Impact of Loading Conditions on the Emission-Economic Dispatch

M. R. Alrashidi, and M. E. El-Hawary

Abstract—Environmental awareness and the recent environmental policies have forced many electric utilities to restructure their operational practices to account for their emission impacts. One way to accomplish this is by reformulating the traditional economic dispatch problem such that emission effects are included in the mathematical model. This paper presents a Particle Swarm Optimization (PSO) algorithm to solve the Economic-Emission Dispatch problem (EED) which gained recent attention due to the deregulation of the power industry and strict environmental regulations. The problem is formulated as a multi-objective one with two competing functions, namely economic cost and emission functions, subject to different constraints. The inequality constraints considered are the generating unit capacity limits while the equality constraint is generation-demand balance. A novel equality constraint handling mechanism is proposed in this paper. PSO algorithm is tested on a 30-bus standard test system. Results obtained show that PSO algorithm has a great potential in handling multi-objective optimization problems and is capable of capturing Pareto optimal solution set under different loading conditions.

Keywords—Economic emission dispatch, economic cost dispatch, particle swarm, multi-objective optimization.

I. INTRODUCTION

GLOBAL warming is partially blamed for some of the natural catastrophes that are taking places in many parts of the world like hurricane Katrina in the USA and the recent floods that hit parts of Asia. The rapid increase in greenhouse gas concentrations is one of the main factors that led to global warming. To reduce the effects of such unfortunate phenomena, a special attention must be made to pollution sources. Thermal power generation plants are major contributors to air pollution. Their main gaseous pollutants are carbon oxides (CO\textsubscript{x}), sulfur oxides (SO\textsubscript{x}), and nitrogen oxides (NO\textsubscript{x}) \cite{1}. In the past few decades, environmental awareness led to impose rigid environmental policies on power utilities to minimize their emissions. The emissions of air pollutants came under US federal regulation in 1963 when the Clean Air Act law was enacted \cite{2}. Consequently, power utilities had to re-adjust their operational practices to meet the new laws. Many solutions were proposed to reduce power plant emissions like installing post-combustion cleaning equipment, changing fuel type to fuel with less pollutants, or dispatching with emission considerations \cite{1}. The latter option is preferred in many cases due to economical reasons since no capital cost is needed and its immediate availability for short term operation.

The electric power industry restructuring has created a highly vibrant and competitive market that altered many aspects of the industry. A new operation philosophy has emerged to cope with these changes. Economic Cost Dispatch (ECD) is one of the areas that was greatly impacted as a result of power industry deregulation. The main goal of ECD is to allocate the optimal power outputs from different generating units at the lowest cost possible while meeting all system constraints. Emission Dispatch (ED) is similar to ECD with the objective to be minimized being emission instead of cost. The two functions are conflicting in nature and they both have to be considered simultaneously to find overall optimal dispatch. Emission-Economic Dispatch (EED) optimization problem is formed by combing the two objective functions. In multi-objective optimization there is no single optimal solution to any problem unless exact preference or “weight” of all objectives is known. This gives rise to finding a set of compromise solutions known as Pareto optimal solutions. When optimizing all objectives simultaneously, Pareto optimal solutions show the tradeoffs among conflicting objective functions.

Different techniques were proposed to solve the EED problem. In \cite{3}, the author utilized evolutionary algorithms to solve the EED problem. The authors of \cite{4} used probability security criteria to compute the solution of EED problem. Nanda et al. approached the same problem by employing linear and non-linear programming in \cite{5}. It is important to note that in all the literature found in \cite{3-5}, the EED problem was tackled considering only one loading condition for a given system. The impact of different loading conditions on the shape of Pareto optimal set was not addressed in the reported literatures.

In this paper, Particle Swarm Optimization (PSO) technique is proposed to solve the EED multi-objective problem by generating the Pareto optimal solution set under different loading conditions. The performance of the proposed technique is validated using a standard test system.

