Neural Network Tuned Fuzzy Controller for MIMO System

Seema Chopra, R. Mitra, Vijay Kumar

Abstract—In this paper, a neural network tuned fuzzy controller is proposed for controlling Multi-Input Multi-Output (MIMO) systems. For the convenience of analysis, the structure of MIMO fuzzy controller is divided into single input single-output (SISO) controllers for controlling each degree of freedom. Secondly, according to the characteristics of the system’s dynamics coupling, an appropriate coupling fuzzy controller is incorporated to improve the performance. The simulation analysis on a two-level mass–spring MIMO vibration system is carried out and results show the effectiveness of the proposed fuzzy controller. The performance though improved, the computational time and memory used is comparatively higher, because it has four fuzzy reasoning blocks and number may increase in case of other MIMO system. Then a fuzzy neural network is designed from a set of input-output training data to reduce the computing burden during implementation. This control strategy can not only simplify the implementation problem of fuzzy control, but also reduce computational time and consume less memory.

Keywords—Fuzzy Control, Neural Network, MIMO System, Optimization of Membership functions.

I. INTRODUCTION

The majority of process industries are nonlinear, Multi-Input Multi-Output (MIMO) systems. The control of these systems is met with a number of difficulties due to process interactions, dead time and process nonlinearities. The difference between MIMO systems control and single-input single-output (SISO) systems control is based on an estimation and compensation of the process interaction among each degree of freedom. It is obvious that the difficulty of MIMO systems control is how to overcome the coupling effects among each degree of freedom. To obtain good performance, coupling effect cannot be neglected. Hence SISO system control scheme is not easy to implement on complicated MIMO systems. In addition, the control rules and controller computation will grow exponentially with respect to a number of considered variables. Therefore, intelligent control strategy is gradually drawing attention.

Fuzzy systems and neural networks-based control methodologies have emerged in recent years as a promising way to approach nonlinear control problems [1], [2], [4], [6]. Fuzzy and neural control, in particular, has had an impact in the control community because of their simplicity and feasibility to use heuristic control knowledge for control problems. The integration of fuzzy logic with neural network techniques has resulted in what is commonly referred to as neuro-fuzzy systems. These systems use fuzzy rules as the underlying structure and then apply neural techniques to learn the rule parameters, e.g., the input region covered by each rule and the output value of each rule.

Recently, many analysis results and design methodologies of fuzzy system and neural network have been reported. Most of the reported research however only focused on SISO systems [11], [13], [17], [21]-[26]. The MIMO systems usually possess characteristics of nonlinear dynamics coupling [1]-[6]. Therefore, the difficulty of MIMO systems control is how to overcome the coupling effects among each degree of freedom [4]. The structure of the MIMO controller can be divided into multi-input single output (MISO) and SISO controllers. Each MIMO controller then consists of many fuzzy logic controllers (FLC). It is clear that the control structure of the fuzzy control system is very complicated when the input variable is multi-degree and the output variable is one degree, or more than one degree. These parameters of a fuzzy control system are not easily decided because the fuzzy control rules will be grow as a geometric series, and much computing time will be required. To minimize the amount of memory used and computational time, we can put constraints on the type of fuzzy controller (e.g., membership functions) or limit the rules. But it will affect the performance of the system; hence we need the solution which exhibits good performance with smallest possible rule base. In our previous research [14], [16], [29]-[32] for the reduction of rules for fuzzy controllers we used Fuzzy curve, Fuzzy subtracting clustering (FSC), neural networks and neurofuzzy techniques. Neurofuzzy learning [30] is one of the fast and effective method to generate the suitable initial membership functions and shortest rule base from input/output data. Due to the above problems, we are motivated to design an effective neural network based tuned fuzzy controller for MIMO systems, which provides acceptable solution.

