In this note, the robust static output feedback stabilisation of an induction machine is addressed. The machine is described by a non homogenous bilinear model with structural uncertainties, and the feedback gain is computed via an iterative LMI (ILMI) algorithm.

Keywords—Induction machine, Static output feedback, robust stabilisation.

I. INTRODUCTION

INDUCTION machine has received particular interest through many researches in industry application systems, since it's intensively used in applications requiring high dynamic performances.

Controllers for such machines must have some characteristics as limiting both currents and flux in their respective nominal ranges, while driving the motor torque along a giving profile.

On other hand, the controller synthesis for drives using an induction machine is a rather difficult problem that must deal with nonlinear dynamics, multivariable inner structure of the system and no availability of flux sensors. Moreover, the most applications must be sensorless to insure required safety, high level availability of devices and low costs implementation, which increase problems difficulty. So, sensorless control for both synchronous and asynchronous machine was the one of the most attractive problems trough the last two decades.

In the last years, the most effective approaches to this problem were based on linearising and decoupling between stator parameters leading to the so called field oriented control (FOC). In such control problem, the knowledge of stator resistance and self induction coefficient is necessary. Those parameters are not constants due to the large heat domain of work and variable saturation level of the machine.

II. INDUCTION MACHINE MODEL

In this section, the classical structure of induction machine is considered. Such machines are driven by variable frequency voltage to provide suitable torque which is a combination of flux and current components.

Then, the considered equations are a two axes representation of induction machine under appropriates simplifying hypothesis. It's a set of a non linear differential equations with current and flux components as variables. The state vector is build with measured variables (currents) and estimated ones (flux).

\[
\begin{align*}
\dot{x} &= [i_{ds}, i_{qs}, \varphi_{ds}, \varphi_{qs}]^T \\
\end{align*}
\]

Where:

- \(i_{ds}\) and \(i_{qs}\) are projections of the components current stator on a (d,q) axes.
- \(\varphi_{ds}\) and \(\varphi_{qs}\) are projections of the components flux stator on a (d,q) axes.

and the control \(u = [v_{ds}, v_{qs}, \omega]^T\) applied by the inverter to achieve a variable speed control.

The dynamic equations describing the motor behaviour in the general form can be written as:

\[
\begin{align*}
x &= f(x) + g(x)u \\
y &= Dx \\
\end{align*}
\]
With
\[
f(x) = Ax = \begin{bmatrix}
\frac{R_c L_c + R_s L_s}{\sigma_d L_s} & 0 & \frac{R_c}{\sigma_d L_s} & \sigma \\
0 & \frac{R_c L_c + R_s L_s}{\sigma_d L_s} & 0 & \sigma \\
-\frac{R_c}{\sigma_d L_s} & 0 & -\frac{R_c}{\sigma_d L_s} & 0 \\
0 & -\frac{R_c}{\sigma_d L_s} & 0 & -\sigma
\end{bmatrix} x
\]
and
\[
g(x) = \begin{bmatrix}
\sigma \\
0 \\
1 \\
0
\end{bmatrix}
\]
where \( \sigma = 1 - \left( M_2^2 / L_c L_s \right) \)
and \( R_c, R_s, L_c, L_s, L_r \) and \( M \) stand for nominal machine electromagnetic parameters.

Such system can be written in the homogenous bilinear form:
\[
\dot{x} = Ax + \sum_{i=1}^{3} u_i B_i x + Cu
\]
With
\[
B_1 = B_2 = 0 ; B_3 = \begin{bmatrix}
0 & 1 & 0 & 0 \\
-1 & 0 & 0 & 0 \\
0 & 0 & 1 & 0 \\
0 & 0 & -1 & 0
\end{bmatrix}
\]
and
\[
C = \begin{bmatrix}
\sigma \\
0 \\
1 \\
0
\end{bmatrix} / \sigma_d \\
D = \begin{bmatrix}
1 & 0 & 0 & 0 \\
0 & 1 & 0 & 0
\end{bmatrix}
\]

limited ranges of stator resistance and/or inductance. Thus the following inequalities hold:
\[
\|\Delta A\| < a \quad \text{and} \quad \|\Delta C\| < c
\]
The maximum values \( a \) and \( c \) can be easily determined from classical identifying tests.

