Redundancy in Steel Frames with Masonry Infill Walls

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Abstract—Structural redundancy is an interesting point in seismic design of structures. Initially, the structural redundancy is described as indeterminate degree of a system. Although many definitions are presented for redundancy in structures, recently the definition of structural redundancy has been related to the configuration of structural system and the number of lateral load transferring directions in the structure.

The steel frames with infill walls are general systems in the constructing of usual residential buildings in some countries. It is obviously declared that the performance of structures will be affected by adding masonry infill walls. In order to investigate the effect of infill walls on the redundancy of the steel frame which constructed with masonry walls, the components of redundancy including redundancy variation index, redundancy strength index and redundancy response modification factor were extracted for the frames with masonry infills. Several steel frames with typical storey number and various numbers of bays were designed and considered. The redundancy of frames with and without infill walls was evaluated by proposed method. The results showed the presence of infill causes increase of redundancy.

Keywords—Structural redundancy, Masonry infill walls frames.

I. INTRODUCTION

STRUCTURAL redundancy is an important concept in seismic design of structures. Redundancy of structures became the focus of research after major structural failure of buildings caused by catastrophic earthquakes such as 1994 Northridge and 1995 Kobe. It has been emphasized in seismic design codes that redundancy of structures plays an important key in seismic performance of structures. As pointed in FEMA356 [1] the configuration of structural system and number of lateral load resisting line of a building, which is referred as redundancy, has significant role in seismic performance of existing structures. Current seismic codes such as UBC1997 [2] and NEHRP97 [3] describe some factors as the term of redundancy and reliability factor which is a function of ground floor area and maximum shear ratio of storey. ATC19 [4] proposed a method to estimate behavior factor (R) in which R is calculated as multiplication of three coefficients of ductility, over strength and redundancy (Rr).

The redundancy coefficient in the equation of R demonstrates the structural redundancy. Although, R, is taken as 1 due to complexity in quantifying it. Moreover, it is predictable that special attendance to be considered to the redundancy factor in performance based seismic design codes as its importance was previously taken into account in FEMA356 and ATC40 [5]. Since a variety of redundancy source in the structures the definition of structural redundancy varies significantly in literature. Moreover, uncertainty and complexity in the source of redundancy has caused difficulty of defining and quantifying the amount and effect of structural redundancy. As the difficulty of quantifying the amount of redundancy has been pointed out by Furuta [6], who used probabilistic and fuzzy interpretations to review several definitions of structural redundancy.

Initially, structural redundancy is referred to the number of equations that are required for solution, in addition to the equilibrium equations. This definition may be inadequate in view of the complicated nonlinear structural behaviors under random earthquake excitations and the effects of uncertainty in demand and capacity. An acceptable theory for the redundancy declares that the redundancy is related to the structural configuration. Generally in a lateral-force-resisting system, which has many lateral-force-resisting components and the lateral load is distributed among a lot of these components there is a less possibility of failure of all components in comparison with the one having less lateral-force-resisting components. Thus, a redundant and reliable system has many components outstanding against failure. This means that the better definition of structural redundancy to be accomplished by addressing redundancy to the failure possibility of a system.

As the other definition, structural redundancy is related to the number of plastic hinges of the structure system when the structure collapses. Bertero and Bertero [7] defined the redundancy degree to investigate the redundancy of frame structures as the number of plastic hinges formed at structural member ends, up to the point of total collapse. They used the ratio of strength capacity over strength demand as an indication of overstrength resulting from redundancy and investigated its effects on the system reliability.

A method for calculating uniform risk redundancy factor was proposed by Wang and Wen [8]. They defined the uniform risk redundancy factor as a ratio of spectral...
displacement capacity for incipient collapse to the spectral displacement corresponding to a specified allowable probability of incipient collapse.

Husain and Tsopelas [9] proposed two indices, a redundancy strength index and redundancy variation index to quantify the effects of redundancy on structural systems. Based on these two indices, the redundancy response modification factor and reliability index were derived. Several important factors such as building height, number of stories, beam span lengths, and vertical lines of resistance are investigated. Since only static pushover analysis is used, further evaluation of structural redundancy considering dynamic response is needed.

