A Distributed Group Mutual Exclusion Algorithm for Soft Real Time Systems

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Abstract—The group mutual exclusion (GME) problem is an interesting generalization of the mutual exclusion problem. Several solutions of the GME problem have been proposed for message passing distributed systems. However, none of these solutions is suitable for real time distributed systems. In this paper, we propose a token-based distributed algorithms for the GME problem in soft real time distributed systems. The algorithm uses the concepts of priority queue, dynamic request set and the process state. The algorithm uses first come first serve approach in selecting the next session type between the same priority levels and satisfies the concurrent occupancy property. The algorithm allows all n processors to be inside their CS provided they request for the same session. The performance analysis and correctness proof of the algorithm has also been included in the paper.

Keywords—Concurrency, Group mutual exclusion, Priority, Request set, Token.

I. INTRODUCTION

THE design of protocols for distributed real time systems is more challenging than that for normal distributed systems because the real time systems must satisfy stringent response time constraints in addition to the logical correctness of the system. Nevertheless, the distributed systems are emerging as a highly promising candidate for implementing the next generation of high performance real time systems. However, the distributed system must be fine tuned before they can be used to monitor and control critical real time systems. The real time systems (RTS) are generally classified as soft real time systems (SRTS) and hard real time systems (HRTS) [22]. In the soft real-time systems, the utility of the system goes down with every unit of time elapsed after missing the deadline. However, missing a deadline does not lead to catastrophic system failure in SRTS. The hard real-time systems are those in which the utility of a system becomes zero in the event of a missed deadline and missing a deadline could lead to a catastrophic system failure. Although, both paradigms namely, shared memory and message passing exist, we have considered the message passing systems only.

Resource sharing is an important aspect of the real time distributed systems. Some resources are inherently non-shareable and must be accessed in a mutually exclusive way. Many algorithms exist in the literature to solve the mutual exclusion problem [1, 10, 18, 19, 20] in message passing distributed systems. Some of these algorithms have been fine tuned to suit the needs of real time systems in [12, 13, 14, 15, 16, 21].

In [2] Joung proposed group mutual exclusion (GME) problem as generalization of classical mutual exclusion problem, and modeled it as congenial talking philosophers (CTP) problem. In group mutual exclusion a process request a session (alternatively called forum), before entering its Critical Section (CS), processes requesting for the same session are allowed to be in their CS simultaneously. However, processes requesting for different sessions must do so in a mutually exclusive way. The readers-writer problem can be considered as a special case of GME problem. In order to achieve this, we can use a common read session for all processes and a unique write session for each individual process.

The requirements for group mutual exclusion problem are:

Mutual exclusion: No two processes, requesting for a different session can be in their critical sections concurrently.

Starvation Freedom: A process attempting to attend a session will eventually succeed.

Concurrent Occupancy: If some process P, has requested for a session X and no other process is currently attending or requesting a different session, then P can attend X without waiting for any other process to leave the session.

The first algorithm for GME problem was given by Joung [2] for shared memory model. In [3] Joung proposed two algorithms RA1 and RA2 based on Ricart - Agrawala algorithm [10] to solve GME problem for message passing systems. Several non token-based algorithms for GME problem have been proposed in the literature [3,6,7,8]. Token-based algorithms for GME problem have been presented in [4,9,11,17]. However, none of these algorithms is suitable for real time systems. Mittal–Mohan algorithm [4] considers the concept of priority in selecting the next session type. However, the priority of a session is decided by the number of processes willing to attend that session. In Mittal-Mohan algorithm a requesting process can not assign priority to a request. Therefore, in its present form Mittal-Mohan algorithm can not be used for real time distributed systems.

The paper presents a token-based algorithms for solving GME problem for soft real time systems (SRTS). Our algorithm is based upon the concept of dynamic request sets.
The concept have been used earlier also [1 18], but to handle some other problem that is comparatively simple. In the proposed scheme, a captain process is responsible for the session initiation and sending start message to other processes requesting for the same session, called followers, in order to allow them to enter in CS.

The rest of the paper is organized as follows. We describe the system model and assumptions in section 2, the data structures and the messages used in our algorithm are explained in section 3 and the description of the algorithm is given in section 4. The correctness proof and performance analysis of the algorithm are given in section 5 and section 6 respectively. The concluding remarks are given in section 7.

