Abstract—This paper deals with the synthesis of fuzzy controller applied to a permanent magnet synchronous machine (PMSM) with a guaranteed $H_\infty$ performance. To design this fuzzy controller, nonlinear model of the PMSM is approximated by Takagi-Sugeno fuzzy model (T-S fuzzy model), then the so-called parallel distributed compensation (PDC) is employed. Next, we derive the property of the $H_\infty$ norm. The latter is cast in terms of linear matrix inequalities (LMI’s) while minimizing the $H_\infty$ norm of the transfer function between the disturbance and the error $(T_m)$. The experimental and simulations results were conducted on a permanent magnet synchronous machine to illustrate the effects of the fuzzy modelling and the controller design via the PDC.

Keywords—Feedback controller, Takagi-Sugeno fuzzy model, Linear Matrix Inequality (LMI), PMSM, $H_\infty$ performance.

I. INTRODUCTION

THE classical methods of control proved their effectiveness in many problems of industrial regulation. The methods of advanced control developed by many theoreticians (adaptive control, predictive control, robust control…) allow replying to the requirements of a number of these highly nonlinear systems. This is in this framework that methods of modelling and fuzzy control are positioned.

Many presented approaches have been proposed to tune fuzzy controllers. Many of these techniques are adapted to SISO (single input, single output) [1]-[5] systems as well as their stability study and their performances. But it is highly desirable to extend this technique for MIMO (multi inputs, multi outputs) systems [6]-[8].

The basic idea in our approach is to represent a non linear process by a Takagi-Sugeno fuzzy linear model. Particularly, an original approach has been proposed by (Morère, 2000) [9] and equivalently by (Chaabane and Souissi, 2004) [10], [11].

The interest of this last representation is its simplicity due to its linear model and then the easy use of Linear Matrix Inequality (LMI) to develop a stabilizing linear control law.

Once stabilization is achieved, we take an interest on the exploitation of the properties of $H_\infty$ norm in order to guarantee some performance criteria such as disturbance rejection and good monitoring of the deposit.

The paper is organized as follows:

In section 2 we define the problem formulation. Section 3 deals with the synthesis of fuzzy control law with $H_\infty$ performance. Finally, in section 4, an experimental study was conducted on a permanent magnet synchronous machine (PMSM) to illustrate the proposed method.

Notations

$\text{sym} \{X\} = X + X^T$ and the symbol $\ast$ means the transposed

II. PROBLEM FORMULATION

A nonlinear system can be approximated by a Takagi-Sugeno (T-S) fuzzy model. The T-S fuzzy model is a piecewise interpolation of several linear models through membership functions. The following fuzzy dynamic model is employed to represent a complex multi-input multi-output continuous-time system with both fuzzy inference If-Then rules and local analytic linear models as follows [14]-[17]:

$$\begin{align*}
\text{Plant Rule i:} \\
\text{If } \{z_1(t) \in F_{i1} \text{ and } z_2(t) \in F_{i2} \ldots \text{and } z_p(t) \in F_{ip}\} \text{ Then } & \quad \dot{x}(t) = A_ix(t) + B_iu(t) + D_i\omega(t) \\
& \quad y(t) = C_ix(t)
\end{align*}$$

for $i = 1,2,…,r$

where $x(t) = [x_1(t), x_2(t),…,x_n(t)]^T \in R^{nx1}$ denotes the state vector; $u(t) = [u_1(t), u_2(t),…,u_n(t)]^T \in R^{nx1}$ denotes the control input; $\omega(t) = [w_1(t), w_2(t),…,w_n(t)]^T \in R^{nx1}$ denotes bounded external disturbance; $F_i$ is the fuzzy set, $A_i \in R^{nxn}$, $B_i \in R^{nxn}$ are system and input matrix respectively, $r$ is the number of If-Then rules and $z_1(t), z_2(t),…,z_p(t)$ are the premise variables.
Each conclusion part represents a sub model of the system. For each sub model is assigned a weight \( \lambda_i(t) \), which depends on the degree of belonging variables premises \( z_i(t) \) to sub- fuzzy sets \( F_i \), and choice of interpretation of the operator “and” connecting the premises.

