coupling by providing a closer current path for both the aggressor and victim lines. Dynamic crosstalk is discussed for coupled RLC interconnect in [17]. The proposal made in [18] presents a method to estimate the crosstalk from the output response using correlation method. Kim et al. [19] considers a linear driver model for the crosstalk calculation for RLC interconnects. In [20], closed form crosstalk modeling is proposed using matrix approximation. But these models either suffer from computational complexity or sacrifice the error by taking some approximations. This paper presents a closed form crosstalk analysis for inductively and capacitively coupled RLC interconnect based on 4-π model. It considers the inductive effect and introduces a new multi-line model that considers non-switching aggressors as well as switching aggressors. The realistic exponential waveforms are considered during victim noise derivations. Our proposed crosstalk noise model is considerably different from the previous models as inductive coupling has been introduced between aggressor and victim lines along with capacitive coupling. It is also accurate in the respect that the passive aggressors are represented as equivalent capacitances to the victim line rather than simple lumped coupling capacitance. Equivalent capacitances represent the loading effect of passive aggressors on victim line and have been formulated by including realistic exponential aggressor waveform and resistive shielding. Similarly, the tree branches are also...
formulated by an equivalent capacitance. Based on this model, first aggressor coupling node waveform is derived. Then after calculating the transfer function between aggressor coupling node and victim receiver, victim noise waveform has been derived. Noise peak and width are the two parameters to determine whether the noise is below the acceptable limit. Therefore, the closed form analytic expressions for peak noise and noise width are also formulated. The results for different random circuits are compared with SPICE results. Simulations are also carried out for multiple switching aggressors and results show good agreement to HSPICE results.

II. THE 4-Π MODEL

The multi-line model has been developed based on 4-π model parameters. In the 4-π model, both victim and the aggressor net are modeled using the 2-π circuits [7]. Finally, we obtain the template circuit, shown in Figure 1. In this model, effective resistances \( R_d \) and \( R_{th} \) model the victim and aggressor drivers, respectively. Drivers are represented by linear resistors, inductors and capacitors using the method described in [13]. The coupling node (node-2) is set to be the center of the coupling portion of the victim net.

![Fig. 1 The 4-π model for two coupled interconnects](image)

\( R_{1a}, L_{1a}, C_{ua} \) are the upstream resistance, inductance and capacitance for the aggressor net, respectively. Similarly for victim net, let’s assume upstream and downstream resistance, inductance, and capacitance at node 5 to be \( R_{1v}, L_{1v}, C_{uv} \) and \( R_{2v}, L_{2v}, C_{dv} \), respectively. Then, for aggressor and the victim, we have:

\[
C_{1a} = \frac{C_{ua}}{2}, C_{2a} = \frac{(C_{ua} + C_{da})}{2} \quad \text{and} \quad C_{la} = \frac{C_{da}}{2} + C_{lda}
\]

\[
C_{1v} = \frac{C_{uv}}{2}, C_{2v} = \frac{(C_{uv} + C_{dv})}{2} \quad \text{and} \quad C_{lv} = \frac{C_{dv}}{2} + C_{ldv}
\]

Here, \( C_{lda} \) and \( C_{ldv} \) represent the load capacitances for aggressor and victim lines, respectively.

III. PASSIVE AGgressor MODELING BY EQUIVALENT CAPACITANCE

A victim can be coupled to many non-switching (passive) aggressors. In the earlier approaches the loading effect of a passive aggressor is simply taken as a coupling capacitor at victim coupling point [6-7]. However, a passive aggressor follows victim waveform and contributes to the stability of the victim line. Therefore, equivalent load capacitance at the victim coupling point is less than the coupling capacitance and can be formulated using coupling and/or branching admittance concept as discussed in [14]. The inductance at node 1 of the aggressor will be the sum of two coupled inductance and twice the mutual inductance between them. In this paper, an equivalent capacitance formula for a passive aggressor is first derived assuming an exponential aggressor waveform. In order to derive the capacitance expression, the passive aggressor is first reduced to the simple circuit as shown in Figure 2b, where,

\[
R_a' = R_{th} + R_{1a}
\]

\[
L_a' = L_{1a} + L_{1v} + 2M
\]

\[
C_a' = C_{2a} + C_{la} + \left( \frac{R_{th}^2}{(R_{th} + R_{1a})^2} \right) C_{1a}
\]

