Abstract—Bendability is constrained by maximum top roller load imparting capacity of the machine. Maximum load is encountered during the edge pre-bending stage of roller bending. Capacity of 3-roller plate bending machine is specified by maximum thickness and minimum shell diameter combinations that can be pre-bend for given plate material of maximum width. Commercially available plate width or width of the plate that can be accommodated on machine decides the maximum rolling width. Original equipment manufacturers (OEM) provide the machine capacity chart based on reference material considering perfectly plastic material model. Reported work shows the bendability analysis of heavy duty 3-roller plate bending machine. The input variables for the industry are plate thickness, shell diameter and material property parameters, as it is fixed by the design. Analytical models of equivalent thickness, equivalent width and maximum width based on power law material model were derived to study the bendability. Equation of maximum width provides bendability for designed configuration i.e. material property, shell diameter and thickness combinations within the machine limitations. Equivalent thicknesses based on perfectly plastic and power law material model were compared for four different materials grades of C-Mn steel in order to predict the bend-ability. Effect of top roller offset on the bendability at maximum top roller load imparting capacity is reported.

Keywords—3-Roller bending, Bendability, Equivalent thickness, Equivalent width, Maximum width.

NOMENCLATURE

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<td>a</td>
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<td>F</td>
<td>top roller load (N)</td>
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<td>K</td>
<td>strength co-efficient (N/mm²)</td>
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<td>K'</td>
<td>(2/3)^n+1·K</td>
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<tr>
<td>l</td>
<td>center distance between bottom rollers (mm).</td>
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<td>L</td>
<td>length of roller barrel</td>
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<td>t</td>
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<td>x</td>
<td>horizontal distance of top roller center from left bottom roller center (mm).</td>
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Greek symbols

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Sub-script

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I. INTRODUCTION

The 3-roller plate bending machine are widely used in heavy engineering industries for the manufacturing of skeleton of oil and gas rigs, the construction of tunnels, cylindrical tanks, boiler equipments, fuel tanks for launch vehicle in space application, industrial buildings, pressure vessel, heat exchangers, tall towers, reactors, etc. Fig. 1 shows the six stages of 3-roller plate bending operation sequence for fixed bottom roller gap.

Top roller load required to bend the plate is the function of various parameters viz. plate thickness, plate width, shell diameter to be rolled, plate material property, gap between bottom rollers etc. Machine capacity is limited by the factors such as the size of rollers and horse power of its motor. In the process of continuous roller bending, maximum load is required during the edge pre-bending stage as top roller is at the offset distance from its mid position. So, top roller offset during the pre-bending decides the maximum width and minimum shell diameter combinations that can be rolled for
the particular plate thickness and material property within machine capacity.

Although the roller bending process is widely used in practice, available literatures focusing the efficient utilization of machine capacity are limited. Bassett & Johnson [1] and Hansen & Jannerup [2] reported the theoretical analysis of 3-roller pyramid type plate bending process. Bassett & Johnson [1] analyzed the top roll vertical force and the torque required in 3-roller pyramid bending process without considering roller plate contact offset. Whereas, mathematical model reported by Hansen & Jannerup [2] considered the contact offset. Hua et al. [3-4] reported mathematical modeling of internal bending moment at the top roll contact in single pass and multiple pass four roll thin plate bending. Hu & Wang [5] suggested upper bound and lower bound methods for the bending mechanism of 3-roller plate bending. Ramamurthi et al. [6] reported design aspects and parametric study to evaluate the performance of machine stands in relation to various design or structural modifications of three roll heavy duty plate bending machines. For the calculation of top roller load, internal bending resistance of the plate material based on yield stress was considered.

Thickness of the material to be bent corresponds to the reference material thickness (which can be bent at the maximum top roller load imparting capacity), is defined as an equivalent thickness. Similarly, width of the material to be bent corresponding to the reference material width, is defined as an equivalent width. Width of the plate for roller bending decides number of cylindrical segments and so number of seams required to achieve the final shell length. Maximum plate width for pre-bending depends on maximum top roller load imparting capacity of machine, width of machine and maximum plate width available commercially. Looking to the above considerations, roller bending to the maximum width controls the welding and assembly cost. Presented work provides the methodology for the analysis of bendability within the machine capacity. Analytical models of equivalent thickness, equivalent width and maximum width based on the power law material model were derived to study the bendability and reported. Effect of top roller offset on the bendability at maximum top roller load imparting capacity is addressed.