There are two challenging design issues to be addressed in designing fuzzy system, viz., structural and parameter identification. The structure identification amounts to determine the proper number of rules needed i.e. finding how many rules are necessary and sufficient to properly model the available data and the number of membership functions for input and output variables. Parameter learning phase is used to
tune the coefficients of each rule, like the shape and positions of membership functions. Fast computation speed is attained by requiring much less tunable parameters. There is a need for effective methods for tuning the membership functions so as to minimize the output error measure or maximize performance index. For structure identification, different researchers [8]-[11], [24]-[26], [28] use different methods to extract initial fuzzy rules from given input-output data. Clustering techniques [8]-[11], [25], [28] have been recognized as a powerful alternative approach to develop fuzzy systems. Clustering of numerical data forms the basis of many classification and system-modeling algorithms. The purpose of clustering is to identify natural grouping of data from a large data set to produce a concise representation of a system’s behavior. Clustering algorithms typically require the user to prespecify the number of cluster centers and their initial locations. The preceding discussion shows that different researchers have used different clustering algorithms to decide the number of rules. A clustering method called subtractive clustering forms the basis of the present work. For parameter identification, a network is trained by hybrid learning algorithm that is the mixture of a back propagation and least square algorithm. This algorithm iteratively learns the parameter of the premise membership functions via back propagation and optimizes the parameters of the consequent equations via linear least-squares estimation. The authors chose hybrid approach because it is much faster and more accurate than gradient decent as described in ANFIS by Jang in 1993 [17].

In this paper, a neural network tuned fuzzy controller is designed for MIMO systems from the given set of input-output data. An appropriate coupling tuned fuzzy controller is incorporated to control the MIMO systems to compensate for the dynamics coupling among each degree of freedom. A tuned fuzzy controller is obtained from data set in two steps. First, the data set is partitioned into a set of clusters based on the similarity of data. Then using subtractive clustering algorithm a fuzzy if-then rule is extracted from each cluster to form a fuzzy rule base. Secondly, a fuzzy neural network is designed accordingly to optimize the parameters of the fuzzy system. After simulation of a two-level mass–spring MIMO system and comparison of results, it can be seen that the computational time and memory is reduced substantially in case of tuned fuzzy controller and the performance is also identical.

II. CONTROLLER STRUCTURE FOR MIMO SYSTEM

Fuzzy set theory and neural network has been successfully applied in a number of control applications [12]-[19], [21], [24], [27], [28] based on the SISO system point of view without system model consideration. In this paper, the fuzzy control strategy is used to control MIMO systems. The block diagram of the MIMO fuzzy control scheme is shown in Fig. 1. The design procedure of the fuzzy control strategy is used to control each degree of freedom of this MIMO system individually. Then, an appropriate coupling fuzzy controller is designed to compensate for the coupling effects of system dynamics among each degree of freedom.

An ordinary fuzzy controller that usually operates with system output error and error change was chosen as the main controller to control each degree of freedom of the MIMO systems. Here, the input variables of the conventional fuzzy controller for among each degree of freedom of a MIMO system were defined individually as

\[
e_i(k) = R_i(k) - Y_i(k)
\]

\[
\Delta e_i(k) = e_i(k) - e_i(k-1)
\]

where \(e_i(k)\) is the position error of the \(i^{th}\) degree, \(\Delta e_i(k)\) is used for change in error of the \(i^{th}\) degree, \(R_i(k)\) is the reference input of the \(i^{th}\) degree and \(Y_i(k)\) represents the \(i^{th}\) position output of each degree of freedom of this MIMO system at the \(k^{th}\) sample.

The relationship between the scaling factors (SFs) \((G_e, G_{\Delta e}, G_u)\) are the input and output variables of the FLC is

\[
e_{iN} = G_e \times e_i, \quad \Delta e_{iN} = G_{\Delta e} \times \Delta e_i, \quad \Delta u_i = G_{\Delta u} \times \Delta u_{iN}
\]

Selection of suitable values for \(G_e, G_{\Delta e}\) and \(G_u\) are made based on the knowledge about the process to be controlled and sometimes through trial and error to achieve the best possible control performance. This is so because, unlike conventional nonfuzzy controllers to date, there is no well-defined method for good setting of SF’s for FLC’s. The SFs are the significant parameters of FLC because changing the SFs changes the normalized universe of discourse, the domains, and the membership functions of input/output variables of FLC.