![Fig. 2 A non-switching aggressor not coupled to the victim line](image)

Then for matching purposes, the victim waveform is assumed to be a normalized exponential voltage, as shown in Figure-3. The equivalent capacitance for the passive aggressor can now be formulated.

The currents coming from the victim node should be same for both the cases and can be calculated as,

\[
I = C_v \left[ \frac{dV_v'(t)}{dt} - \frac{dV_{A}'(t)}{dt} \right] = C_{eq} \frac{dV_v(t)}{dt}
\]
Assuming zero initial condition and exponential waveform, we can calculate the equivalent capacitance by integrating (4) over the interval $0 \leq t \leq t_{r}$. Here, $t_r$ is the exponential rise time constant.

$$C_{eq} = C_c \left[1 - V_A(5t_r)\right]$$

Now considering left part of Figure-3,

$$C_c \frac{dV_A(t)}{dt} = \left(C_c + C_a\right) \frac{dV_A(t)}{dt} + V_A(t) + \frac{1}{R_a} \int V_A(t) dt$$

Taking Laplace’s transform of (6) yields,

$$V_A(s) \left[sC_c + sC_a + \frac{1}{R_a + sL_a}\right] = sC_c V_A(s)$$

Now,

$$V_A(s) = \left[\frac{1}{s} + \frac{1}{s + \frac{1}{t_r}}\right]$$

So,

$$V_A(s) = \frac{k\left(s + \frac{R_a}{L_a}\right)}{\left(s + \frac{1}{t_r}\right)\left(s + \frac{R_a}{L_a}\right) + \frac{1}{L_a\left(C_c + C_a\right)}}$$

Where,

$$k = \frac{C_c}{t_r\left(C_c + C_a\right)}$$

Taking inverse Laplace’s transform of (9),

$$V_A(t) = A_1 e^{\frac{-t}{t_r}} + A_2 e^{-\alpha t} + A_3 e^{-\beta t}$$

Where, $\alpha, \beta, A_1, A_2, A_3$ are derived as follows:

$$\alpha = \frac{-R_a}{2L_a} + \sqrt{\left(\frac{R_a}{2L_a}\right)^2 - \frac{1}{L_a\left(C_c + C_a\right)}}$$

$$\beta = \frac{-R_a}{2L_a} - \sqrt{\left(\frac{R_a}{2L_a}\right)^2 - \frac{1}{L_a\left(C_c + C_a\right)}}$$

IV. RLC TREE AND BRANCH MODELING

The model proposed in [7] treats aggressor net branches simply as lumped capacitances at the branching point. However, the capacitance seen at the branching node is less than the total branch capacitance due to resistive shielding effect. Hence, the approach in [7] is incorrect. In this paper, an equivalent capacitance formula for tree branches is derived noting the exponential aggressor waveform. First, tree branches are reduced to a simple $\pi$ model using the moment matching method as demonstrated in [14]. Then, this model reduces to an equivalent branching capacitance $C_{eq-br}$ (Figure 4) considering an exponential waveform on input node $A$.

$$A_1 = \frac{kt_r\left(R_a - L_a\right)(C_c + C_a)}{t_r^2 - R_a t_r \left(C_c + C_a\right) + L_a \left(C_c + C_a\right)}$$

$$A_2 = \frac{k\left(R_a - aL_a\right)t_r}{L_a\left(1 - t_r\alpha\right)\left(\beta - \alpha\right)}$$

$$A_3 = \frac{k\left(R_a - \beta L_a\right)t_r}{L_a\left(1 - t_r\beta\right)\left(\alpha - \beta\right)}$$

