Effect of a Linear-Exponential Penalty Function on the GA’s Efficiency in Optimization of a Laminated Composite Panel

A. Abedian, M. H. Ghiasi, B. Dehghan-Manshadi

Abstract—A stiffened laminated composite panel (1 m length × 0.5m width) was optimized for minimum weight and deflection under several constraints using genetic algorithm. Here, a significant study on the performance of a penalty function with two kinds of static and dynamic penalty factors was conducted. The results have shown that linear dynamic penalty factors are more effective than the static ones. Also, a specially combined linear-exponential function has shown to perform more effective than the previously mentioned penalty functions. This was then resulted in the less sensitivity of the GA to the amount of penalty factor.

Keywords—Genetic algorithms, penalty function, stiffened composite panel, finite element method.

NOMENCLATURE

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
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<tbody>
<tr>
<td>P(\mathbf{X})</td>
<td>Penalty function</td>
</tr>
<tr>
<td>F(\mathbf{X})</td>
<td>Fitness function</td>
</tr>
<tr>
<td>\alpha</td>
<td>Penalty factor (P.F)</td>
</tr>
<tr>
<td>GN</td>
<td>Generation number</td>
</tr>
<tr>
<td>\phi(\mathbf{X})</td>
<td>Objective function (Obj.)</td>
</tr>
<tr>
<td>\mathbf{X}</td>
<td>Design variable vector</td>
</tr>
<tr>
<td>N_{con}</td>
<td>Number of constraints</td>
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</table>

ABBREVIATIONS

<table>
<thead>
<tr>
<th>Abbreviation</th>
<th>Description</th>
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<tbody>
<tr>
<td>G.N.:</td>
<td>Generation number</td>
</tr>
<tr>
<td>P.F:</td>
<td>Penalty Factor</td>
</tr>
<tr>
<td>R.P:</td>
<td>Reproduction Period</td>
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I. INTRODUCTION

Using advanced materials, tools, and technologies in the design of real engineering problems makes the design more and more complex. For instance, using composite materials in design of structural components has significantly increased the complexity of the design process. The variety in ply thickness and orientation have widened the design space and thus prohibited the use of general design rules. The optimum design of laminated composite plates and panels that widely used in aerospace and marine applications, involves large, non-convex, and integer-programming problem that is discrete in nature and includes a substantial number of design variables like the geometry parameters, ply composition, number and shape of stiffeners, layers stacking sequences, and so on. In these applications, engineers can neither tolerate the additional weight required for larger safety factors nor the dangers implied by small safety factors. High reliability of the components must be assured by detailed analysis and accurate optimization in order to meet minimum weight requirements, while satisfying strength constraints [1, 2].

The first important characteristic of design and optimization of stiffened composite panels is that the solution sought should be the global optimum solution. However, it is usually obscured among large number of local optimums. Also, the discrete nature of the design variables and the non-linearity of the constraints add to the difficulties involved. Therefore, for easy dealing with the problems of this kind, the GA is suggested as the best choice for searching the global optimum in such a large, nonlinear and discrete space [3-5]. The Genetic algorithm, which is a searching technique based on the Darwinian Principle of Natural Selection (firstly introduced by J. Holland in 1975) is now used as a tool for searching the large and poorly-understood spaces that arise in many areas of science and engineering [6]. Also, with attention to these facts and reviewing the available literature, the GA is found well suited for the design and optimization of laminated composite plates [7-10].
Normally, real engineering problems are considered as constraint problems, however application of Genetic Algorithm (GA) to such problems is often a challenging endeavor. Thus, some external penalty functions have been traditionally used to convert a constrained optimization problem into an unconstrained one. This approach requires a somewhat arbitrary selection of penalty draw-down coefficients. These factors are strongly problem-dependent and there is not any defined standard way to select the best values for them. In this research work, in order to find the best fitted penalty factor for these kinds of problems, several static and dynamic penalty functions are utilized with variety of values for the draw-down coefficients. Finally, a combination of the linear and exponential penalty function is suggested, in order to introduce a penalty factor that is less sensitive to the amount of penalty factors.