Infill masonry walls are commonly constructed in the exterior frames of steel frames buildings. Their effects on the behavior of the steel frame buildings typically are ignored during design process. This study investigates the effect of inﬁlls on the redundancy of the steel frames. A method based on the concept of redundancy components which includes strength index and redundancy variation index was proposed to investigate the effect of infill walls on the redundancy. The relation of redundancy components for steel frame with infill walls were derived and utilized to measure the redundancy of several typical infill walls frames.

II. STEEL FRAMES WITH MASONRY INFILL WALLS

Masonry infill walls are found in most existing steel frame building systems. This type of infill walls was common in some county such as Iran. The masonry infill walls which are constructed after completion of steel frames are considered as non-structural elements. Although they are designed to perform architectural functions, masonry infill walls do resist lateral forces with substantial structural action [10]. In addition to this, infill walls have a considerable strength and stiffness and they have significant effect on the seismic response of the structural system. There is a general agreement among of the researchers that infill frames have greater strength as compared to frames without infill walls. The presence of the infill walls increases the lateral stiffness considerably. Due to the change in stiffness and mass of the structural system, the dynamic characteristics change as well.

In conventional analysis of infill frame systems, the masonry infill wall may be modeled using an equivalent strut model [1] as shown in Fig. 1. According to this idealization the elastic in-plane stiffness of a masonry infill wall prior to cracking is represented with an equivalent diagonal compression strut of width, w, given by Equation 1.

\[
W = 0.25\left[\lambda \cdot h_{inf}\right]^{0.75} \quad (1)
\]

Where

\[
\lambda = \left[\frac{10E_{inf}}{E_{fe} \cdot I_{col}} \cdot \frac{\sin 2\theta}{\cos \theta}ight]^{0.25} \quad (2)
\]

In which, \(h_{col}\) and \(I_{col}\) are the height and the moment inertia of columns, respectively, \(h_{inf}\), \(r_{inf}\) and \(t_{inf}\) are the height, diagonal length and thickness of infill wall, respectively. \(E_{fe}\) and \(E_{me}\) are expected modulus of elasticity of frame and infill material. \(\theta\) is angle whose tangent is the infill height to length aspect ratio.

To investigate the redundancy of masonry inﬁlls steel frames, diagonal compression strut idealization model of infill walls is utilized.

III. REDUNDANCY COMPONENTS

A. Redundancy Variation Index

The proposed method to investigate redundancy is based on derivation of two statistical moments of the system strength, namely, the mean or expected value, and the standard deviation or coefficient of variation with the corresponding values of the external loads. The system strength is to be formulated in terms of the strengths of its members [11]. For the frame with infill walls under lateral load, which vertically distributed along the frame height in an inverted triangular shape, by assuming the overall collapse is sway mechanism, as showing in Fig 2, the following relation can be derived:

\[
S = \theta \left(c \sum_{i=1}^{n} M_{i}^c + b \sum_{j=1}^{m} M_{j}^b + \frac{1}{cos \theta} \sum_{k=1}^{n} P_{k}^w \right) \quad (3)
\]

Where \(S\) is frame strength, \(M_{i}^c\) is yield moment of column at joint \(i\), \(M_{j}^b\) is yield moment of beam at joint \(j\), \(P_{k}^w\) is axial yield load of infill wall at joint \(k\) and \(n\), \(m\), \(k\) are the number of plastic hinges of columns, beams and infill walls respectively.
The standard deviation of the overall strength of the frame, \( \sigma_f \), can be obtained from Equation 4 as follows:

\[
\sigma_f = \sqrt{\left\{ \sum_{i=1}^{n} \sum_{j=1}^{n} \rho_{ij} \sigma_{M_i} \sigma_{M_j} + \sum_{i=1}^{n} \sum_{j=1}^{n} \rho_{ij} \sigma_{M_i} \sigma_{M_j} \right\}}
\]

\[
= \left\{ \sum_{i=1}^{n} \sum_{j=1}^{n} \rho_{ij} \sigma_{M_i} \sigma_{M_j} \cos \alpha \right\}^{1/2}
\]

\[
= \left( \sum_{i=1}^{n} \sum_{j=1}^{n} \rho_{ij} \sigma_{M_i} \sigma_{M_j} \cos \alpha \right)^{1/2}
\]

In which, \( \sigma_{M_i} \) is standard deviation of yield moment of column at joint i, \( \sigma_{P_i} \) is standard deviation of axial yield load of infill wall, \( \rho_{ij} \) is correlation between strengths of joints i, and j, equal 1, for \( i=j \).