Substituting the value of $V_A(5t_r)$ in (5), $C_{eq}$ can be represented as,

$$C_{eq} = C_c \left[1 - \left(A_1 e^{\frac{-t}{t_r}} + A_2 e^{-\alpha t} + A_3 e^{-\beta t}\right)\right]$$
In Figure-4, we can equate the currents in node A for both circuits:

\[
C_{eq-br} \frac{dV_A(t)}{dt} = C_{2a} \frac{dV_A(t)}{dt} + C_{la} \frac{dV_B(t)}{dt}
\]  

(18)

Assuming a rising exponential voltage on input node and zero initial condition, we can obtain an equivalent branching capacitance after integrating both sides of above equation over 0 ≤ t ≤ 5t,

Then by applying KCL on node B, one obtains the relation as given in (20).

\[
V_A(t) = 1 - e^{-\frac{t}{r}}
\]  

(21)

Finally, this value can be inserted in (19), which results,\n
\[
C_{eq-br} = C_{2a} + C_{la} V_B(5t_i)
\]  

(19)

\[
\frac{V_A(t) - V_B(s)}{R_{2a} + L_a} s = sC_{la} V_B(s)
\]  

(20)

\[
V_A(t) = 1 - e^{\frac{t}{r}}
\]

(21)

\[
V_B(s) = \frac{1}{s + \frac{1}{t_r} + \left( \frac{R_{2a}}{L_a} \right) s + \frac{1}{C_{la} L_a}}
\]  

(22)

\[
A_4 = 1
\]  

(25)

\[
A_5 = \left( \frac{L_a}{C_{la} + R_{2a} C_{la} t_r - t_r^2} \right) - \left( \frac{1}{C_{la} L_a} \right)
\]  

(26)

\[
A_6 = \frac{1}{\alpha_1 C_{la} t_r^n (1 - \alpha_1 (1 - \beta_1))}
\]  

(27)

\[
A_7 = \frac{1}{\beta_1 C_{la} t_r^n (1 - \beta_1 (1 - \alpha_1))}
\]  

(28)

Upon solving (23) and inserting t=5t yields,\n
\[
V_A(t) = u(t) + A_2 e^{-\frac{t}{t_r}} + A_6 e^{-\alpha_1 t} + A_2 e^{-\beta_1 t}
\]

(29)

\[
V_B(5t_i) = A_1 e^{\frac{5t_i}{t_r}} + A_6 e^{-\alpha_1 t} + A_2 e^{-\beta_1 t}
\]

(30)

Finally, this value can be inserted in (19), which results,\n
\[
C_{eq-br} = C_{2a} + C_{la} \left( u(t) + A_2 e^{\frac{t}{t_r}} + A_6 e^{-\alpha_1 t} + A_2 e^{-\beta_1 t} \right)
\]

(31)

V. AGGRESSOR WAVEFORM CALCULATION AT COUPLING NODE

Our proposed model uses a reduced transfer function between aggressor coupling node and the victim node, hence results in small accuracy loss compared to the method in [7]. In the previous work, the direct transfer function between aggressor input and victim output is first calculated, then dominant pole approximation is hired over the whole transfer function to reduce complexity. However, too much use of dominant pole approximation always reduces model accuracy.

In the proposed model, the aggressor waveform at the coupling node is first calculated and then entered to the transfer function between the coupling node and the victim output to obtain victim noise voltage. Compared to [7], the dominant pole approximation is used moderately which results in increased accuracy.

In order to model the coupling node aggressor waveform correctly, victim-loading effect on the aggressor node needs to be calculated. The loading effect is smaller than the coupling capacitor due to resistive shielding. The victim line can be reduced to an equivalent capacitor Ceqv using the quiet aggressor/victim net reduction techniques which are summarized in Section 3. The aggressor branches after the coupling point are also reduced to an equivalent capacitance Ceq-br using the tree branch reduction techniques discussed earlier.

After application of reduction techniques, the 4-π network, shown in Figure 1, reduces to Figure 5 for aggressor coupling node voltage calculation.