II. GENETIC ALGORITHMS (GAS)

The genetic algorithm is a searching technique based on ideas from the science of genetics and processes of natural selection. A simple genetic algorithm includes a number of solutions for the problem under consideration, which are decoded into binary strings called chromosomes. Each particular element of a chromosome is called a gene and represents a value of a specific design variable. These variables are then used to evaluate the corresponding fitness value that is found from the fitness function, which is itself related to the objective and penalty functions. Then a particular selection method based on the fitness value of the chromosomes is used to select the next population. The selection method is conducted in a way that obtains more selection probability for the better chromosomes. The reproduction and selection process will be continued until the stopping criteria are met and the optimum or near-optimum solution is found.

III. PENALTY FUNCTION IN EVOLUTIONARY METHODS

In a typical evolutionary optimization problem, constraints are usually handled by means of penalty functions, which penalize infeasible solutions by reducing their fitness values (ensured by reduction in the probability of selection) in proportion to their amount of constraint violation.

The penalty functions for problems with inequality constraints are divided into two categories, namely, the exterior and the interior functions. For the interior penalty functions, the penalty term is chosen such that the constraints act as barriers during the optimization process, that force the generated searching points to always lay down within the feasible domain [11]. This is why the interior penalty functions could not be used with GA optimization methods. But, for the exterior penalty functions a penalty value is added to the violating solutions considering the number of violated constraints and their distance from the feasible domain. By means of these penalty values it will be possible to use the infeasible solutions in order to steer the optimization progress toward the optimum solution(s). Eq.1 depicts the general form of a penalizing function,

$$F(\vec{x}) = \phi(\vec{x}) + \sum_{i=1}^{N_i} \alpha_i \times P_i(\vec{x})$$  \hspace{1cm} (1)

where $F(x)$ is the fitness function (or Penalized Objective Function), $P_i(\vec{x})$ is the penalty function, and $\alpha_i$ are positive constants (or rising factors) normally called “penalty factors”.

In all available penalty schemes, the degree of penalty can be further controlled by the way that various coefficients ($\alpha_i$) in penalty functions are set. Most of these coefficients are treated as constants during the calculation and their values have to be specified at the beginning of the process. The penalty functions with these coefficients are normally referred to as the “Static External Penalty Functions (SEPF)”. “Death Penalty Function” (DPF) is the simplest and most common form of the “SEPF”, and corresponds to infinite penalty value for violating chromosomes. Regarding to the probability of selection, which is calculated based on the penalized objective values; these chromosomes find no chance to remain in the optimization process. The main idea in using the static and dynamic penalty factors instead of “Death Penalty” is to give this opportunity to the least optimum solutions to find the opportunity to transform their desirable aspects to the subsequent reproduced populations. The population directly ensured by this methods, would increase the probability of having wider variety in the newly reproduced children.

Since, the static penalty schemes do not provide appropriate penalty strength during the optimization, the dynamic penalty factors have been suggested to balance the strength of the penalty function [5, 12]. One of these schemes is called "Linear Dynamic External Penalty Function" (LDEPF). In this scheme, the penalty factor is linearly increased with the generation number. In case of selecting an inappropriate penalty factor, the GA may converge to either non-optimal feasible solution (for high penalty values) or to infeasible solutions (for very small penalty values) [12, 13].

IV. PROBLEM DESCRIPTION

The design problem under consideration consists of a laminated composite plate of (1 m length × 0.5m width), which is reinforced with stiffeners and subjected to a concentrated load (P=1000 N) at the midpoint of the plate as depicted in Fig. 1. The symmetric laminated composite plate is assumed to be made up of Glass-Epoxy layers. Each ply in the stacking sequence is allowed to be oriented at any angle between 0° and 90° in increments of 15 degrees. The stiffeners are considered to be running along the length of the plate. The number, shape, and thickness of the stiffeners are considered as design parameters. Fig. 2 illustrates the assumed cross sections of the stiffeners, and the corresponding mechanical properties of the constituents are
presented in Table 1. Also, Table 2 summarizes the meaning of each gene in a typical chromosome.

<table>
<thead>
<tr>
<th>Allowable Range</th>
<th>Lower Limit</th>
<th>Increment</th>
<th>Upper Limit</th>
<th>Binary Length</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1</td>
<td>1</td>
<td>14</td>
<td>4</td>
</tr>
<tr>
<td>2</td>
<td>0.1</td>
<td>0.1</td>
<td>1.6</td>
<td>4</td>
</tr>
<tr>
<td>3</td>
<td>1</td>
<td>-</td>
<td>1</td>
<td>0</td>
</tr>
<tr>
<td>4</td>
<td>L, Z, T, Hat</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>5</td>
<td>2</td>
<td>1</td>
<td>9</td>
<td>3</td>
</tr>
<tr>
<td>6</td>
<td>0.1</td>
<td>0.1</td>
<td>1.6</td>
<td>4</td>
</tr>
<tr>
<td>7-20</td>
<td>0°</td>
<td>15°</td>
<td>90°</td>
<td>3</td>
</tr>
</tbody>
</table>