Therefore the closed loop system is:
\[
\begin{align*}
\dot{x} &= (A_0 + C_0 K_D) x + \sum_{i} K_i D x B_i x + (\Delta A + \Delta C \cdot K \cdot D) x \\
y &= D x
\end{align*}
\]  

III. ROBUST OUTPUT FEEDBACK STABILISATION

Consider a linear time-invariant system described by:
\[
\begin{align*}
\dot{x} &= Ax + Bu, \\
y &= Cx
\end{align*}
\]  

where \( x \in \mathbb{R}^n \) is the state vector, \( u \in \mathbb{R}^r \) is the control vector and \( y \in \mathbb{R}^m \) is the output vector.

Definition: system (4) is said to be stabilizable via static output feedback if there exists \( K \) such that the closed loop systems:
\[
\dot{x} = (A + BK C)x
\]
is stable. And it’s said to be \( \alpha \)-stabilizable via static output feedback if \( K \) places the closed-loop poles to the left of a vertical line \( \text{Re}(s) = -\alpha \) (for some real \( \alpha \)) in the complex plane.

Theorem:
Under the next hypothesis:

i. The system (1) linear part is \( \alpha \)-stabilizable via static output feedback for some known real number \( \alpha \).

ii. The state initial conditions are in some ball of radius \( R \).

the output feedback \( u(t) = Ky(t) \) exponentially stabilise the system (2).

Proof:
First, we will focus on the linear part of the system.

The nominal closed-loop matrix is defined as:
\[
\tilde{A} = A_0 + C_0 K_D
\]

Where a static gain feedback \( K \) is chosen so that allows the matrix \( \tilde{A} \) to have all its eigenvalues with negative real part.

Then the matrix \( \tilde{A} \) is called stable. And the linear part of the system is output feedback stabilizable.
Cao et al. [3] give a theorem on the necessary and sufficient condition for static output feedback stabilizability of a linear system as follows:

\[ A^T P + P A - PBB^T P + [B^T P + KC] \] \( T (B^T P + KC) < 0 \) \( \text{with: } f_1(s) = M(a + c)K\|D\| = M\beta \) and

\[ f_2 = M\sum_i K_i B_i x_0 \|e^{-\omega s}\| \]

The application of the Gronwall-Bellman lemma in its general form [4] leads to the second hypothesis, which arises from the following ones (see the theorem in the Appendix). The same result can be obtained from the classical form of Gronwall-Bellman lemma.

We define \( R \) as:

\[ R = -\frac{\omega + M\beta}{M^2 \sum_i K_i B_i D} \]

Hence the second hypothesis of the theorem is:

\( (H) \Rightarrow 1 - \int_0^\infty Mg_2(t) dt > 0 \)

\[ g_2(t) = f_2(t) \exp\left(\int f_1(s) ds\right) \]

\[ = M\left|\sum_i K_i B_i\right| x_0 \right| e^{\omega s} \cdot \left(e^{M(a + c)\|P\|\|D\|} \right) \]

\[ (H) \Rightarrow 1 - M^2 \left|\sum_i K_i B_i\right| x_0 \int_0^\infty e^{(\omega + M\beta)s} dt \]

\[ = 1 - M^2 \left|\sum_i K_i B_i\right| x_0 \left[ e^{(\omega + M\beta)s} \right]_0^\infty \]

\[ \text{The hypothesis H is written as:} \]

\[ 1 - M^2 \left|\sum_i K_i B_i\right| x_0 \left[ \frac{1}{\omega + M\beta} \right] > 0 \]

\[ \Rightarrow M^2 \left|\sum_i K_i B_i\right| x_0 \left( \frac{1}{\omega + M\beta} \right) > -1 \]

Hence the initial conditions must satisfy:

\[ \|x_0\| \leq -\frac{(\omega + \beta)}{M^2 \sum_i K_i B_i} \]

It follows:
then the system is exponentially stable.