From Figure-5,

\[
V_1(s) = \frac{z}{z + R_{th}} V_{in}(s)
\]

(32)

Where, \[ z = \frac{1}{R_{la} + s L_{eq} + \frac{1}{s C}} \]

(33)

\[
C = C_{2a} + C_{eq-br} + C_{eqv}
\]

(34)

\[
L_{eq} = L_{la} + L_{lv} + L_{2a} + L_{2v} + 4M
\]

Then we have,

\[
V_2(s) = \left( \frac{1}{L_{eq} C_s^2 + R_{la} C_s + 1} \right) V_1(s)
\]

(35)

\[
V_2(s) = \left( \frac{1}{L_{eq} C_s^2 + R_{la} C_s + 1} \right) \left( \frac{z}{z + R_{th}} \right) V_{in}(s)
\]

(36)

Finally, the transfer function between the input and coupling node 2 can be derived as,

\[
V_2(s) = \left( \frac{1}{L_{eq} C_s^2 + R_{la} C_s + 1} \right) \left( \frac{z}{z + R_{th}} \right) V_{in}(s)
\]

(37)
We equate these two waveforms from (44) and (45), have,

\[ V_2(s) = \frac{1}{t_x} \left[ \frac{a_2}{t_x^2} + \frac{a_1}{t_x} \right] s^2 + \left( \frac{a_1}{t_x} - 1 \right) s + \frac{1}{t_x} \]

where, \( a_2 = \left( R_{ia} + R_{ib} + L \right) \)
\( a_1 = R_{ia} + R_{ib} \)

The dominant pole approximation method [6] [15-16] is used to reduce the complexity of the transfer function. Finally we have,

\[ V_2(s) = \frac{1}{t_x} \left[ \frac{a_2}{t_x^2} + \frac{a_1}{t_x} \right] s^2 + \left( \frac{a_1}{t_x} - 1 \right) s + \frac{1}{t_x} \]

Where, \( A = 1 \)
\( B = \frac{1}{\alpha_2 t_x (\alpha_2 - \beta_2)} \)
\( C = \frac{1}{\beta_2 t_x (\beta_2 - \alpha_2)} \)

Taking inverse Laplace transform of (41),

\[ V_2(t) = 1 + \frac{1}{t_x} \left( \frac{1}{\alpha_2} e^{-\alpha_2 t} - \frac{1}{\beta_2} e^{-\beta_2 t} \right) \]

This waveform when plotted represents a delayed exponential waveform as expected. However, it contains two exponential terms, and should be reduced to only one term for simplicity. We assume the delayed waveform at coupling node to be

\[ V_2(t) = 1 - e^{-\frac{t}{t_x}} \]

We equate these two waveforms from (44) and (45),

\[ 1 + \frac{1}{t_x} \left( \frac{1}{\alpha_2} e^{-\alpha_2 t} - \frac{1}{\beta_2} e^{-\beta_2 t} \right) = 1 - e^{-\frac{t}{t_x}} \]

The area under both exponential terms should be same:

\[ \int_{0}^{\infty} \frac{1}{\alpha_2} e^{-\alpha_2 t} dt - \int_{0}^{\infty} \frac{1}{\beta_2} e^{-\beta_2 t} dt \]

So,

\[ t_x = \frac{\alpha_2 \beta_2}{\alpha_2 + \beta_2} \]

The new calculated rising exponential time constant \( t_x \) has been verified by plotting the function given in (45) simultaneously with HSPICE result. The following parameter values used for the verification: \( R_{th}=200 \) Ohm, \( R_{a}=250 \) Ohm, \( R_{b}=50 \) Ohm. The coupling capacitance \( C_c \) is taken as 150fF. Other capacitances are given as follows: \( C_{u}=C_{d}=100 \) fF, \( C_{uv}=C_{dv}=100 \) fF. Let the load capacitances for aggressor and victim line be 50 fF each. \( L_{1a} = L_{2a} = L_{1v} = L_{2v} = M=100nH. \) Also a normalized aggressor voltage is assumed and aggressor rise time \( t_r \) is chosen as 150 ps.