II. BENDING WITHOUT TENSION

As reported by Marciniak & Dunkun [7], for simple bending without applied tension and where the radius of curvature is more than several times the sheet thickness, the neutral surface approximately coincides with the middle surface. The internal bending moment for the power law material model can be derived to the form:

\[ M_{\text{int}} = K' \left( \frac{r}{n+2} \right) \frac{t^{n+2}} {t^{n+2}} b_{r} \]

For the perfectly plastic material \((n=0)\), internal bending moment \((M_{\text{int}})\) is given by [7]:

\[ M_{\text{int}} = \frac{\sigma_{y} t^{2}} {4} b_{r} \]

A. Equivalent Thickness for Perfectly Plastic Material Model

Generally OEM provides machine capacity relating to one reference material. But in actual practice, different materials having high or low strength, compare to reference material, are to be processed on machine. So it is important to check the bendability of new material with respect to reference material with the help of equivalent thickness for working within specified safe limits of machine.

Equivalent thickness of new material can be obtained by comparing the material property of new material with reference material specifying machine capacity. Equivalent thickness using perfectly plastic material behavior can be obtained by equating internal bending moment for reference material and new material as per (2). So, for reference material:

\[ M_{r,\text{int}} = \frac{\sigma_{y} y_{r} t^{2}} {4} b_{r} \]

And, for new material:

\[ M_{n,\text{int}} = \frac{\sigma_{y} y_{n} t^{2}} {4} b_{n} \]

If the width of the new material to be bent \((b_{n})\) is assumed to be same as that of the width of reference material \((b_{r})\), then from (3) and (4), equivalent thickness of the new material to be bent, as given by OEM [8], can be derived to the form \((M_{n} = M_{r})\):

\[ t_{n} = \sqrt{\frac{t^{2} \sigma_{y} y_{r}} {\sigma_{y} y_{n}}} \]

OEM provides \((5)\) to find equivalent thickness considering perfectly plastic material behavior. Equation \((5)\) does not
include strain hardening characteristics, where as in field many material exhibits strain hardening characteristics. If power law material model is considered, then equivalent thickness can be calculated as explained in following paragraphs.

B. Equivalent Thickness for Power Law Material Behavior

Equivalent thickness, for the power law material model can be obtained by equating the bending moment equations for reference material and new material to be bent. So, rewriting (1) for reference material, and new material:

\[ M_{r, int} = K_r \frac{t_r^{(n_r+2)}}{\rho_r^{n_r(n_r+2)}} b_r \]  
(6)

\[ M_{n, int} = K_n \frac{t_n^{(n_n+2)}}{\rho_r^{n_n(n_n+2)}} b_n \]  
(7)

Equating (6) and (7), and assuming constant width (b_r = b_n):

\[ t_n^{(n_n+2)} = \frac{K_r (n_n+2)\rho_r^{(n_r-n_n)}t_r^{(n_r+2)}(n_r-n_n)\left(\frac{2}{\sqrt{3}}\right)}{(n_r+2)K_n} \]  
(8)

Equation (5) and (8) gives the equivalent thickness for perfectly plastic material and power law behavior respectively. The equivalent thickness calculated using (5) is the function of yield stress only. It does not have any influence of post yield stress material behavior, whereas (8) depicts the influence of strain hardening behavior of the material.

C. Equivalent Width for Perfectly Plastic and Power Law Material Models

Assuming thickness of new material to be bent \( t_n \) is same as that of the reference material thickness \( t_r \), equation for the equivalent width can be derived to the form given by (9) and (10) for perfectly plastic and power law material models respectively.

\[ b_n = b_r \frac{\sigma_{yr}}{\sigma_{yr}} \]  
(9)

\[ b_n = b_r \frac{K_n t_r^{(n_r-n_n)}(n_n+2)\rho_r^{(n_r-n_n)}}{K_r (n_r+2)} \]  
(10)

As explained earlier, equivalent width based on power law material model is the function of strain hardening behavior of the material.