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II. PROBLEM FORMULATION

EED problem is composed of mainly two types of objective functions, ECD and ED subject to equality and inequality constraints. Each problem is detailed as follows [6]:

A. ECD Problem

The input/output fuel cost function of all generating units is typically modeled as a quadratic function as follows:

$$F(P_i) = \sum_{j=1}^{N} (a_j + b_j P_i + c_j P_i^2) \text{$/hr}$$  \hspace{1cm} (1)

where
- $a_j$, $b_j$, and $c_j$ represent to the cost function coefficients of the $i$-th generating unit.
- $P_i$ is the generating unit real power output.
- $N$ is number of generating units.

The ultimate goal of the ECD problem is to minimize the overall fuel cost function subject to the following constraints:
1. Generating unit capacity limits as inequality constraints

$$P_i \min \leq P_i \leq P_i \max$$  \hspace{1cm} (2)

2. Generation-demand balance as an equality constraint

$$\sum_{i=1}^{N} P_i - P_L - P_D = 0$$  \hspace{1cm} (3)

where
- $P_i$ is the overall system real power losses.
- $P_D$ is the total system real power demand.

Note that the system loss function is approximated by

$$P_L = \sum_{i=1}^{N} \sum_{j=1}^{N} (P \cdot B_{ij} \cdot P_j) \text{ MW}$$  \hspace{1cm} (4)

where
- $B_{ij}$ are elements of the B-coefficients loss matrix.

Equation (3) states that the total units generation shall meet system load demand and losses.

B. ED Problem

Different mathematical models were proposed to represent the emission function of thermal generating units [7;8]. In this study, the following emission function will be considered to model the total emissions of all generating units [3]:

$$E(P) = \sum_{i=1}^{N} \left( \alpha_i + \beta_i P_i + \gamma_i P_i^2 + \zeta_i e^{\delta_i P_i} \right) \text{ton/hr}$$  \hspace{1cm} (5)

where
- $\alpha_i$, $\beta_i$, $\gamma_i$, and $\lambda_i$ are the emission function coefficients of the $i$-th generating unit.

The aim of ED problem is to minimize total emission of all thermal units such that constraints (2) and (3) above are satisfied.

C. EED Multi-Objective Optimization Problem

The EED is formulated as a multi-objective optimization problem as follows:

$$\text{Min } z = [F(P), E(P)]$$  \hspace{1cm} (6)

subject to

$$g(P) = 0$$  \hspace{1cm} (7)

$$h(P) \leq 0$$  \hspace{1cm} (8)

One way to deal with multi-objective optimization problem is by assigning different weights for each objective. Then, weights are changed such that the entire set of Pareto optimal set is computed. Mathematically, the overall objective can be stated as follows:

$$\text{Min } z = w_i F(P) + w_j E(P)$$  \hspace{1cm} (9)

Subject to equations (2) and (3). Note that $\sum_{i=1}^{2} w_i = 1$.

III. PARTICLE SWARM OPTIMIZATION

Traditional optimization methods such as those described in references [9;10] are by far the most common optimization tools used in the industry. However, these techniques can encounter some difficulties such as getting trapped in local minima, increasing computational complexity, and being not applicable to certain objective functions. This led to the need of developing a new class of solution methods that can overcome these limitations. Heuristic optimization techniques are fast growing tools that can overcome most of the shortcomings found in derivative-based techniques.

Two scientists, Kennedy and Eberhart, first introduced Particle Swarm Optimization (PSO) in 1995 as a new heuristic method [11;12]. The original objective of their research was to graphically model the social behavior of bird flocks and fish schools. As their research progressed, they discovered that with some modifications their social behavior model can serve as a powerful optimizer. The first version of PSO was intended to handle only nonlinear continuous optimization problems. However, many advances in PSO development elevated its capabilities to handle a wide class of complex optimization problems involved in engineering and science. Summaries of recent advances are presented in [13] and [14].