Figure 6 shows the result from HSPICE at aggressor coupling node. For the given parameter values above, the model predicts the new rise time constant \( t_x \) as 20.967μSec, while HSPICE calculates as 21.985μSec. The model error is only 4.6%. For several random circuits, the model has been verified and error corresponding to each case is calculated and it has been found that the absolute error value remains less than 7%.

VI. OUTPUT VOLTAGE FORMULATION

In the previous Section, the aggressor waveform at the coupling node is formulated by considering the exponential aggressor input. Now, the aggressor waveform at coupling location needs to be entered to the transfer function to calculate the noise as shown in Figure-7.
Referring to Figure-7 we have,
\[
\frac{1}{z_1} = \frac{1}{R_d} + sC_{Iv}
\]
(49)
\[
\frac{1}{z_2} = \frac{1}{R_{2v} + L_v''} + \frac{1}{sC_{hv}} + \frac{1}{z_1 + R_{IV} + sL_v}
\]
(50)
\[
V_z(s) = \frac{z_2}{z_2 + 1} V_{agg}(s)
\]
(51)
\[
V_{\text{noise}}(s) = \frac{1}{C_h L_v' s^2 + R_{2v} + C_{hv} s + 1}
\]
(52)
\[
z_2 = \frac{a_1 s^4 + a_3 s^3 + a_2 s^2 + a_1 s + a_0}{b_2 s^4 + b_2 s^3 + b_1 s^2 + b_1 s + b_0}
\]
(53)
where,
\[
a_1 = L_c' C_{hv} R_v R_p a_1 = \left[ L_c' C_{hv} (L_v' + C_v R_v R_p) + L_c' C_{hv} R_v R_p \right],
\]
(54)
\[
a_2 = \left[ L_c' C_{hv} (R_v + R_p) + L_c' C_{hv} R_v \right],
\]
(55)
\[
a_3 = \left[ L_c' C_{hv} (R_v + R_p) + L_c' C_{hv} R_v \right],
\]
(56)
\[
V_{\text{noise}}(s) = \frac{C_c (a_1 s + a_0)}{s + a_3} \quad \text{where,}
\]
(57)
\[
\alpha_3, \beta_3 = \frac{a_2 + a_1}{2} \quad \text{and,}
\]
(58)
\[
V_{\text{noise}}(s) = \frac{A_8}{s + a_3} + \frac{A_9}{s + \beta_3}
\]
(59)
\[
V_{\text{noise}}(t) = A_8 e^{-\alpha_3 t} + A_9 e^{-\beta_3 t}
\]
(60)
\[
A_8 = \frac{C_c}{t_r} \left( a_0 - a_1 \alpha_3 \right)
\]
(61)
\[
A_9 = \frac{C_c}{t_r} \left( a_0 - a_1 \beta_3 \right)
\]
(62)
If we insert the exponential function in (44) as the aggressor voltage, we obtain the following noise waveform:
\[
V_{\text{noise}}(s) = \frac{1}{t_r} \left[ \frac{a_2}{s + \frac{a_1}{t_r}} + \frac{a_1}{s + \frac{a_1}{t_r}} \right] + \frac{p_1 + \frac{p_2}{t_r}}{s + \frac{1}{t_r}}
\]
(56)
\[
V_{\text{noise}}(s) = \frac{C_c (a_1 s + a_0)}{s^2 + \left( \frac{a_2}{s + \frac{a_1}{t_r}} + \frac{a_1}{s + \frac{a_1}{t_r}} \right) s + \frac{1}{t_r}}
\]
(57)
Again applying dominant pole approximation method to the above equation, it becomes,
\[
V_{\text{noise}}(s) = \frac{C_c (a_1 s + a_0)}{s + a_3}
\]
(58)
where,
\[
\alpha_3, \beta_3 = \frac{a_2 + a_1}{2} \quad \text{and,}
\]
(59)
\[
V_{\text{noise}}(t) = A_8 e^{-\alpha_3 t} + A_9 e^{-\beta_3 t}
\]
(60)
\[
A_8 = \frac{C_c}{t_r} \left( a_0 - a_1 \alpha_3 \right)
\]
(61)
\[
A_9 = \frac{C_c}{t_r} \left( a_0 - a_1 \beta_3 \right)
\]
(62)
Applying inverse Laplace transform to (58), we will get,
\[
V_{\text{noise}}(s) = \frac{1}{t_r} \left[ \frac{p_1 + \frac{p_2}{t_r}}{s + \frac{1}{t_r}} \right]
\]
(57)
\[
V_{\text{noise}}(s) = \frac{C_c (a_1 s + a_0)}{s^2 + \left( \frac{a_2}{s + \frac{a_1}{t_r}} + \frac{a_1}{s + \frac{a_1}{t_r}} \right) s + \frac{1}{t_r}}
\]
(57)
\[
V_{\text{noise}}(t) = A_8 e^{-\alpha_3 t} + A_9 e^{-\beta_3 t}
\]
(60)
\[
A_8 = \frac{C_c}{t_r} \left( a_0 - a_1 \alpha_3 \right)
\]
(61)
\[
A_9 = \frac{C_c}{t_r} \left( a_0 - a_1 \beta_3 \right)
\]
(62)
By differentiating $V_{\text{noise}}(t)$ with respect to $t$, the time when the noise voltage reaches its peaks, $t_{\text{peak}}$, can be found:
The noise peak voltage $V_{\text{peak}}$ is found by substituting (63) in (62):