D. Bending Force Analysis and Bendability

OEM provides the machine capacity in terms of thickness vs. width combinations for fix shell diameter and reference material property parameters at the maximum top roller load imparting capacity of the machine. As the maximum bending load is required during the pre-bending mode, top roller load for pre-bending can be obtained by considering plate as a simply supported beam, carrying a concentrated load at offset distance \((l/2)-x\) from the centre of span as shown in Fig. 2. For simply supported beam, external bending moment can be derived to the form [9]:

\[ M_{ext} = F \left[ 1 - \left( \frac{x}{7} \right)^2 \right] x \]  
(11)

Top roller load \( F \) for pre-bending assuming perfectly plastic material model can be derived by equating internal and external bending moments from (2) and (11) to the form:

\[ F = \frac{S_y}{4} \left( \frac{l}{l-x} \right) b \]  
(12)

Where as, if power law material model is used, then top roller load \( F \) can be derived using (1) and (11) as:

\[ F = \frac{K \rho^{n+2} b}{l^{n+2}} \left( \frac{l}{l-x} \right) b \]  
(13)

Equation (12) shows that the force required to bend the plate considering perfectly plastic material model is the function of material property parameter \( \sigma_{yr} \). It does not shows any influence of post yield behavior of material, whereas, (13) shows that the force required to bend the plate considering power law material model is the function of material property parameters \( K, n \). This shows the influence of strain hardening characteristics.

Plate material, thickness and shell diameter are the constrained by design requirement. However, overall shell length can be obtained by welding the shell segments of different width. Width of bending decides the number of circumferential seams required to complete the cylinder of overall length. As welding seams increases, assembly and fabrication cost also increases. Further, it demands proper alignment of individual shell segments edges with other similar adjoining shells. So, to minimize the number of seams, it becomes important to check maximum width; the machine is capable of bending within the safe working limit specified by OEM. Maximum width \( b_{max} \) considering the perfectly plastic material model can be obtained by rewriting (12) in terms of width \( b_{max} \) for maximum top roller load imparting capacity \( F = F_{max} \) specified by OEM.
Similarly, from (13) maximum width \( b_{\text{max}} \) for power law material model can be derived to the form:

\[
b_{\text{max}} = \frac{4F_{\text{max}} \left(1 - \frac{x}{l}\right)x}{\sigma_y t^2}
\]

(14)

Similarly, from (13) maximum width \( b_{\text{max}} \) for power law material model can be derived to the form:

\[
b_{\text{max}} = \frac{F_{\text{max}} \left(1 - \frac{x}{l}\right)xp^n (n+2) 2^{n+1}}{K' t^{n+2}}
\]

(15)

### III. RESULTS & DISCUSSION

Equivalent thickness, equivalent width and maximum width equations derived using two different material models were used to analyze the bendability of the heavy duty 3-roller plate bending machine. Figure 4 shows the one of the capacity chart of the 3-roller heavy duty plate bending machine specified in Table I. Capacity charts provided by the OEM shows the plate thickness vs. plate width for pre-bending of reference material at shell diameter of 2000 mm. Under the pre-bending stage the top roller is at offset distance of \((l/2)-x\) as shown in Fig. 2. It is general practice to keep the value of \(x\) in the range of 1.4 to 1.75 times the plate thickness \( [8] \). However, value of \(x\) can be decided based on practical limitations. For the machine under consideration, the maximum top roller load imparting capacity is 5000 tons. From the machine capacity chart, at 5000 tons, 135 mm thick and 4500 mm wide reference material plate can be pre-bent to the shell diameter of 2000 mm. From Fig. 4 it can be observed that at maximum top roller load imparting capacity, as the plate thickness increases, the plate width of the reference material corresponding to the same pre-bent shell diameter and offset value decreases. Capacity charts provided by the OEM are for specific (reference) material, whereas in actual practice different materials as per the design specification are required to be bent. So, equivalent thickness of the new material to be bent corresponding to the reference material, for the same width and shell diameter combinations can be obtained from (5) and (8). Similarly, equivalent width for the new material can also be obtained from the equation of equivalent width (i.e. (9) and (10)).