Finding the minimum weight and deflection are the main goals (objectives) of the optimization problem considered here. The panel is designed for resisting failure under excessive strains caused by a particular set of design loads and boundary conditions. The weighting parameters in multi-objective function are chosen regarding to the significance of each state variable in a real-engineering problem. Hence, the objective function considered here has the following form;

\[
\varphi(x) = 1 - \left(0.8\bar{w} + 0.2\delta\right);
\]

\[
\bar{w} = w / w_{\text{max}} = w / 100(\text{kg});
\]

\[
\delta = \delta / \delta_{\text{max}} = \delta / 10(\text{mm});
\]

The constraints imposed are as follows:
1. Safety factor must be greater than 1.2
2. The weight must remain under 100 kg
3. The maximum allowable deflection is 10 mm (at the midpoint of the panel)

Here, different ways are utilized to deal with the violating designs. First of all, the DPF is applied to eliminate the infeasible chromosomes. For the second method, some penalty factors are utilized to slightly penalize the infeasible chromosomes. The penalty factors are considered to remain constant during the entire evolutionary process (SEPF). However, for the third method the current generation number is involved in the computation of the corresponding penalty factors (LDEPF), and the amount of the penalty factor is linearly increased in subsequent generations with increasing the generation number. Finally, for the fourth method, an especially combined form of the penalty factor is utilized. Here, the mentioned penalty factor is linearly increased in early generations related to the "Reproduction Period" (R.P) and subsequently in higher generation numbers which is referred to the “Filtration Period” (F.P) the amount of the penalty factors is exponentially increased. In this way, in R.P the variety of the searching domain and in F.P the feasibility of the final population are increased.

V. RESULTS AND DISCUSSION

The standard form of the defined constrained optimization problem and the related penalty factors are shown in Eq. 3, 4, and 5. The weighting factors in the penalty function are selected regarding the advantages and disadvantages caused by violation of the constraints, where the details could be found in [14].

In the present study, for better understanding of the effects of the penalty factor in leading toward the optimum design in a GA process, the DPF, SEPF (with three constant values of \(\alpha\)), LDEPF (with two linearly changing values of \(\alpha\)), and LEDEPF (with three linear-exponential changes in \(\alpha\)) are considered.

It is noted that for the optimization analysis, a GA code has been developed where the details could be found in [15]. This code is capable of linking with Ansys FEM software for calculating the stress and deformation analysis. As for the meshing, the 100 layers composite element of Ansys “Shell99” was employed.

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It is noted that whenever the chromosome is feasible, the penalty would be zero and as a result the fitness and objective values would be the same (Eq. 4).

A. DPF IN COMPARISON WITH SEPF

In the first step the performance of the (SEPF) method (with the constant penalty factor of unity) is compared with the simple and widely used death penalty function (DPF). Fig. 3 presents the maximum values of the penalized objective values for the DPF and SEPF with different penalty factors during 25 generation cycles, and Fig. 4 shows the objective value of all the feasible chromosomes obtained in generation 25th for both of the mentioned penalty cases. These figures confirm the superiority of the SEPF method over the DPF in obtaining a better local optimum.

B. SEPF WITH THE DIFFERENT PENALTY FACTORS

In order to find the effects of the value of the constant penalty factor in the SEPF method, three different values of \( \alpha = 0.5, 1, \) and 2 are considered, which the corresponding analyses are marked here as SEPF I, SEPF II, and SEPF III, respectively. Fig. 5 depicts the maximum value of the objective function obtained in each generation cycle, and Fig. 6 shows the objective value for the feasible chromosomes in the last population for the three cases. Comparing these figures reveals that the amount of the fitness function is improved by lowering the value of the penalty factors (see Eq. 4), however the number and objective value of the feasible chromosomes in the last population are clearly decreased. The results indicate that finding the best penalty factor for a problem is not clearly defined or it is very much problem dependent. For instance in this case, the SEPF II performs more efficient regarding to its gradually improvement in max objective value than the SEPF III (\( \alpha = 2 \)) which achieved a better solution regarding the amount of objective value, see Figs. 5 and 6.