II. THE SYSTEM MODEL

We assume an asynchronous distributed system. The system has $N$ sites, numbered as 1, 2, 3, … , $N$. The sites do not share any memory or global clock, and the only way of communication between sites, is through message passing. The system is fully logically connected, i.e. every site can send message to every other site. We assume that, at each site $i$, there exists exactly one process $P_i$. Once a process has requested for a session, it will not make new requests unless the old request is serviced. Each process $P_i$, also announces its priority $Z_i$ while requesting a session. A higher value of $Z_i$ indicates higher priority level. The lowest priority level is one and the highest priority level is $K$.

III. NOTATIONS

Each process may be in any one of the following 6 states:

(i) $R$ - requesting for a session.
(ii) $N$ - not requesting.
(iii) $EC$ - process is executing in its CS as captain.
(iv) $EF$ - process is executing in its CS as follower.
(v) $HI$ - process is holding token because, no pending request is there.
(vi) $HS$ - process is holding token because, some followers are still in their CS.

Every process $P_i$ stores following local variables-

- state - stores the current state of process $P_i$.
- $RS_i$ - stores the ids of all the processes, to which $P_i$ must send its request, in case it wishes to attend a session and not possessing the token.
- $SN_i$ - where $SN[i]=k$ denotes that $P_i$ knows about $k$ requests made by $P_i$.
- captain$_i$ - stores the id of the captain of the current session, if $P_i$ is in its CS as follower. Otherwise captain$_i$ is set to NULL.

The token in our algorithm contains following variables-

- $token.queue$ - $token.queue$ is a priority queue to store all pending requests. The requests for the same session are grouped together, and are treated as single entry in the queue. A priority level is associated with each entry in $token.queue$. The priority level of an entry is assigned equal to the priority of the highest priority process, requesting for the session, associated with the entry. The entry with highest priority level always remains at the head of the $token.queue$.

- $token.type$ - stores the type of the current session
- $token.followers$ - stores the number of follower processes still in their CS.

In our algorithm various messages are exchanged among processes in order to solve GME problem. We briefly describe each message.

request $(i, SN_i, X, Z_i)$ – When a process $P_i$ wishes to attend a session $X$ with priority $Z_i$, and $P_i$ is not holding the token then it sends a request message containing its id, sequence number of request, type of session requested and the priority of the process $P_i$ to all processes in its request set

- start $(i)$ - start message is sent to process to allow it to enter in CS as follower of $P_i$.
- complete $(i)$ - When a process $P_i$ executing in its CS as follower, comes out of CS, it sends a complete message to its captain.

$token$ $(token.queue, token.type, token.followers)$ - A unique token exists in the system and only the process holding the token can enter in its CS as captain. Whenever a session finishes and next session is selected, the token is passed to the new captain.

IV. DESCRIPTION OF THE ALGORITHM

The complete pseudo code of our algorithm is given in Appendix A; however, brief description of the algorithm is given in this section. Initially all processes are in state $N$, having their captain as NULL, all entries of $SN$ are zero and the Request set of each process contains ids of all other processes except itself. Only exception is process $P_1$. We assume that $P_1$ holds the token initially, therefore, the variable state$_1$ is set to HI and $RS_1$ is initialized to empty set.

A process $P_i$ wishing to attend a session $X$ with priority $Z_i$ and not possessing the token, sends its request to all members in its request set, changes its state to $R$ and waits for the token or start message. Upon receiving the token, $P_i$ initiates a new session and enters in its CS as captain along with its followers. In case $P_i$ receives a start message, it enters in its CS as a follower. If $P_i$ possesses an idle token it enters in its CS as captain. However, if $P_i$ is holding token in $HS$ state, it enters in its CS again only if the requested session is the same as the current session and the $token.queue$ is empty. Otherwise, the request is added in $token.queue$.

A procedure add_request $(i, X, Z)$ is called to accommodate a new request in $token.queue$ according to its priority level and the requested session, where $i$ is the id of the requesting process, $X$ is the session requested and $Z$ is the priority level of the request. There exists only one entry for a session. Also, a priority level is associated with each entry in $token.queue$. If the entry for the requested session $X$ already exists in $token.queue$, $P_i$ is also added in the list of processes requesting for session $X$. If the priority level of the newly arrived request is greater than the priority level of the entry for session $X$, the priority level of the entry for session $X$ is set
equal to the priority of the newly arrived request. After that
the entry for session X is moved forward according to its new
priority level in token.queue. On the other hand, if there is no
entry corresponding to session X then a new entry is created
and added in the token.queue according to its priority level.

