Abstract—The objective of this study is to examine the validity of Wagner’s law and relationship between economic growth, population and export for Pakistan. The ARDL Bounds cointegration and ECM are utilized for long and short run equilibrium for the period of 1972-2007. Population has considerable role in an economy and exports are the main source to raise the GDP. With the increase in GDP, the government expenditures may or may not increase. The empirical results indicate that the Wagner’s Law does hold, as economic growth is significantly and positively correlated with government expenditures. However, population and exports have also significant and positive impact on government expenditures both in short and long run. The significant and negative coefficient of error correction term in ECM indicates that after a shock, the long run equilibrium will again converge towards equilibrium about 70.82 percent within a year.

Keywords—ARDL Cointegration, Growth, Pakistan, Wagner’s law

I. INTRODUCTION

ECONOMIC growth and population are closely related since the beginning of mankind. Diverse thoughts are there in different era. Currais [1] found that population depended upon per capita income, in Malthusian and fertility theory. On the other hand the association between population and revenue did not persisted as narrated in Malthusian and fertility theory. In fact, Malthus [2] overturned the influence of mercantilists that the level of population resolute the nation’s resources.

Thirlwall [3] elaborated that the relation between economic growth and population was complicated. Economic growth led population growth, or population growth was a required condition for economic development. Impact of population on economic growth was evaluated by a large number of economists. Both gloomy and bright approaches were found. Malthus [2] found that population would increase more than economic growth and mankind had to suffer severe problems. No doubt these thoughts were not proven due to technological changes and with the invention of new resources. Population might enhance the labour force and in this way output would increase. However population growth is a foremost factor in shaping and acquiring higher rise in agriculture production [4].

Developing countries want to achieve higher level of growth rate but one of the root causes to their low growth rate is population. Kelly and Schmidt [5] found that low population growth would be favourable for economic growth. Population growth rate at a low level would help a country to achieve high economic growth. Increase in per capita income shows that the growth rate of a country is increasing. It is now becoming a common method to measure the economic growth of a country. Stockwell [6] elaborated the relationship between population growth in the developing countries and the increase in per capita income. It was found that population had no significant relation with the per capita income of the country. G.D.P. was also used to show that with the increase in G.D.P. the government expenditures would also increased. Wagner’s input to the government expenditure theories is chiefly significant. When earlier than Wagner’s law, the existing observation was that as a country grew wealthier, the government expenditures would decline [7]. Peacock and Wiseman [8] version of Wagner’s law is used in this study.

Wagner [9] evaluated the relationship between public expenditure and the government revenue. It was natural that when the government income increased the public expenditures would also enhance. Wagner elaborated three main reasons, first, in the era of industrialization, the organizational functions of the state would replace with public sector, second, economic growth would direct to the increase in per capita income. It was found that population growth would be favourable for economic growth. Kelly and Schmidt [5] found that low population growth rate but one of the root causes to their low growth rate was population. Kelly and Schmidt [5] found that low population growth rate but one of the root causes to their low growth rate was population. Kelly and Schmidt [5] found that low population growth rate but one of the root causes to their low growth rate was population. Kelly and Schmidt [5] found that low population growth rate but one of the root causes to their low growth rate was population. Kelly and Schmidt [5] found that low population growth rate but one of the root causes to their low growth rate was population.

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developing countries. Pakistan population is increasing at distressing rate. Hussain et al. [11] found that Pakistan was one of the top10 countries which population is greater than 100 million and in this way Pakistan was the 7th heavily populated kingdom of the globe. The projections of the United Nations would depict that Pakistan would become the 3rd populous country of the world up to the year of 2050. Pakistan’s population growth, GDP, government expenditures and exports are shown in the table I.

<table>
<thead>
<tr>
<th>Years</th>
<th>GDP (million $)</th>
<th>Population (million)</th>
<th>Government Expenditures (million $)</th>
<th>Exports (million $)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1972-1980</td>
<td>10103385.6</td>
<td>661.02</td>
<td>969744.43</td>
<td>1033452.8</td>
</tr>
<tr>
<td>1981-1990</td>
<td>20403684.76</td>
<td>962.57</td>
<td>2214276.57</td>
<td>2472960.55</td>
</tr>
<tr>
<td>1991-2000</td>
<td>32937345.34</td>
<td>1240.82</td>
<td>3156095.12</td>
<td>4897399.77</td>
</tr>
<tr>
<td>2001-2007</td>
<td>52247157</td>
<td>1064.09</td>
<td>2963723</td>
<td>5661038</td>
</tr>
</tbody>
</table>


The organization of the study is as follows. Section II contains literature review; methodology is elaborated in section III, section IV presents empirical results and conclusion and policy implications are given in section V.