IV.  ITERATIVE LMI

It can be noted that the hypothesis of α-stabilizability of the linear part system arises from the condition that the uncertainties and the closed-loop system must verify:

\[ \omega + M\beta < 0. \]

Where \( \omega \) stand for the maximum eigenvalues of the closed-loop matrix (A-CKD).

Corollary.

The linear system (2) is α/2-stabilizable via static output feedback if and only if there exist two matrices \( P > 0 \) and \( K \) satisfying the following matrix inequality:

\[
A^T P + PA - PBB^T P + [B^T P + KC]^T (B^T P + KC) - 2\alpha P < 0
\]

Cao et al. propose an iterative algorithm to solve the above QMI as an ILMI. In the next, we use the same algorithm where we verify the α-stabilizability according to the hypothesis (ii) of our theorem.

Then the algorithm can be written as:

Step 1. Select \( Q > 0 \), and solve \( P \) from the following algebraic Riccati equation

\[
A^T P + PA - PCC^T P + Q = 0
\]

Set \( i = 1 \) and \( X_1 = P \).

Step 2. Solve the following optimization problem for \( P_i, K \) and \( \alpha_i \).

\[ \text{OP1 : Minimize } \alpha_i \text{ subject to the following LMI constraints (6) and (7) with } \alpha_i = \alpha^*_i. \text{ Denote } P_i^* \text{ as the } \alpha_i \text{ that minimized trace}(P_i). \]

Step 3. If \( \alpha_i^* < M\beta \), \( K \) is a stabilizing static output feedback gain. Stop.

Step 4. Solve the following optimisation problem for \( P_i \) and \( K \).

\[ \text{OP2 : Minimize } \text{trace}(P_i) \text{ subject to the above LMI constraints (6) and (7) with } \alpha_i = \alpha^*_i. \text{ Denote } \]

Step 5. If \( \|X_i - P_i^*\| < \delta \), a prescribed tolerance, go to Step 6.

Step 6. The system may not be stabilizable via static output feedback. Stop.

V. CONCLUSION

In this paper, the problem of the robust static output feedback stabilization of an induction machine is studied. We improve the results available on the SOF for the linear systems to the case of non homogenous bilinear model of the induction machine by using a generalization of the Bellman-Gronwall lemma. Then we proposed to follow the same iterative LMI algorithm developed in Cao et al. to compute the gain feedback.

APPENDIX

A generalization of Bellman-Gronwall lemma:

If \( a, b, n, k \in \mathbb{R} \), where \( a < b \), \( n > 1 \) and \( k > 0 \) and

\[ \alpha_i \in [a, b], \text{ a set of integrable functions that verify :} \]

\[ \forall \alpha, \beta \in [a, b], \alpha < \beta : \]

\[ g_i(t) = f_i(t) \exp \int_a^t (n-1)f_i(s)ds \]

\[ \int_a^b g_i(t)dt > 0 \]

\[ x : [a, b] \rightarrow \mathbb{R}^+ a \text{ bounded function that satisfies } \forall t \in [a, b] : \]

\[ x(t) \leq k + \int_a^t \sum_{i=2}^n k^{-1} g_i(t)dt > 0 \]

Then under the following hypothesis:

\[ 1 - (n-1) \int_a^b \sum_{i=2}^n k^{-1} g_i(t)dt > 0 \]

for \( t \in [a, b] : \]

\[ x(t) \leq k \exp \int_a^t f_i(s)ds \frac{1}{1 - (n-1) \int_a^b \sum_{i=2}^n k^{-1} g_i(t)dt} \]
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