When a process \( P \) receives a request \((j,SN,X,Z)\), it discards
the old request without taking any action. However, if the
request is new, \( P \) updates the value of \( SN[j] \). If \( P \) is not in its
request set, \( P \) adds \( P \) in its request set. \( P \) also sends a request
to \( P \), if it is requesting for a session. If \( P \) is holding an idle
token, it immediately sends it to \( P \). However, if \( P \) is holding
token in state EC or HS, it passes a start message to \( P \) only if
token.queue is empty and the session requested is the same as
the current session. Otherwise procedure \textit{add request}
is called to add the request of \( P \) in token.queue.

When a follower process comes out of its CS, it sends a
complete message to its captain; it changes its state to N and
sets its captain to NULL. However, when a captain process
comes out of its CS, it checks the number of followers still in
CS. If there are still some follower processes in their CS, the
captain changes its state to HS. If no follower process is in CS
and no pending requests are there, the captain process changes
its state to HI. However, if there are pending requests in the
token.queue, the captain process changes its state to R or N ,
depending upon whether its request is in token.queue or not,
removes next captain and its followers from the token.queue,
sends token, sends token to the next captain, and sends start messages to all
followers. Before sending the token to the next captain the
priority level of all entries in token.queue are incremented by
one.

Upon receiving a complete message the captain decrements
the variable token.followers by one. If the state of the captain
is HS and token.followers is zero, the captain changes its state
to HI if token.queue is empty. However, if token.queue is not empty,
the captain process changes its state to R or N depending upon whether its request is in token.queue or not,
removes the next captain and its followers from token.queue,
sends token, sends token to the next captain, and sends start messages to all
followers. The priority level of each entry is also incremented
by one before transferring the token in order to remove the
possibility of starvation.

The captain process on receiving token changes its state
to EC and enters in its CS. Upon receiving a start message, a
process changes its state to EF, sets the variable captain and
enters in its CS.

V. CORRECTNESS OF THE ALGORITHM

In this section we will show that our algorithm satisfies all
requirements, which are necessary for a solution of group
mutual exclusion problem.

A. Safety

The mutual exclusion requirement in GME problem says
that, no two processes requesting for a different session, must
be in their CS simultaneously. There exists only one token in
the system, and only the process holding the token can initiate
a session as a captain. The process holding the token can send
the start message to only those processes requesting for the
same session. Further the token is not transferred to another
process, until the current captain and all its followers have
come out of their CS. Therefore, no two processes requesting
for a different session, can be in their CS at the same time.

B. Freedom from Starvation

A priority queue is associated with the token to store the
pending requests. A priority level and a session type are
associated with each entry in the token.queue. The entry with
the highest priority level is always at the front of the
token.queue in order to favor sessions associated with higher
priority levels. However, an FCFS approach is used to select a
session, among sessions having same priority levels. Further,
the priority of long waiting processes is gradually enhanced
using the idea of aging [Silberschatz] in order to completely
remove the possibility, if any, of starvation. Whenever a new
session is selected the priority level of all sessions, whose
requests are stored in token.queue, is incremented by one.
Therefore, the process having lowest priority level will also be
able to attain highest priority level after K-1 session switches.

If a request for the current session type arrives at the
captain, it first checks whether the token.queue has any
pending requests or not. The captain sends start message to the
requesting process, only if the token.queue is empty. However,
if the token.queue is not empty, the request is added in the
token.queue. This entry policy reduces the concurrency
and hence the resource utilization, however, it removes the
possibility that the processes of a particular group keep on
requesting for the current session and not allowing other
processes to enter in their critical sections. Therefore, we can
say that, the sessions in our algorithm are served in a
starvation free manner.

C. Concurrent Occupancy

In the proposed algorithm, when a process starts a session
as a captain, it captures all the processes (requesting for the
same session), whose requests are stored in the token.queue, at
the time of entry in its CS. When the captain process is in state
EC or state HS and a request for the current session arrives, it
checks whether the token.queue is empty or not. If the
token.queue is empty, it immediately sends a start message to
the requesting process. The requesting process enters in its CS
upon receiving the start message. Hence, it is proved that our
algorithm satisfies the concurrent occupancy property.

VI. PERFORMANCE ANALYSIS OF THE ALGORITHM

In this section we analyze the performance of our
algorithms using following performance parameters: message
complexity /CS request, average message size, forum switch
complexity, maximum concurrency, and synchronization
delay. Forum switch complexity [2] and maximum
concurrency are applicable only for GME algorithms, not for
mutual exclusion algorithms.