**E. Comparison of Equivalent Thickness and Equivalent Width by Different Material Behavior**

Equivalent thickness and width of new material to be bent can be calculated as explained earlier. Table II shows the comparison of equivalent thickness and equivalent width, calculated using perfectly plastic and power law behavior for four different grades of C-Mn steel (as per ASME sec II part A) \([11]\). These equivalent thickness and width are calculated for 135 mm thick and 4500 mm wide plate of reference material SA-516 Gr60, for pre-bending to shell diameter of 2000 mm at top roller load of 5000 Tones. Equivalent thicknesses for \“Mat 1\” as per perfectly plastic and power law material models are found to be 139.9 mm and 124.7 mm respectively.

Fig. 5 shows the comparison of equivalent thicknesses as per two different material models for \“Mat 1\” corresponding to different thicknesses of the reference material. Referring to Table II and Fig. 5, for \“Mat 1\” and \“Mat 2\” equivalent thicknesses as per the perfectly plastic material behavior is found to be higher than that of the reference material thickness i.e. 135 mm, as yield stress of these materials are less than the yield stress of the reference material. Whereas, due to the higher value of the yield stress for \“Mat 3\” and \“Mat 4\” equivalent thickness as per the perfectly plastic material behavior is found to be less than the reference material thickness. For \“Mat 2\” equivalent thickness as per the power law material model is found to be higher than reference material thickness where as, for \“Mat 1\”, \“Mat 3\” and \“Mat 4\” it is found to lesser than the reference material thickness. As per (8), equivalent thickness based on the power law material model is the function of material property parameters such as ‘\(K\)’ and ‘\(n\)’. So, combined effect of these two material property parameters is seen in the equivalent thickness reported in Table II.

![Fig. 4 Capacity chart for SA-516 Gr60, 5000 MT top roll capacity, shell \(\varphi\) 2000 mm, \(x = 214\) mm](image)

**Table II and Fig. 5, for \“Mat 1\” and \“Mat 2\” equivalent thicknesses as per the perfectly plastic material behavior is found to be higher than that of the reference material thickness i.e. 135 mm, as yield stress of these materials are less than the yield stress of the reference material. Whereas, due to the higher value of the yield stress for \“Mat 3\” and \“Mat 4\” equivalent thickness as per the perfectly plastic material behavior is found to be less than the reference material thickness.**

![Fig. 5 Comparison of Equivalent thickness](image)
As it is already discussed earlier, maximum width of the plate that can be pre-bent is the crucial factor in controlling the overall shell cost.

As the maximum top roller load imparting capacity \( (F_{\text{max}}) \) is known, maximum width of the plate which can be bent at that load can be easily calculated from (14) and (15) for the given material property parameters, plate thickness and shell diameter combinations. Maximum width for “Mat 1” as per perfectly plastic and power law material models are found to be 4494 mm and 2385 mm respectively for thickness of 180 mm and shell diameter of 5000 mm at maximum top roller load imparting capacity of 5000 tones. From (14) it can be observed that as per perfectly plastic material model, maximum width \( (b_{\text{max}}) \) is the function of yield stress, plate thickness, shell diameter, top roller offset and maximum top roller load imparting capacity, whereas according to power law material model, maximum width \( (b_{\text{max}}) \) is the function of material property parameters \( K \) and \( n \), plate thickness, top roller offset, maximum top roller load imparting capacity and shell diameter. Equation of maximum width based on the perfectly plastic material model do not consider the effect of strain hardening due to the increased degree of deformation, which may be contradicting the concept of increase of yield point due to strain hardening. Further, for the material under consideration (i.e. “Mat 1”) equivalent width calculated using perfectly plastic material model is considered then, cost of welding & fabrication is high as more number of seams will be required due to lower width of pre-bending & vice versa for other materials (i.e. “Mat 2”, “Mat 3” & “Mat 4”).

![Fig. 6 Comparison of equivalent width](image)

**Fig. 6 Comparison of equivalent width**

If thickness for new material to be bent is same as that of the reference material then the width of new material which can be bent at maximum top roller load imparting capacity of 5000 tones can be calculated as explained earlier. Figure 6 shows the comparison of equivalent widths for two different material models for “Mat 1” at shell diameter 2000 mm and plate thickness of 135 mm. Equivalent width of “Mat 1” is found to be 4829.3 mm and 4087.8 mm for perfectly plastic and power law behavior respectively. If the equivalent width exceeds the maximum plate width which can be accommodated on machine \( (b_{\text{machine}}) \) i.e. 4500 mm, then equivalent plate width for the new material to be bent should be taken as 4500 mm.