II. LITERATURE REVIEW

There is extensive literature on economic growth, population, Wagner’s law and exports. Meier and Baldwin [12] found that sustained increase in national income is determined by resources, techniques, structure of markets, institutions and characteristics of population. Population and economic growth had significant relation up to 1980s. Bloom and Freeman [13], Kelley and Schmidt [14], and Brander and Dowrick [15] elaborated that population and economic growth are positively related. But Mankiw, Romer and Weil [16] found a considerably negative relation. Population in developing countries becomes serious issue. Higher population growth rate may cause to slower economic growth. Cincotta and Engelman [17] elaborated that higher population growth become difficult to overcome in developing countries and their institutions had yet to realize this problem for economic growth. No nation can achieve higher growth until population is controlled. Population fertility rate should be minimized for higher growth.

Lower the pace of population growth will help to enhance economic growth at a higher rate. Kelley [18] elaborated that economic growth would be higher in the situation of slower population growth even though the impact of population growth in many countries was insignificant. Population and per capita income are closely associated to depict the picture of economic growth. Lower the population growth and higher the per capita income show that nation achieve their growth targets. Stockwell [6] found that countries with population growth under 1 percent, their per capita income could increase at the rate of 2.5 percent annually. Countries with population growth more than 2 percent had a little increase in per capita income of less than 2 percent.

Wagner’s law got much importance after the Second World War. Public expenditure increased with the increase economic growth. Ram [19] examined the Wagner’s law. Countries were separated according to their growth level. Different time series techniques like Ordinary Least Square (OLS) was used and it was found that the result supported the law. Vatter and Walker [20] and Ganti and Kolluri [21] found strong relationship between public sector expenditures and economic growth for U.S.A. Masuo [22] had tested the Wagner’s law for Japan for the period of 1960-1991. Two Stage Least Squar (2SLS) method was used and no significant relation was found between the increased economic activities and the government expenditures.

Ashworth [23], Henrekson [7], Murthy [24], Ahsan et al. [25], Hayo [26], Biswal et al. [27], Islam [28], Burney [29] and Wahab [30] used the Engle and Granger methodology and the Johansen techniques to test the Wagner’s Law. Different results, in favour and against, were found.

The theoretical association between trade and economic growth had been argued for over two centuries. Iyoha [31] elaborated that a number of theories had been put forward to change characteristics of the modern theory of trade. It comprised the Linder theory of external trade, the size and distance theory of external trade proposed by Linnemann and Tinbergen, and the Vent- For-Surplus Theory, originated by Adam Smith but it was transformed and applied to developing countries by Hla Myint. Thirlwall [32] presented three main models of export-led growth hypothesis: the Neoclassical Supply-Side Model; the Balance of Payments Constrained Model and the Virtuous Circle Model.

There are large number of empirical studies that validate the well-built relationship between exports and economic growth. Pragmatic studies can be separated into two parts to evaluate the association between export and economic growth. First group used cross-country analysis, of which key contributions are: Feder [33], Ram [34], Sheehy [35], Edwards [10], and Ngoc et al. [36]. Most of the studies are inclined to verify the significance of exports for emerging economies. However, Clarke and Kirkpatrick [37] employed pooled data for 80 developing countries from 1981-1988 to calculate the effect of trade reform on the economic growth and concluded that trade reform did not affect economic performance. Akbar and Naqvi [38] analyzed the relationship between export diversification and economic growth in Pakistan from 1972-1998. There was a significant positive relationship between export diversification and economic growth. Though, the direction of causation ran from economic growth to export diversification.

III. METHODOLOGY

The study is planned to find out short and long run impact of the government expenditure on the GDP [Wagner’s law, (Peacock and Wiseman, [8] version)], population and export.
For this purpose cointegration and causality tests among these variables will be analyzed. It is important to check each time series for stationarity. If a time series in non-stationarity, the regression analysis done in a conventional way will produce spurious results. Therefore, in order to examine this property of time series, the unit root test is conducted.

A time series is considered to be stationary if its mean and variance are self-regulating of time. If the time series is non-stationary, i.e., having a mean and or variance changing over time, it is said to have a unit root. Therefore, the stationarity of a time series is examined by conducting the unit root test.

### A. Unit Root Test

Dickey-Fuller (DF) [39], Augmented Dickey-Fuller (ADF) [40], Phillip-Perron (PP) [41] and NG Perron [42] test methods are generally used to check the stationarity of a time series. In this study Kwiatkowski, Phillips, Schmidt, and Shin (KPSS) [43] method is used to test the stationarity.