$$
V_{\text{peak}}(t) = \frac{C_r}{t_r(\beta_1 - \alpha_1)} \left[ \left( a_o - a_1 \alpha_3 \right) e^{-\alpha_3 t} - \left( a_o - a_1 \alpha_1 \right) e^{-\beta_3 t} \right] - \left( a_o - a_1 \beta_3 \right) \left[ \left( a_o - a_1 \alpha_3 \right) e^{-\alpha_3 t} - \left( a_o - a_1 \alpha_1 \right) e^{-\beta_3 t} \right]
$$

(64)

Noise peak has been traditionally used as a metric to determine whether the noise is at an acceptable level. However, the noise width is also a necessary metric in determining whether a noise pulse can go through a receiver. If noise peak exceeds the threshold, but does not carry sufficient width, the noise may not be received at the receiver output at all. Therefore, the noise width should also simultaneously be considered. The noise peak expression is derived in (64). For noise width, the threshold is usually taken as 50% of $V_{\text{peak}}$. Considering (62) and the threshold, one can obtain a function $f(t)$ which can be used in Newton’s Iteration method to solve for $t_1$ and $t_2$ time instances.

$$
\frac{dV_{\text{noise}}(t)}{dt} = 0
$$

$$
t_{\text{peak}} = \frac{1}{\alpha_3 - \beta_3} \ln \left[ \frac{\alpha_1 \left( a_o - a_1 \alpha_3 \right)}{\beta_3 \left( a_o - a_1 \beta_3 \right)} \right]
$$

(63)

Then, the noise width is defined by,

$$
t_{\text{width}} = t_2 - t_1
$$

(70)

The algorithm converges very rapidly after some iteration.

VII. VALIDATION OF THE PROPOSED MODEL

The model has been tested extensively and its accuracy has been compared with SPICE simulation results. Several circuits with different parameter values have been taken and tested. The parameter ranges were taken as follows: $R_d$ and $R_{\text{th}}$ are 10-1500 Ω; load capacitances for victim and aggressor lines are 5-50 fF; aggressor and victim wire resistances are 10-250 Ohms; aggressor and victim line capacitances are 0.5-100 nF; the mutual inductance between aggressor and victim are 150 nH and finally $t_r$ is chosen in the range between 20-500 ps. After substituting these values in (64) and (70), noise peak and width have been calculated. In Table-1, noise peak and noise width of the proposed model is compared with SPICE result, and the average error for noise peak is found to be 4.707% and for noise width 6.1523%.