**G. Top Roller Offset Analysis**

Top roller offset plays major role in machine capacity as the equivalent thickness; width and maximum width are function of top roller offset. With the shifting of top roller towards the center of span, top roller vertical travel required to pre-bend the same shell diameter increases. But load required for marginal increase of top roller vertical travel is less compared to the load increase due to marginal shift of top roller in center. This in turn reduces the pre-bending load. As the top roller shift towards the center of the span, pre-bending load decreases and so bendability increases. On the other side, material wastage increases due to larger straight length at the end portions of plate. At top roller offset of 214 mm, 180 mm
thick and 2893 mm wide plate of “Mat 1” can be bent to the shell inside diameter of 5000 mm. As top roller shifts towards the center, maximum width that can be pre-bent increases as shown in Fig. 7. For the top roller offsets of 1.4t i.e. 252 mm, maximum pre-bending width is 3215 mm whereas at the top roller offset of 1.75t i.e. 315 mm, maximum pre-bending width increases to 3628 mm. Similarly at top roller offset of 450 mm, maximum pre-bending width further increases to 3986 mm.

Fig. 8 Capacity graph for (a) 214 mm offset distance and (b) 450 mm offset distance

Fig. 8(a) and 8(b) are the graphs of the machine capacity for bending of reference material at two different top roller offsets of 214 mm and 450 mm respectively. The point on the surface shows the plate thickness, width and shell diameter combinations at the maximum top roller load imparting capacity of 5000 tones. Figure 8 shows that, on “Point 1” at the top roller offset of 214 mm, it is unsafe to pre-bend the 156 mm thick and 4500 mm wide plate of reference material to the shell diameter of 2000 mm, as required top roller load is 6887 tones. Whereas, for the same combination of thickness, width, and diameter by increasing the top roller offset from 214 mm to 450 mm (center position) required top roller pre-bending load reduces to 4993 tones. By increasing the top roller offset from 214 mm to 450 mm, maximum pre-bending width (at the top roller load imparting capacity of 5000 tones) increases from 3267 mm to 4506 mm. Flat top portion of the graph shows that machine width constrain of 4500 mm. Fig. 8(a) shows that, on “Point 2” at the top roller offset of 214 mm, it is unsafe to pre-bend 180 mm thick and 4500 mm wide plate of reference material to the shell diameter of 10000 mm, as required top roller load is 6673 tones. Whereas, for the same combination of thickness, width and diameter by increasing the top roller offset from 214 mm to 450 mm (center position) required top roller pre-bending load reduces to 4801.5 tones. So, by increasing the top roller offset from 214 mm to 450 mm, maximum pre-bending width at top roller load imparting capacity of 5000 tones increases from 3360 mm to 4635 mm. Comparison of Fig. 8(a) and 8(b), shows that as top roller shifts towards the centre, graph shift upward showing the increase of maximum pre-bending width for given plate thickness and shell diameter combination. This shows improvement in bendability by reducing cost of welding & fabrication. However, there will be increase in material wastage due to increase in end flat portion of plate with shifting of top roller towards center.

IV. CONCLUSION

Presented work analyzes the bendability in terms of maximum width for bending of C-Mn steel plate on 3-roller heavy duty plate bending machine. Following important conclusion can be derived based on investigation;

1) The equivalent thickness, equivalent width and maximum width calculated based on power law material model includes the effect of strain hardening and can lead to the more realistic prediction of machine capability.

2) It is very crucial to predict bendability based on one plate material because capacity of 3-roller plate bending machine is the complex function of plate thickness, plate width, plate material, maximum top roller load imparting capacity, top roller offset and shell diameter to be bend.

3) Maximization of the width of the plate for bending is the important factor which controls the overall cost of fabrication. So, in place of capacity chart for the reference material supplied by OEM, the maximum width at top roller load imparting capacity for plate material, thickness and shell diameter combinations to be bent should be specified to define the machine capacity.

4) Top roller offset for the pre-bending should be analyzed from the point of view of maximization of machine capability and minimization of material wastage.

5) Capacity of heavy duty 3-roller plate bending machine in practice can be fully exploited using the bendability analysis which may results into better utilization and will help either in reduction of fabrication cost or by reducing material wastage.

REFERENCES