For simplifying the expressions assume the beams of frame are composed of identical elements in all stories. This assumption is also applied for columns and consequently for infill walls. Therefore the strength of beams, columns and infill walls with mean values \( M_i \), \( M_j \), and \( P_i \), respectively, and the standard deviation of member strength and average correlation coefficient between member strengths are considered, \( \sigma_{M} \), and \( \rho_{M} \), respectively. Then the mean value of the frame strength, \( S \), and its standard deviation, \( \sigma_f \), can be written in the following form:

\[
S = \theta \left( cnM + cm + \frac{1}{\cos \alpha} \right) P_r
\]

\[
\sigma_f = \theta \left[ c^2 \sigma_{M}^2 \left( n + m(m-1) \rho_{M} \right) + c^2 \sigma_{M}^2 \left( m + m(m-1) \rho_{M} \right) \right]
\]

\[
\times \left\{ \sum_{i=1}^{n} \sum_{j=1}^{n} \rho_{ij} \sigma_{M_i} \sigma_{M_j} \cos \alpha \right\}^{1/2}
\]

\[
= \theta \left[ c^2 \sigma_{M}^2 \left( n + m(m-1) \rho_{M} \right) + c^2 \sigma_{M}^2 \left( m + m(m-1) \rho_{M} \right) \right]
\]

\[
\left\{ \sum_{i=1}^{n} \sum_{j=1}^{n} \rho_{ij} \sigma_{M_i} \sigma_{M_j} \cos \alpha \right\}^{1/2}
\]

By dividing Eq. 5, by Eq. 3 and considering the strength of each member of frame to be the coefficient of the strength of its one element, namely, \( M_i = \gamma M_0 \) and \( P_i = \beta M_0 \), the coefficient of variation of the frame strength, \( \nu_f \), can be expressed in terms of the coefficient of variation of element strength, \( \nu_M \) (\( \sigma_M / M_0 \)) as shown below:

\[
\nu_f = \nu_M \sqrt{A}
\]

\[
\times \left( \sqrt{cnf + cm + \frac{1}{\cos \alpha} \beta l} \right)
\]

Where,

\[
A = c^2 \left( n + m(n-1) \rho_{M} \right) + c^2 \left( m + m(m-1) \rho_{M} \right) + \left( \frac{1}{\cos \alpha} \right)^2 \left( l + l(l-1) \rho_{M} \right)
\]

From Eq. 7, a redundancy variation index, \( r_{nu} \), defined as the ratio between \( \nu_f \) and \( \nu_M \), by which the effect of frame redundancy on the coefficient of variation of frame strength can be assessed, may be written as:

\[
r_{nu} = \frac{S}{S_r}
\]

\[
= \frac{\sqrt{B}}{cnf + cm + \frac{1}{\cos \alpha} \beta l}
\]

Where,

\[
B = c^2 \left( n + m(m-1) \rho_{M} \right) + c^2 \left( m + m(m-1) \rho_{M} \right) + \left( \frac{1}{\cos \alpha} \right)^2 \left( l + l(l-1) \rho_{M} \right)
\]

The redundancy variation index is a measure of the probabilistic effects of redundancy on system strength. It is a function of the number of plastic hinges, the ratio of column strength to beam strength, the ratio of infill wall strength to beam strength and their average correlation coefficient. It measures the effect of redundancy on reducing the coefficient of variation of the system strength, which improves the reliability of the structure. It takes values between 1.0 and 0.0, where \( r_{nu} = 1.0 \) represents non-redundant structure, lower values of \( r_{nu} \) indicate highly redundant structures, and \( r_{nu} = 0.0 \) represents infinitely redundant structures.