$$
t_{1,i} = t_{1,i} - \frac{f(t_{1,i})}{f'(t_{1,i})}
$$

(68)

and,

$$
t_{2,i} = t_{2,i} - \frac{f(t_{2,i})}{f'(t_{2,i})}
$$

(69)

This method converges very rapidly if the initial guesses are taken carefully. The initial guesses of $t_1$ and $t_2$ are taken as $\frac{1}{4} t_{\text{peak}}$ and $4t_{\text{peak}}$, respectively. The values of $t_1$ and $t_2$ are updated using the iteration formula given below:
### Table I

#### EXPERIMENTAL RESULTS FOR NOISE PEAK AND NOISE WIDTH

<table>
<thead>
<tr>
<th>Sr. No.</th>
<th>$T_r$ (psec)</th>
<th>$R_d/R_{th}$ (Ohm)</th>
<th>$R_{1a}/R_{2a}$</th>
<th>$L_{1a}/L_{2a}$</th>
<th>$C_{1a}$ (SPICE)</th>
<th>$V_{peak}$ (SPICE)</th>
<th>$V_{peak}$ (Proposed Model)</th>
<th>Relative Error (%)</th>
<th>$T_{width}$ (SPICE) (μSec)</th>
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#### VIII. MULTIPLE ACTIVE AGGRESSORS

In a real circuit, a given victim line can be coupled to many switching aggressors. In this case, superposition theorem can be applied to calculate the total cross-coupling noise. With superposition, each active aggressor is switched at a time while holding other aggressor drivers quiet. The noise contributions are summed at the end to calculate total noise at the victim end. If there are $N$ switching aggressors, it is necessary to calculate noise for $N$ times to obtain the final result, hence time complexity is linear. Prior to any noise calculation, an equivalent capacitance value should be calculated for each aggressor using (17). The equivalent capacitance values are utilized for superposition to represent non-switching aggressors and this reduces the complex multi-line network into a manageable 4-π template shown in Figure-1 during each superposition step. Table-II shows experimental results obtained for multiple aggressors’ case. Experiments are performed upto 5 aggressors. $R_g/R_{th}=500Ω$, $R_{1a}=R_{2a}$ $=R_{1v}=R_{2v}=150Ω$, $L_{1a}=L_{2a}=L_{1v}=L_{2v}=50nH$, $M=150nH$, $C_{1a}=C_{1v}=50fF$. The noise peak and width values of the previous approach in [7] are used, and the proposed approach has been compared with SPICE simulation results. The proposed approach has an average error of 4.89% for the noise peak and 5.215% for the noise width. The inclusion of victim loading effect, the equivalent capacitance representation for passive aggressors and moderate use of dominant pole approximation method makes our approach superior in terms of accuracy.

### Table II

#### EXPERIMENTAL RESULTS OBTAINED FOR MULTIPLE AGGRESSOR LINES COUPLED TO VICTIM LINE

<table>
<thead>
<tr>
<th>No of aggressors</th>
<th>Noise Peak (mV)</th>
<th>Noise Peak (mV)</th>
<th>Relative Error (%)</th>
<th>Noise Width (μSec)</th>
<th>Noise Width (μSec)</th>
<th>Relative Error (%)</th>
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<td>Estimated Value</td>
<td></td>
<td>SPICE Result</td>
<td>Estimated Value</td>
<td></td>
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<td>13.8875</td>
<td>4.95</td>
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</table>
The exponential aggressor waveform and formulation included resistive shielding effects. Then closed form expression for noise peak and width has been derived and compared against SPICE results and results are very promising. Results show that the average error for noise peak is 4.89% and for the noise width is 5.2% while allowing very fast analysis time. This 4-π model will be useful in many applications at various levels to guide noise aware DSM circuit designs.

REFERENCES