Various versions of PSO algorithms were proposed but the most standard one is the one introduced by Shi and Eberhart [15]. Key attractive feature of PSO is its simplicity as it involves only two model equations. In PSO, the coordinates of each particle represent a possible solution associated with two vectors, the position ($x_i$) and velocity ($v_i$) vectors. The size of vectors $x_i$ and $v_i$ is equal to the problem space dimension. A swarm consists of number of particles “or possible solutions” that proceed (fly) through the feasible solution space to explore optimal solutions. Each particle updates its position based on its own best exploration; best swarm overall experience, and its previous velocity vector according to the following model:

$$v_{i+1} = \mu v_i + c_1 (p_{\text{best}_i} - x_i) + c_2 (g_{\text{best}} - x_i)$$  \hspace{1cm} (10)
where
- $c_1$ and $c_2$ are two positive constants
- $r_1$ and $r_2$ are two randomly generated numbers with a range of $[0,1]$
- $\mu$ is the inertia weight and it is defined as a function of iteration index $i$ as follows:

$$\mu(i) = \mu_{\text{max}} - \left( \frac{\mu_{\text{max}} - \mu_{\text{min}}}{\text{Max. Iter.}} \right) \cdot i$$

$pbest_i$ is the best position particle $i$ achieved based on its own experience
- $gbesti$ is the best particle position based on overall swarm experience

The PSO algorithm can be best described, in general, as follows:
1. For each particle, the position and velocity vectors are randomly initialized with the same size as the problem dimension.
2. Measure the fitness of each particle ($pbest$) and store the particle with the best fitness ($gbest$) value.
3. Update velocity and position vectors according to equations (10) and (11) for each particle.
4. Repeat steps 1-3 until a termination criterion is satisfied.

### A. Constraints Handling Mechanism

There are two types of constraints associated with the EED problem; equality and inequality constraints. The equality constraints in particular represent a challenge to most stochastic optimization algorithms since it is often hard to satisfy throughout the optimization process. In the context of PSO, constraints are handled as follows:

#### 1. Equality Constraints

A novel mechanism is proposed in this paper to handle this type of constraints for the EED problem. At each iteration, equation (3) is satisfied by following the simple yet effective algorithm:
1. Ignore network losses at first and randomly generate all unit’s power levels within their bounds except for the last unit, i.e. $[P_1, P_2, \ldots, P_{N-1}]$.
2. Calculate the last unit’s power level according to equation (13).

$$P_N = P_D - \left[ P_1 + P_2 + \ldots + P_{N-1} \right]$$

3. Calculate the network losses in accordance with equation (4).
4. Incorporate losses into power generation by adjusting the last unit’s power level as follows:

$$P_N = P_D + P_L - \left[ P_1 + P_2 + \ldots + P_{N-1} \right]$$

#### 2. Inequality Constraints

Particle’s position (i.e. power level) is checked after each iteration to ensure its compliance with bounds in equation (2). If any particle flies outside its bounds, its current position will be restored to its previous best position ($pbest$).

### IV. SIMULATION RESULTS AND DISCUSSION

The PSO program was written using Matlab and simulations were performed utilizing HP desktop with AMD Athlon 64 X2 dual core processor. Extensive testing was conducted to tune the parameters of the proposed approach in order to reach acceptable convergence characteristics. The selected tuned parameters are:
- Maximum velocity: 2.
- Population size: 20 particles.
- $C_1=C_2=1.25$.

The Inertia weight was kept between 0.4 and 0.9.

The PSO technique was tested on the 30-bus standard test system with six generating units and 41 interconnected transmission lines. Economical and environmental characteristics of all generation units are tabulated in Tables I and II respectively [3]. Four different loading conditions, namely $P_{D1} = 283.4$ (base load), $P_{D2} = 340.08$ MW, $P_{D3} = 396.76$ MW, and $P_{D4} = 453.44$ MW, were selected to test its impacts on the trade-off curves. Note that the selected loading conditions are spaced at 20% increments from the base load.