The KPSS test is change from the other unit root tests in terms of hypotheses. The KPSS assumes stationary under the null hypothesis and vise versa. The statistic is based on the residuals from the OLS regression of $y_t$ on the exogenous variables $x_t$.

$$y_t = x_t/\delta + \mu_t$$  \hspace{1cm} (1)

The LM statistic is elaborated as:

$$LM = \sum S(t)^2 / (T^2 \hat{f} 0)$$  \hspace{1cm} (2)

where $\hat{f} 0$ is an estimator of the residual at zero occurrence and $S(t)$ is a collective residual function:

$$S(t) = \sum \mu^r$$  \hspace{1cm} (3)

Based on the residuals $\mu^r = y_t - x_t /\delta (0)$

### B. Cointegration

The main objective of the study is to analyze the impact of the GDP, Exports and Population on government expenditure as a GDP ratio for the economy of Pakistan. For this purpose, the study of the long run relationships among time series is necessary in exposing the exact representation of this impact. This part is dedicated to briefly elucidate the methodological framework to examine the short and long-term relationship among different time series connected to this study. This section discusses the concept of cointegration and the causes for utilizing the autoregressive distributed lag (ARDL) modeling approach to cointegration.

The idea of cointegration is related with the long run equilibrium relationship between two or more variables. The economic understanding of cointegration was that if two or more variables were related to form an equilibrium relationship spanning the long run, even though the series themselves in the short run may deviate from the equilibrium, they would move closer together in the long run equilibrium [44].

There are several methods available for conducting the cointegration test. The most widely used methods include the residual based Engle-Granger [45], and maximum likelihood based Johansen [46], [47] and Johansen-Juselius [48] test. Due to the low power and other problems associated with these tests, the OLS based ARDL approach to cointegration has become popular.

The ARDL based cointegration was proposed by Pesaran and Pesaran [49], Pesaran and Smith [50] and Pesaran and Shin [51]. The main advantage of this approach is that it can be applied irrespective of whether the regressors are I(0) or I(1) [49]. Another advantage of this approach is that the model takes sufficient numbers of lags to capture the data generating process in a general to specific modeling framework [52]. Moreover, a dynamic error correction model (ECM) can be derived from ARDL through a simple linear transformation [53]. The ECM integrates the short-run dynamics with the long-run equilibrium without losing long-run information. It is also argued that using the ARDL approach avoids problems resulting from nonstationary time series data [52].

In order to observe the impact of GDP growth, exports and population on government expenditure as a ratio of GDP, the following model is developed:

$$(GEXP)_t = \alpha_0 + \alpha_1(GDP)_t + \alpha_2(POP)_t + \alpha_3(X)_t + \epsilon_t$$  \hspace{1cm} (4)

where

- $t$ is time period
- $GEXP_t$ is Government Expenditure
- $GDP_t$ is Gross Domestic Product
- $POP_t$ is Population
- $X_t$ is Export
- $\alpha_0$ is intercept, $\alpha_1$, $\alpha_2$, $\alpha_3$ and are coefficients of explanatory variables
- $\epsilon_t$ is usual error term and independent from all explanatory variables.

The ARDL specification of above equation is as under to find empirical evidence of long run equilibrium:

$$(1 - L)(GEXP)_t = \alpha_0 + \sum_{i=1}^q \alpha_{i1}(1 - L)(GEXP)_{-i} + \sum_{i=1}^q \alpha_{i2}(1 - L)(GDP)_{-i} + \sum_{i=1}^q \alpha_{i3}(1 - L)(POP)_{-i} + \sum_{i=1}^q \alpha_{i4}(1 - L)(X)_{-i} + \lambda_0(GEXP)_{-i} + \lambda_1(GDP)_{-i} + \lambda_2(POP)_{-i} + \lambda_3(X)_{-i} + \epsilon_t$$  \hspace{1cm} (5)

where $\alpha, \lambda$ are intercept and coefficients of first differenced variables, $\lambda$s contain long run information and $\epsilon_t$ is usual error term in above equation. The first step in the ARDL based cointegration approach is to estimate equation (5) using OLS. The second step is to trace the presence of cointegration...
by restricting all estimated coefficients of lagged level variables equal to zero. The following hypothesis is tested for cointegration in above equation by the mean of F-statistic with an asymptotic non-standard distribution:

For equation (5) \[ H_0: \lambda_{10} = \lambda_{11} = \lambda_{12} = \lambda_{13} = 0 \text{ (no cointegration)} \]
\[ H_1: \lambda_{10} \neq \lambda_{11} \neq \lambda_{12} \neq \lambda_{13} = 0 \text{ (cointegration)} \]

Two asymptotic critical value bounds give a test for cointegration when the independent variables are \( I(d) \) with 0 ≤ \( d \) ≤ 1. The lower bound presupposes that all the regressors are \( I(0) \) and the upper bound assumes that they are \( I(1) \). If the computed F-statistics lies greater than the upper level of the bound, the null hypothesis is rejected, indicating cointegration. If the computed F-statistics lies beneath the lower level band, the null cannot be rejected, supporting the nonexistence of cointegration. If the F-statistics fall within critical bounds, conclusion would be inconclusive.