C. LDEPF IN COMPARISON WITH SEPF

As it was previously mentioned, the dynamic penalty function is used in order to adjust the amount of penalty factor during the optimization process. Therefore, in addition to the previous cases two other penalty factors, which are linearly increased with increasing the generation number, are employed in this step. Equation 6 presents these dynamic penalty factors.

\[
\text{LDEPF I} \rightarrow \alpha = \text{G.N.} / 10 \\
\text{LDEPF II} \rightarrow \alpha = \text{G.N.} / 5
\]

The maximum fitness value of the population during the 50 optimization generations and the objective values of the feasible chromosomes of the last generation are illustrated in Figs. 7 and 8, respectively. Also, the best results obtained in the case of SEFP (i.e. SEPF II) are added to these graphs in order to provide a better comparison between the results of the mentioned methods. The graphs illustrate that the fitness
values for the two dynamic cases are gradually increased with increasing the generation number, but in comparison to the results of the best static case (shown in Fig. 7) the obtained fitness values remain under the statically found values. However, the comparison between the objective values of the feasible chromosomes of the last population as shown in Fig. 8 indicates that the second dynamic case (i.e. LDEPF II) obtain the better solution than the other cases.

It is very important to note that the better performance of the second dynamic case is over shadowed by its high sensitivity to amount of the penalty factor. It is best explained when one compares the results in Fig. 8 for the two dynamic cases.

The objective value of the feasible chromosomes for the three cases of $\alpha$ for LDEPF are presented in Fig. 10. Also, for a better comparison, the values of the same quantity for the two linear penalty factors that previously discussed for the case LDEPF are shown in Fig. 11. The figures clearly show that the sensitivity to the amount of penalty factor is highly reduced by means of the newly introduced function for the penalty factor. Interestingly, the number of feasible chromosomes in the last population also shows a big increase. In fact, as Fig. 10 shows, all the chromosomes in the last population are feasible.
Fig. 12 clearly highlights the superiority of the LEDEPF method over the other methods. The maximum objective values obtained by the three cases of the LEDEPF method reach the optimum value after 20 generations. A review of the feasibility of the chromosomes at generation 40 shows that only a few of them are feasible. Interestingly, the results at generation 50 show that all the chromosomes are feasible. This could be attributed to the fact that from this stage, the exponential part of the penalty factors comes into effect.

It must also be mentioned that based on Figs. 9 and 10, a better optimum value for the objective function is obtained by the LEDEPF method, as well. This is more understood when one compares the weight obtained for the stiffened panel under all of the applied penalty factors. As table 3 shows, the weight for SEPF reduces to 18.09 kg from 25.59 with increasing the value for $\alpha$. This reduction in weight continues when the dynamic penalty factors are applied, see table 4. For example, the weight reduces to 16.5 kg for $\alpha = 0.5$. Applying the linear-exponential penalty factor reduces the weight even further. The obtained weight in this way reaches to 15.45 kg, see table 6.

VI. CONCLUSION

Comparison of the results obtained by the four methods (i.e. DPF, SEPF, LDEPF, and LEDEPF) shows that:

1. The DPF reduces the variety in characteristics of the chromosomes in the early generations due to the sever filtration performed by this method, and as a result the chance for production of better chromosomes in subsequent generations highly reduced.

2. Application of penalty functions with static penalty factors (SEPF) could greatly affect the results. It is shown that the SEPF method could, for example, reduce the weight considerably and with increasing the penalty factors this trend continues even further.

3. Though the SEPF method could secure the variety in the characteristics of the chromosomes during the subsequent generations, however, production of feasible chromosomes due to the constant value of the penalty factor is not guaranteed.

4. The above shortcoming of the SEPF method is partially overcome using LDEPF penalty factor. In this way
better optimum solution and also higher number of feasible chromosomes is obtained, but the dependency of the results to the value of $\alpha$ is highly pronounced.

5. The influence of $\alpha$ on the results could highly be reduced by incorporating LEDEPF method. Since in the final generation of the process the exponential part of the penalty factor comes into the effect, the very high values of $\alpha$ obtained in this stage could greatly guarantee the feasibility of all the chromosomes in the last population.

6. The main and very interesting result obtained in this study could be attributed to the reduction of the sensitivity of the results to the values of $\alpha$.

7. Considering the above results, it could be mentioned that the existed blindness regarding the way of selection of $\alpha$ for an unknown problem is now a bit brightened.

REFERENCES