In an investigations of the correlation coefficients between structural elements, some structural members found to have as a strong correlation 0.8 or more, and others show weak correlation of about 0.2 or less, which suggests an average values of \( \rho_{M} \) between 0.2 and 0.5.

B. Redundancy Strength Index

Another major benefit of system redundancy on structures has been investigated by many researchers is the reserve strength or the over-strength effects. The over strength effect needs to be quantified if the redundancy effects on structures is to be totally realized. The over-strength effects can be obtained through the use of nonlinear pushover analysis, under incrementally increasing external loads up to the overall collapse. The results of this analysis is to be presented in the form of the base shear lateral drift (of frame tip) curves. The ratio between the ultimate base shear at collapse to the base shear at the yield point, that in fact the ultimate base shear of the non-redundant frame, is to be called redundancy variation index, and symbolized by \( r_s \) :

\[
r_s = \frac{S_y}{S_u}
\]

Where, \( S_y \) and \( S_u \) is the base shear value at yielding and collapse (ultimate strength), respectively. The \( r_s \) index is the deterministic measure of the improvement of the of the frame strength due to its ability withstand many events of local damage, while continues to support increasing lateral loads. The values of redundancy-strength index are greater than 1.0, where higher values stand for higher redundancy levels, It is noted that \( r_s \) equal 1.0 represents the non-redundant system.
C. Seismic Redundancy Factor

The overall effects of redundancy on the structural strength can be completely described by the ratio of the ultimate strength of a structural system over the strength of the same, but non-redundant structure. Thus the formulation of the redundancy factor, $R_R$, in this study is based on the following expression:

$$R_R = \frac{S_u}{S}$$

(10)

Where $S_u$ is structural system strength which includes all the effects of redundancy and $S$ is strength of the same, but non-redundant structural system.

The system strength, $S$, can be written as a function of its mean value, $\overline{S}$, and its reliability limit as follows:

$$S = \overline{S} - \sigma_{\overline{S}} \lambda$$

(11)

Where $\sigma_{\overline{S}}$ is standard deviation of the system elements strength without redundancy; and $\lambda$ is reliability index, depends on the reliability of the strength value, and the shape of the density distribution of the strength.

The system strength including the redundancy effects, $S_u$, can be written as follows:

$$S_u = \overline{S} - \sigma_{\overline{S}} r \lambda$$

(12)

In which $\sigma_{\overline{S}}$ is the standard deviation of the system strength.

The relation between $\overline{S}$ and $\overline{S}$ is defined by the redundancy strength, $r \overline{S}$, in other words as the following:

$$\overline{S}_u = r \overline{S}$$

(13)

Furthermore, $\sigma_{\overline{S}}$, and $\sigma_{\overline{S}}$ are related by the redundancy variation index, $r$:

$$\sigma_{\overline{S}} = r \sigma_{\overline{S}}$$

(14)

By substituting from Eq. 9 through into Eq. 12, the following relation can be obtained:

$$R_R = r \left( \frac{1 - \sigma_{\overline{S}}}{1 - \epsilon} \right)$$

(15)

Where $\epsilon = \nu \lambda$

(16)

The value of $\lambda$ ranges between 1.5 and 2.5, depending on the reliability percentage limit (93%-97%). The values of the coefficient of variation of system strength, $\nu$, are as low as 0.08 for high levels of quality control constructions, up to 0.20 for low quality control. Moderate quality control constructions have $\nu$ between 0.12 and 0.15, hence an average value of can be used with reasonable accuracy.

IV. MODELS

To implement proposed method for quantifying redundancy in infill walls steel frames, three types of steel frames including 3-, 5-, and 7-storey were considered. These are typical numbers of storey, are chosen to cover low- to medium-rise framed buildings. To investigate the number of bays, it was assumed each type of the frames has the bays of 1, 2, 4 and 6. The storey height and bay length of models are fixed to 3.2m and 4m, respectively. It was also assumed that all bays of the models were constructed by masonry infill walls. Fig. 3 shows the configurations of a 5 storey, 4 bays model. All models were located on zone I (high seismic risk zone) and base shear values were computed based on Iranian seismic design code No.2800 [12].