IV. EMPIRICAL RESULTS

A. Unit Root Test

KPSS unit root test is utilized for the stationarity of \( GEXP_t, GDP_t, POP_t, \) and \( X_t \) at level and then first difference of each series in table II. The results of KPSS unit root test indicate that at level \( GEXP_t, GDP_t, \) and \( POP_t \) are stationary and both \( GDP_t \) and \( POP_t \) contain unit root. However, all these variables are stationary at first difference. As the variables have mixed order of integration at level, the most suitable methodology is ARDL cointegration for long run equilibrium.

B. The Existence of Short and Long Run Equilibrium

The long run equilibrium can be checked using standard F-statistic. The calculated F-Statistic of equation 5 is 14.1864 to see the long run equilibrium based on ARDL bounds cointegration technique. The critical lower and upper bounds at 95% and 90% are 3.5934, 4.8908 and 2.9357, 4.0778 respectively. The calculated F-statistic exceeds the upper critical bounds both at 95% and 90% level of significance which indicates a strong long run relationship between \( GEXP_t, GDP_t, POP_t, \) and \( X_t \). The long run cointegrating vector is reported in table III. The variable \( GDP_t \) is significant which depicts that Wagner’s Law does hold in the economy of Pakistan. On the other hand, \( POP_t \) and \( X_t \) are significant and have positive correlation with \( GEXP_t \).

The diagnostic tests carried out on long run vector indicate that there exists serial correlation in the model. As the time series data is stationary at mixed order \( I(0) \) and \( I(1) \), so the presence of serial correlation does not effect the robustness of model [52].

The results of short run equilibrium are reported in table IV. The coefficient of \( ECM_{t-1} \) is statistically significant at 1% level and has correct sign (negative). This significance also confirms the short run cointegration between \( GEXP_t, GDP_t, POP_t, \) and \( X_t \). The coefficient of \( ECM_{t-1} \) is -0.7082, which shows speed of adjustment to the long run equilibrium after a short run shock about 70.82 percent (approximately 1 year and 5 months).

<table>
<thead>
<tr>
<th>Variables</th>
<th>Intercept</th>
<th>Trend and Intercept</th>
<th>Intercept</th>
<th>Trend and Intercept</th>
</tr>
</thead>
<tbody>
<tr>
<td>( GEXP_t )</td>
<td>0.95868*</td>
<td>0.07513*</td>
<td>0.18403</td>
<td>0.09414*</td>
</tr>
<tr>
<td>( GDP_t )</td>
<td>1.84347*</td>
<td>0.30254*</td>
<td>0.94552*</td>
<td>0.61348**</td>
</tr>
<tr>
<td>( POP_t )</td>
<td>1.86919*</td>
<td>0.46046*</td>
<td>0.94445*</td>
<td>0.15427**</td>
</tr>
<tr>
<td>( X_t )</td>
<td>0.93547*</td>
<td>0.14399*</td>
<td>0.32113**</td>
<td>0.062205*</td>
</tr>
</tbody>
</table>

Note: * and ** the variables are significance at 10 percent level and 5 percent level.

V. CONCLUSION

In this study, the relationship between government expenditures, population, economic growth and exports is examined. This study contributes in two important ways. First, following the modern inclination, Wagner’s Law, population, and export is evaluated for the period of 1972 to 2007 and provide wide-ranging analysis. Second, a comparatively better time series technique i.e. ARDL bounds testing for cointegration is competent of finding relationships that might not be found using more traditional methods.

The product of bound testing shows that there is a long run cointegration between under investigation variables. Population and exports have significant relation with the government expenditures. With the increase in population and exports, the government expenditure is increased. However, it is found that Wagner’s law does hold as GDP has positive and significant relation with the government expenditures. The coefficient of \( ECM_{t-1} \) is statistically significant at 1 percent level of significance. It has correct coefficient sign which depicts that the speed of adjustment to the long run equilibrium is 70.82 percent within a year.
In the light of the empirical results, it may cautiously suggest that the increase of public expenditures in the case of Pakistan does rely on economic growth as Wagner’s law states. So, the public expenditures are the results of many assessments in the light of changing economic circumstances. In the same direction, Klein [53] as well suggests about the decisions that how public expenditures should be distributed among competing groups, whether geographically concentrated or aggregated in organized interests.

REFERENCES