#### TABLE I

<table>
<thead>
<tr>
<th>Generator</th>
<th>a</th>
<th>b</th>
<th>c</th>
<th>$P_{\text{min}}$(p.u.)</th>
<th>$P_{\text{max}}$(p.u.)</th>
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<tr>
<td>1</td>
<td>0.04091</td>
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<td>0.06490</td>
<td>0.000200</td>
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<td>3</td>
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<td>-0.05094</td>
<td>0.04586</td>
<td>0.000001</td>
<td>8.0</td>
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<td>0.002000</td>
<td>2.0</td>
</tr>
<tr>
<td>5</td>
<td>0.04258</td>
<td>-0.05094</td>
<td>0.04586</td>
<td>0.000001</td>
<td>8.0</td>
</tr>
<tr>
<td>6</td>
<td>0.06131</td>
<td>-0.05555</td>
<td>0.05151</td>
<td>0.000010</td>
<td>6.667</td>
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#### TABLE II

<table>
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<th>Generator</th>
<th>$\alpha$</th>
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<th>$\gamma$</th>
<th>$\zeta$</th>
<th>$\lambda$</th>
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<tr>
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<td>0.000001</td>
<td>0.000001</td>
<td>0.000001</td>
<td>0.000001</td>
<td>0.000001</td>
</tr>
<tr>
<td>4</td>
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<td>0.000001</td>
<td>0.000001</td>
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<tr>
<td>5</td>
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<td>0.000001</td>
<td>0.000001</td>
<td>0.000001</td>
<td>0.000001</td>
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<tr>
<td>6</td>
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<td>0.000001</td>
<td>0.000001</td>
<td>0.000001</td>
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</tbody>
</table>

The two extreme points of Pareto front were computed first by minimizing each objective separately for each loading condition. Table III shows the optimization results for each loading condition.

#### TABLE III

<table>
<thead>
<tr>
<th>Loading (MW)</th>
<th>Fuel Cost ($/hr$)</th>
<th>Emission (ton/hr)</th>
</tr>
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<tr>
<td>$P_{D1}$ = 283.40</td>
<td>600.1118</td>
<td>0.19420</td>
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<tr>
<td>$P_{D2}$ = 340.08</td>
<td>729.2915</td>
<td>0.19656</td>
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<tr>
<td>$P_{D3}$ = 396.76</td>
<td>865.2600</td>
<td>0.20542</td>
</tr>
<tr>
<td>$P_{D4}$ = 453.44</td>
<td>1008.9449</td>
<td>0.22355</td>
</tr>
</tbody>
</table>

Fig. 1 shows the convergence characteristics of the proposed optimizer. It seems that PSO tends to have steady convergence characteristics as it approaches the optimal
solution or near optimal within reasonable number of iterations.

Then, the PSO Algorithm was used to handle the EED multi-objective problem by adopting the weighting method. Note that each given weight provides a single solution in Pareto optimal set. Fig. 2 shows the trade-off curves for all loading conditions. It is clear that Pareto fronts maintained similar patterns regardless of the loading conditions. This pattern seems to change somewhat proportionally among loading conditions. This indicates that dispatchers in power utilities might be able to forecast the shape of Pareto front for any intermediate loading condition other than those considered previously in early studies.

Fig. 1 PSO Convergence Characteristics

Fig. 2 Pareto Fronts for Different Loading Conditions

V. CONCLUSION

This paper presents a new solution method of the economic dispatch problem while accounting for the environmental impacts of generating units. The problem is formulated as a multi-objective optimization problem with two competing objectives. PSO based approach is developed to efficiently solve the problem with special emphasis on studying the impact of loading conditions on the shape of the trade-off curves. Results indicate that trade-off curves maintained their pattern regardless of the system loading conditions. This observation gives indication that Pareto fronts can be interpolated from base loading condition.

ACKNOWLEDGMENT

The first author would like to thank the Public Authority of Applied Education and Training for their moral and financial support (research grant: TS-07-14).

REFERENCES