In this type of model, the product is chosen to model this operator:

\[
\lambda(z(t)) = \prod_{j=1}^{n} F_j(z(t))
\]

With :

\[
\begin{align*}
\sum_{i=1}^{n} \lambda_i(z(t)) &> 0 \\
\lambda_i(z(t)) &\geq 0
\end{align*}
\]

The final output of fuzzy models is inferred by a barycentric defuzzification [18]-[23]:

\[
\begin{align*}
\dot{x}(t) &= \sum_{i=1}^{n} \lambda_i(z(t))(A_i x(t) + B_i u(t) + D_i w(t)) \\
y(t) &= \sum_{i=1}^{n} \lambda_i(z(t)) \cdot \text{f}(x(t))
\end{align*}
\]

(2)

If we set, \( h_i(z(t)) = \frac{\lambda_i(z(t))}{\sum_{i=1}^{n} \lambda_i(z(t))} \), satisfying \( \forall t \) a convex sum property:

\[
\sum_{i=1}^{n} h_i(z(t)) = 1 , \text{ the final output can be rewritten:}
\]

\[
\dot{x}(t) = \sum_{i=1}^{n} h_i(z(t))(A_i x(t) + B_i u(t) + D_i w(t)) \\
y = \sum_{i=1}^{n} h_i(z(t)) C_i x(t)
\]

(3)

In this study, we assume that \( w(t) \) is unknown but bounded. However, the effect of \( w(t) \) will deteriorate the control performance of control system. Therefore, how to eliminate the effect of \( w(t) \) to guarantee the control performance is an important issue in the control systems. Since \( H_\infty \) control is the most important control design to efficiently eliminate the effect of \( w(t) \) on the control system, it will be employed to deal with the robust performance control.

A. Physical model of the PMSM

By considering the classical simplifying assumptions, the dynamic model of the Permanent Magnet Synchronous Machine, in the synchronously d-q reference frame [12], can be described as [13]:

\[
\dot{x}(t) = f(x(t)) + g(x(t))u(t) + w(t)
\]

(4)

With:

\[
x(t) = \begin{bmatrix} I_d & I_q \\
\frac{R}{L_d} I_d + \frac{L_s}{L_d} w_r I_q & -\frac{L_s}{L_d} w_q - \frac{\Phi}{L_d} w_r
\end{bmatrix}, \quad u(t) = \begin{bmatrix} v_d \\
v_q
\end{bmatrix} \\
f(x(t)) = \begin{bmatrix}
\sum_{j=1}^{n} h_i(z(t)) C_j x(t) \\
\sum_{j=1}^{n} h_i(z(t)) C_j x(t)
\end{bmatrix}
\]

\[
g(x(t)) = \begin{bmatrix}
\frac{1}{L_d} & 0 & 0 \\
0 & \frac{1}{L_q} & 0
\end{bmatrix}
\]

\[
w(t) = \begin{bmatrix} 0 \\
- \frac{p \Phi}{J}
\end{bmatrix}
\]

Where \( w_r \) is the rotor speed, \( \{I_d, I_q\} \) are the d-q axis stator currents \( \{v_d, v_q\} \) are the d-q axis stator voltages. The load torque \( c \) is a known step disturbance. The motor parameters are the moment of inertia of the rotor \( J \), the stator winding resistance \( R \), the d-q axis inductances \( \{L_d, L_q\} \), the friction coefficient \( f \) relating to the rotor speed, the flux linkage of the permanent magnets \( \Phi \) and the number of poles pairs \( p \).

B. T-S fuzzy model of the PMSM

The concept of flux oriented control is used to define a decoupled model of the PMSM, its principle is similar to the field oriented control for an induction motor.

Note that for synchronous machines there is no need for a flux model. Consequently, the rotor position is the angle of reference. Moreover, as we have a smooth poles machine, the best choice for its operation is obtained for a value where the internal angle of the machine is equal to \( \frac{\pi}{2} \) that is to say \( I_d = 0 \).

Then the nonlinear model of the PMSM can be written as the state space form:

\[
\begin{cases}
\dot{x}(t) = A x(t) + B u(t) + D w(t) \\
y = C x(t)
\end{cases}
\]

(5)

where:

\[
A = \begin{bmatrix}
-\frac{R}{L_d} & \frac{L_s}{L_d} w_r & 0 \\
-\frac{L_s}{L_q} w_r & -\frac{R}{L_q} & \frac{\Phi}{L_q} \\
0 & \frac{p \Phi}{J} & -\frac{1}{J}
\end{bmatrix}, \quad B = \begin{bmatrix} 1 \\
0 & 0 \\
0 & 0
\end{bmatrix}
\]
To transform the nonlinear model of the machine into a fuzzy linear Takagi-Sugeno model, the adopted method is to use a transformation on functions of one variable [9], [24], as follows:

Lemma: for \( x \in [-a, b] \), \( a > 0, b > 0 \) consider \( f(x) \), \( R \to R \), a bounded function, then there are always two functions \( F_{i1}(x) \), \( R \to R \), and \( F_{i2}(x) \), \( R \to R \) and two scalars \( \alpha \) and \( \beta \) verifying the following properties:

\[
F_{i1}(x) + F_{i2}(x) = 1, \quad F_{i1}(x) \geq 0, \quad F_{i2}(x) \geq 0 \quad \text{and} \quad f(x) = \alpha F_{i1}(x) + \beta F_{i2}(x)
\]

Proof of Lemma: Considering that \( f(x) \) is bounded, we can always write:

\[
f(x) = \alpha F_{i1}(x) + \beta F_{i2}(x)
\]

With:

\[
F_{i1}(x) = \frac{f(x) - f_{\min}}{f_{\max} - f_{\min}}, \quad F_{i2}(x) = 1 - F_{i1}(x)
\]

\[
f_{\max} = \alpha, \quad f_{\min} = \beta.
\]

By applying this decomposition to the PMSM, we obtained local models defined by equation (1). The model state (5) of the machine has three variables, their products with the state variable vector generates the nonlinearity of the model machine. In our study, we have a linearity, resulting from the product of \( r w \) with \( d-q \) axis currents. Thus the vector of premise variables may be defined by:

\[
z(t) = z_i(t) = w_i(t)
\]

With:

\[
w_{r_{\min}}(t) < w_r(t) < w_{r_{\max}}(t)
\]

From lemma 1, we can write:

\[
z(t) = F_{i1}z_{max}(t) + F_{i2}z_{min}(t)
\]

\[
\begin{align*}
F_{i1} &= \frac{z(t) - z_{min}(t)}{z_{max}(t) - z_{min}(t)} \\
F_{i2} &= \frac{z_{max}(t) - z(t)}{z_{max}(t) - z_{min}(t)}
\end{align*}
\]

III. SYNTHESIS OF CONTROL LAW

In the goal, to design a fuzzy linear state feedback controller, the so-called parallel distributed compensation (PDC) is proposed. This concept uses a fuzzy linear controller for each sub-model. Then, the regulator thus produced shares the same basic rules as those of the fuzzy T-S model. The linear control rule \( i \) is derived based on the state equation (1) in the consequent part of the \( i \)th model rule.

Control Rule \( i \)

if \( (z_i(t) \text{ is } F_{0i}) \) and \( (z_r(t) \text{ is } F_{1i} \) and \( \ldots \) and \( (z_q(t) \text{ is } F_{ri}) \)

then \( u(t) = K_i x(t) \)

(9)

where \( K_i \) is the local feedback gain matrix.

The final control \( u \) is inferred using the Sum-Product reasoning method:

\[
u(t) = \sum_{i=1}^{r} h_i(z(t))K_i x(t)
\]

(10)

By substituting (10) in (3), the state equation of the error:

\[
\epsilon(t) = x(t) - x_r:
\]

\[
\epsilon(t) = \sum_{i=1}^{r} \sum_{j=1}^{r} h_i(z(t))h_j(z(t)) \left[ G_i \epsilon(t) + \overline{D} \omega(t) \right] + 2 \sum_{i<j} h_i(z(t))h_j(z(t)) G_{ij} \epsilon(t)
\]

(11)

Where

\[
G_{ij} = A_i + B_i K_i + A_j + B_j K_j
\]

\( x_r \): the reference

\[
\overline{D} = \begin{bmatrix} A & D \end{bmatrix}; \quad \omega = \begin{bmatrix} x_r \end{bmatrix}
\]

Consider the \( H_{\infty} \) norm of the transfer function \( \{T_{r \omega}\} \) between the error and the disturbance:

\[
\|T_{r \omega}\|_{\infty} = \sup_{\|\epsilon\|_{\infty} = 1} \|\epsilon(t)\|_{\infty}
\]

(12)

Where

\[
\|\epsilon(t)\|_{\infty} = \int_{0}^{\infty} e^T(t) \epsilon(t) dt
\]

We can state the following lemma:

Lemma: Consider the system defined by the equation (11). The \( H_{\infty} \) norm of the transfer function \( T_{r \omega} \) is inferior to \( \gamma \) if and only if there exist a symmetric defined matrix \( Q > 0 \) and \( Y \) such as the following inequalities are satisfied:

\[
sym\{AQ + BY_{i}\} \begin{bmatrix} D & QC \end{bmatrix} \begin{bmatrix} \gamma & 0 \\ 0 & -\gamma I \end{bmatrix} < 0
\]

(13.a)
The matrices $Q$ and $Y_i$ verifying (13) have been computed.