All membership functions (MFs) for controller inputs (i.e., \(e_i\) and \(\Delta e_i\)) and incremental change in controller output (i.e., \(\Delta u_i\)) are defined on the common normalized domain \([-1,1]\). We use symmetric triangles (except the two MFs at the extreme ends) with equal base and 50% overlap with neighboring MFs as shown in Fig. 2. This is the most natural and unbiased choice for MFs.

By way of the above design process, the actual control input
voltage for the main fuzzy controller can be written as

\[ u_i(k) = u_i(k-1) + \Delta u_i(k) \]  

(4)

In (4), k is the sampling instant and \( \Delta u_i(k) \) is the incremental change in controller output, which is determined by the rules of the form If \( e_i \) is \( E_i \) and \( \Delta e_i \) is \( \Delta E_i \), then \( \Delta u_i \) is \( \Delta U_i \). The rule base for computing is a standard one [27] as shown in Table I.

The fuzzy control rules of the coupling fuzzy controller are similar to the main fuzzy controller. The output of the coupling fuzzy controller is chosen directly as the coupling control input voltage. The main reason is that there is a different coupling effect for each sampling interval and it does not have an accumulating feature. The coupling effect is incorporated into the main fuzzy controller for each step to improve system performance and robustness.

![Diagram of the MIMO Fuzzy Control Scheme](image-url)
Therefore, the total control input voltage of the MIMO fuzzy controller is represented as

\[ U_i(k) = u_i(k) + U(k)_{i \rightarrow M}, \quad i \neq l \] (5)

where \( u_i(k) \) expresses the system control input voltage of the \( i \)th degree of a main fuzzy controller. \( U(k)_{i \rightarrow M} \) represents the coupling effect control of the \( i \)th degree relative to the \( l \)th degree of the coupling fuzzy controller.

\[
\begin{bmatrix}
\dot{x}_1 \\
\dot{x}_2 \\
\dot{x}_3 \\
\dot{x}_4
\end{bmatrix} =
\begin{bmatrix}
0 & 0 & 0 & 0 \\
-K_2 & -B_2 & 0 & 0 \\
M_1 & M_1 & 0 & 0 \\
0 & 0 & 0 & 0
\end{bmatrix}
\begin{bmatrix}
x_1(k) \\
x_2(k) \\
x_3(k) \\
x_4(k)
\end{bmatrix} +
\begin{bmatrix}
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0
\end{bmatrix}
\begin{bmatrix}
u_1(k) \\
u_2(k) \\
u_3(k) \\
u_4(k)
\end{bmatrix}
\] (6)

\[
\begin{bmatrix}
Y_1(k) \\
Y_2(k)
\end{bmatrix} =
\begin{bmatrix}
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0
\end{bmatrix}
\begin{bmatrix}
x_1(k) \\
x_2(k) \\
x_3(k) \\
x_4(k)
\end{bmatrix}
\] (7)

where \( x_1 \) and \( x_3 \) are the displacements of the main and secondary masses \( M_1 \) and \( M_2 \), respectively, \( x_2 \) and \( x_4 \) are the velocities of the mass \( M_1 \) and \( M_2 \), respectively, and \( B_1 \) and \( B_2 \) are the damping coefficients of the main system and the secondary system, respectively. \( u_1 \) and \( u_2 \) are the inputs, \( \Omega \) is a constant, and \( d_1 \) and \( d_2 \) are the disturbance forces applied to the main and the secondary systems, respectively.

The state space of the discrete time system form can be described as

\[
\begin{bmatrix}
Y_1(k) \\
Y_2(k)
\end{bmatrix} =
\begin{bmatrix}
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0
\end{bmatrix}
\begin{bmatrix}
x_1(k) \\
x_2(k) \\
x_3(k) \\
x_4(k)
\end{bmatrix}
\] (7)

where \( Y_1(k) \) and \( Y_2(k) \) are the displacement of the first mass and second mass, respectively. The system parameters of state space model are listed in Table II.