The steel moment frames were designed without considering the contribution of the infill walls as it is conventional to ignore their effects through design process of these types of buildings.

A. Nonlinear static analysis of models

For evaluating the redundancy and its components in infills frames, nonlinear static analysis (pushover analysis) was developed to determine the frames response curves. The nonlinear static analysis is an incremental iterative method to obtain the base shear versus roof displacement (as a control point) of a structure. The inverse triangular load distribution was adopted for the pushover analysis in this study.

As mentioned previously, to model infill walls the single strut model is used because of its simplest model. Thus, the steel frames with unreinforced masonry walls are modeled as equivalent braced frames (EBF) with infill walls replaced by equivalent struts.
The beams, columns and infill walls were modeled with respect to behavior models that proposed in FEMA356. To model unreinforced masonry infill be equivalent strut element, the stress-strain relationship for unreinforced masonry infill struts is based on the strut models shown in Fig. 4, using $f_{\text{Strut}}=20 \text{ kg/cm}^2$, $\varepsilon_s=0.001$ and $\varepsilon_s=0.01$. A residual stress of 20% of the ultimate is used.

After preparing requirement of a pushover analysis, nonlinear static analysis was performed for the frames with and without infill walls. The results obtained from analysis the models involving the load-displacement can demonstrate the behavior of each frame under applied load. For instance, load-displacement curves of 1 and 2 bays 3 storey frames are presented in Fig. 5 and 6, respectively.

**B. Effect of infill walls on redundancy strength index**

The values of the redundancy strength index frame models are calculated by substituting the output of the pushover analysis, namely, base shear at the first significant yield point (the yield strength) and the base shear at the collapse (ultimate strength) into Eq. 9. In this equation the yield point for the both of with and without infill walls was assumed base shear in the first yield point of the one without infill. In fact collapse strength of non-redundant structure was the same for the models in order to having a proper comparison.

In Figs. 7, 8 and 9 the values of redundancy-strength index for three, five and seven storey frames with and without infill walls are plotted versus the number of bays.

The comparison of the figures shows that the presence of infill led to an increase of strength-redundancy index in the all structures. By considering infill for all of the bays the strength-redundancy index increased 50% averagely.
C. Effect of Infill Walls on Redundancy Variation Index

The number of plastic hinges developed during the pushover response analysis of the frame models was obtained. Assuming the value of average correlation coefficient between member strengths equal to 0.5 and using the number of plastic hinges the redundancy variation index values were calculated using Eq. 8.

Redundancy variation index for the frames with and without infill walls are presented in the Figs. 10, 11 and 12. It can be seen that in the three-storey frames with 1 and 2 bays the presence of infill walls led to the reduction of redundancy variation index. For the five-storey frames, except for the one with 1 bay which has no change of redundancy variation index, considering the infill increased the redundancy variation index. In the seven-storey frames the presence of the infill walls led to the increase of redundancy variation index.

D. Effect of infill walls on redundancy Modification Factor index

The effect of infill walls on redundancy modification factor were calculated in this section. The values of redundancy modification for three, five and seven storey frames with and without infill walls are plotted versus the number of bays in Figs. 13, 14 and 15. It can be observed that with considering of infill walls, the redundancy factor increases. Averagely, the 50% increase can be seen for any number of bays in case infill walls.
V. CONCLUSION

The influence of infill walls on the redundancy of the steel frames was studied. To aim to the purpose of this study, a proposed method to quantify redundancy was used to achieve the effect of adding masonry walls to the steel frames on the redundancy. The redundancy strength index and redundancy variation index, which account for both deterministic and the probabilistic nature of structural redundancy were derived for steel frame with infill walls. The obtained results can be assembled as:

1. The presence of infill walls led to an increase in ultimate strength and reducing the ultimate displacement.
2. By adding infill walls in steel frames the strength-redundancy index increased.
3. The presence of infill walls increases the redundancy factor. An increase of 50% in the results is seen.
4. In the three-storey frames with 1 and 2 bays the presence of infill walls led to the reduction of redundancy variation index, in others it increases.

REFERENCES