$$Q = \begin{bmatrix} 0.0091 & -0.0000 & 0.0000 \\ -0.0000 & 0.0092 & 0.0119 \\ 0.0000 & 0.0119 & 0.4655 \end{bmatrix}$$

$$Y_i = 10^3 \begin{bmatrix} -4.8854 & -0.3185 & 0.0086 \\ 0.2760 & -4.3620 & 0.0003 \end{bmatrix}$$

$$Y_2 = 10^3 \begin{bmatrix} -4.8855 & 0.3178 & -0.0083 \\ -0.2764 & -4.3620 & 0.0003 \end{bmatrix}$$

The obtained state feedback controllers are:

$$K_1 = \begin{bmatrix} -44.4298 & -2.8269 & 0.2042 \\ 2.5118 & -40.1178 & -51.5752 \end{bmatrix}$$

$$K_2 = \begin{bmatrix} -44.4301 & 2.8265 & -0.1053 \\ -2.5118 & -40.1178 & -51.5728 \end{bmatrix}$$

IV. SIMULATION AND EXPERIMENTAL RESULTS

In this section, the performance of the control scheme will be verified by numerical simulations and experiments in order to verify the effectiveness of our proposed fuzzy controller. The PMSM is driven by a PC-based Dspace 1104 controller card. The overall system consists of a two permanent magnet synchronous machines fitted by a 1024 pulse/revolution incremental encoder and powered by a voltage inverter; one was used as a motor and the other as a load. The three-phase 1Kw PMSM is characterized by the following parameters:

- Moment of inertia: $J = 2.08 \times 10^{-3}$ Kg.m²
- Stator resistance: $R_s = 0.56$ Ω
- Friction coefficient: $f_0 = 3.9 \times 10^{-3} Nm/s\cdot rad^{-1}$
- d-axis inductance: $L_d = 4.5 mH$
- q-axis inductance: $L_q = 4.5 mH$
- Magnet flux constant: $\Phi = 0.064 wb$
- Pole pair number: $p = 2$

The proposed control algorithm was executed by the software Matlab / SIMULINK, and then compiled and implemented on Dspace 1104 controller card. The digital sampling period was taken equal to 0.1ms.

The fuzzy control law has been implemented on Dspace card DS1104 with a TMS320F240 processor. This card of control is composed of two processors. The master processor permits to manage the application while the slave processor generates the pulse width modulation (PWM) signals of control. It constitutes the «hardware » part of the Dspace card. The software used in the experiment laboratory is first based on MATLAB/Simulink program which permits an easy programming of the application real time under Simulink environment by the use of specific blocks included in the Real Time Interface (RTI) toolbox. The second program, Control Desk, permits to transfer the code of the Simulink program on the card, to create a control graphic interface of the process in real-time and to observe the evolution of the measured data. A set of I/O modules are constructed in the card « Connector panel CP1104 » for the voltage/current measurement, encoder...
interface and the protection of the power inverter. The switching frequency of PWM for the IGBT chopper is 5kHz. In order to verify the effectiveness of the designed controller, a series of measurements has been accomplished.

Figures (1) to (7) illustrates the dynamic behaviour of the PMSM for enslavement speed vacuum.

The results show that the closed-loop system with the synthesized fuzzy controller has a good deportment: indeed, the measured speed and dq-axis currents track well the trajectory of reference one with good reliability over the whole speed range. Furthermore, the dq-axis currents responses are satisfying. From the results we can conclude the performance of the proposed control scheme.

Fig. 1: d-axis current and its reference

Fig. 2: q-axis current and its reference

Fig. 3: speed and its reference

Fig. 4: Photo of the test bench

Fig. 5: speed and its reference
Fig. 6: d-axis current and its reference

Fig. 7: q-axis current and its reference

Fig. 8: Fuzzy control structure of PMSM
V. CONCLUSION

This paper develops a methodology to design a combined fuzzy state feedback controller and a $H_\infty$ attenuation technique to achieve robust performance for nonlinear systems. The proposed robust fuzzy controller can be applied to any robust control design of the nonlinear system. With the aid of linear fuzzy approximation algorithm and LMI technique, the robust $H_\infty$ control design can be extended from exactly known linear systems toward nonlinear systems. By employing the $H_\infty$ attenuation technique, the performance of linear fuzzy control design for the nonlinear systems can be significantly improved. Furthermore, the robust fuzzy control scheme is also developed to eliminate as possible the effect of external disturbance. The proposed design method is simple, and the number of membership functions for the proposed control law can be extremely small. However, because of the use of fuzzy approximation technique and $H_\infty$ control scheme, the results are less conservative than the other robust control methods. The controller developed has been implemented with success on a permanent magnet synchronous machine (PMSM).

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