<table>
<thead>
<tr>
<th>( a_{11,1} )</th>
<th>( a_{11,2} )</th>
<th>( a_{12,1} )</th>
<th>( a_{12,2} )</th>
<th>( a_{22,1} )</th>
<th>( a_{22,2} )</th>
<th>( a_{21,1} )</th>
<th>( a_{21,2} )</th>
<th>( b_{21} )</th>
<th>( b_{42} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>-1.9665</td>
<td>0.9681</td>
<td>0.0009</td>
<td>-0.0002</td>
<td>-1.9442</td>
<td>0.9453</td>
<td>0.0061</td>
<td>-0.0052</td>
<td>0.0076</td>
<td>0.0051</td>
</tr>
</tbody>
</table>

The structure of the fuzzy control scheme to control this MIMO active vibration system is shown in Fig. 3. The input variables of the fuzzy controller are defined same as in (4) and (5) when indices \( i=1;2 \) represent the main mass and second mass, respectively.

\[
u_1(k) = u_1(k - 1) + \Delta u_1(k),
\] (8)

\[
u_2(k) = u_2(k - 1) + \Delta u_2(k)
\]

and the total control input voltage of this plant is represented as
In parameter identification, the neural network techniques are used to refine the parameters of the initial fuzzy rules. A neural network with three layers is designed based on the fuzzy rules obtained in first phase. To realize the described fuzzy inference mechanism, the operation of a neural network is shown in Fig 5 and described below:

Layer 1: Units in this layer receives the input value (X1, X2, ..., Xn) and acts as the fuzzy sets representing the corresponding input variable. Nodes in this layer are arranged into j groups; each group representing the IF-part of a fuzzy rule. Node (i, j) of this layer produces its output $O_{ij}^{(1)}$, by computing the corresponding Gaussian membership function:

$$O_{ij}^{(1)} = A_{ij}(X_j)$$

Layer 2: The number of nodes in this layer is equal to the number of fuzzy rules. A node in this layer represents a fuzzy rule; for each node, there are n fixed links from the input term nodes representing the IF-part of the fuzzy rule. Node $O_{ij}^{(2)}$ of this performs the AND operation by product of all its inputs from layer 1. For instance,

$$O_{ij}^{2} = \prod_{i=1}^{n} O_{ij}$$

Layer 3: This layer contains only one node whose output $O_{ij}^{(3)}$ represents the result of centroid defuzzification, i.e.,

$$O_{ij}^{3} = \frac{\sum_{j=1}^{J} O_{ij}^{2} c_j}{\sum_{j=1}^{J} O_{ij}^{2}}$$

Here, $c_j$ is the class of data as discussed above and it is also called the fuzzy singletons defined on output variables. Apparently, $m_{ij}$, $\sigma_{ij}$, and $c_j$ are the parameters that can be tuned to improve the performance of the system. After that a hybrid learning algorithm which combines gradient descent and least square estimator method is used to refine these parameters. Each epoch of the hybrid learning procedure is composed of a forward pass and backward pass. In the forward pass, input data is supplied and functional signals go forward to calculate each node output. The consequent parameters are identified by least square estimator method. After identifying the parameters, the functional signals keep going forward till the error measure is calculated. In the backward pass, the error rates (derivative of the error measure w.r.t. each node output) propagate from the output end towards the input and the premise parameters are updated by gradient method. The details of Hybrid learning algorithm is given by Jang in [17] and we are using the same procedure.

$$U_1(k) = u_1(k) + U(k)_{2 \rightarrow 1}$$

$$U_2(k) = u_2(k) + U(k)_{1 \rightarrow 2}$$

(9)
Each fuzzy controller (main and coupling) in Fig. 3 uses 49 rules and 7 membership functions to compute output. Hence, the design procedure of this main fuzzy controller and coupling fuzzy controller should be simplified to reduce the computing burden during implementation using tuned fuzzy controller. Next, we investigate the following. Given some data describing the output ($\Delta u_i$) as a function of Inputs (i.e., $e_i$ and $\Delta e_i$), now main aim is to extract a smaller set of rules using neurofuzzy learning to do the same. Then, the performance of the simple controller (identified system) is compared with the original one. Now the following steps are followed:

A. Data Generation

To identify the fuzzy controller, some data is needed, i.e., a set of two-dimensional input vectors $X=\{X_1, X_2, \ldots, X_n\}$ and the associated set of one-dimensional output vectors as $Y=\{Y_1, \ldots, Y_n\}$ where $X=\{e_i, \Delta e_i\}$ and $Y=\{u_i\}$ is required. Here, the training data has been generated from first fuzzy controller (with 49 rules) by sampling input variables $e_1$ and $\Delta e_1$ uniformly at the step size of 0.1 and computing the value of $\{u_i\}$ for each sampled point using Matlab programming and fuzzy toolbox of Matlab. The number of data points generated is 441.

B. Rule Extraction and Membership Functions

After generating the data, the next step is to estimate the initial rules. Then after applying Subtractive Clustering algorithm, four clusters (rules) are extracted. The unit step response using these four rules is not so close to the identified system [14,16]. Hence, there is a need of optimization of these rules. Parameter optimization is used for tuning of membership functions to minimize the output error measure or maximize performance index using neural networks. Hybrid learning algorithm is used for training to modify the above parameters after obtaining the Fuzzy inference system from subtracting clustering. This algorithm iteratively learns the parameter of the premise membership functions via back propagation and optimizes the parameters of the consequent equations via linear least-squares estimation. The training is continued until the error measure becomes constant. As the value of these parameters change, the Gaussian membership function varies accordingly. The membership functions after optimization for $e$ are $m_{f1}, m_{f2}, m_{f3}$ and $m_{f4}$, and for $\Delta e$ are $m_1, m_2, m_3$ and $m_4$ shown in Fig 6. Finally the rules are written in the form of: Rule $i$: If $e$ is $m_{fi}$ & $\Delta e$ is $m_i$ then class is $c_i$ when indices $i=1$ to 4. The same set of rule base and membership functions is used for each fuzzy controller in Fig. 3.

C. Results

The neurofuzzy learning has been tested on a variety of linear and nonlinear processes in case of SISO systems [30]. The objective here is to investigate whether the tuned fuzzy controller for MIMO system with less number of rules and membership functions can provide the same level of performance as that of the system with ordinary fuzzy controller. To demonstrate the effectiveness of the proposed combination, the results are reported for system with optimized rule base (system with 4 rules for each controller) and ordinary fuzzy controller (system with 49 rules for each controller). After reducing the rules the computation become fast and it also consumes less memory. The memory is calculated using Windows Task Manager and the computational time is calculated using the Process Explorer–Sysinternals software [33]. The result has been shown in Table III.
In case of MIMO controller, the system with 49 rules for each controller (196 rules) is denoted by Ordinary FLC and system with 4 optimized rules (total 16 rules) is denoted by Tuned FLC. Here it is emphasize that tuned MIMO is called satisfactory only with respect to its closeness to the ordinary MIMO. The level of closeness measured here is on the bases of ISE (Integral Square Error) and closeness of response characteristics as shown in Fig 7 and 8. The value of ISE measured up to 100 sec is 3.7 in case of Ordinary FLC and 3.8 in case of tuned FLC.

Response characteristics in both cases (Tuned FLC and Ordinary FLC) are very close. Even the values of ISE in both cases are also very close.

![Fig. 7 Unit Step Response of output 1 (main mass) of MIMO System](image)

The overall performance of the MIMO Fuzzy Controllers with 4 rules for each fuzzy controller (total 16 rules) is compared with those of 49 rules for each fuzzy controller (196 rules).

### V. CONCLUSION

This paper has described a neural network based tuned fuzzy controller for controlling the each degree of freedom of MIMO systems. The coupling effect is added into the main fuzzy controller for each step to improve system performance. A data set generated is partitioned into a set of clusters based on subtractive clustering method. A fuzzy IF-then rule is then extracted from each cluster to form a fuzzy rule base from which a fuzzy neural network is designed. The neural network designed in this paper is very simple and contains only three layers. A hybrid learning algorithm is used to refine the parameters of fuzzy rule base. The advantages of the discussed tuned fuzzy controller are that it improves performance, decreases complexity, reduces computational time and consumes less memory. Furthermore, the proposed method is used to simulate the two-level mass–spring damper and is able to reduce 196 rules to 16 rules maintaining almost the same level of performance.

### REFERENCES

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