Message complexity: The messages exchanged, during the
The execution of the algorithm are, request, token, start and complete. The request messages are sent by a requesting process to all processes in its request set. The maximum cardinality of a request set can be $n-1$; therefore a requesting process can send at the most $n-1$ request messages. Therefore, if a process enters in CS as captain, in the worst case, $n$ messages ($n-1$ ‘request’ messages and one token message), needs to be exchanged. However, in case of a follower process, in the worst case, $n+1$ message are required ($n-1$ ‘request’ messages, one ‘start’ message, and one ‘complete’ message). However, in the best case no message needs to be exchanged. If a process holding token in HI state, wish to attend a session, in that case a new session will be started immediately and the state of the process changes from HI to EC. No message exchange is required in this case.

**Average message size:** The Table I describes the various messages used in our algorithm and their sizes.

<table>
<thead>
<tr>
<th>Message Type</th>
<th>Size</th>
</tr>
</thead>
<tbody>
<tr>
<td>'request'</td>
<td>$O(1)$</td>
</tr>
<tr>
<td>'token'</td>
<td>$O(N)$</td>
</tr>
<tr>
<td>'start'</td>
<td>$O(1)$</td>
</tr>
<tr>
<td>'complete'</td>
<td>$O(1)$</td>
</tr>
</tbody>
</table>

Among the messages used in the algorithm, only the token has the size $O(N)$. However, the token is exchanged, only when a new session is initiated. Therefore, in the best case (all processes requesting for the same session), the average message size will be $O(1)$, because one token, $N-1$ ‘start’, $N-1$ ‘complete’ and some ‘request’ messages (depending upon the cardinality of the request sets at each site), will be exchanged. However, in the worst case (all processes requesting for a different session); $N$ token messages will be exchanged, besides the ‘request’ messages. In this case the average message size will be $O(N)$.

**Maximum concurrency:** In our algorithm the request of a process requesting for the current session can be fulfilled by the captain process, if no request for some other session is pending in the token.queue. Therefore, if all the processes are requesting for the same session, they can be in their CS concurrently. Hence, the maximum concurrency of our algorithm is $n$.

**Forum switch complexity:** The pending requests for a particular session in token.queue are grouped together and the requests for one session are treated as a single entry in token.queue. Therefore, at any point of time there can be at most $\min(n,m)$ entries in token.queue. If a process requests for a new session, which has no entry in token.queue till now, then a new entry is created and added at the tail of the queue. If we assume only one priority level, after a process has made a request, at most $\min(n,m)$ forum switchs can take place. However, in a prioritized environment where $k$ priority levels exist, a process with higher priority can be placed ahead of a lower priority process, even if the lower priority process entered the queue before the higher priority process. However, due to aging the priority of lower priority process(es) will increase with each forum switch and would succeed in attaining the highest priority level after at most $K-1$ session switches. Therefore, the forum switch complexity of the algorithm is $\max\left\{\min(n,m), (K-1)\right\}$.

**Synchronization delay:** The heavy load synchronization delay of the algorithm is $2T$ in the worst case and $T$ in the best case, where $T$ is the maximum message propagation delay.

Under heavy load conditions, there will always be some pending requests in token.queue, therefore, as soon as a captain comes out of CS and no follower is in its CS, the token is passed to the next captain, and the, heavy load synchronization delay is $T$. However, if the last process to come out is a follower, it will first send a complete message to the captain, which in turn finish the session and passes the token to next captain. Therefore, the synchronization delay in this case will be $2T$.

**VII. CONCLUSION AND FUTURE WORK**

In the present paper, we proposed a token-based algorithm for the group mutual exclusion problem which favors the requests with higher priority levels. This feature of the algorithm makes it suitable for soft real time distributed systems also. The introduction of priority makes a system susceptible to starvation problem. It has been taken care by using the idea of aging while maintaining the strongest fairness requirement that is FCFS, among sessions having same priority levels. The algorithm satisfies the mutual exclusion and concurrent occupancy. The algorithm has reduced forum switch complexity keeping maximum concurrency as $n$. To the best of our knowledge, the proposed work is the first algorithm on group mutual exclusion that allows a process to declare a priority level along with its request for a session and the algorithm favors the sessions with higher priority levels. Although, Mittal-Mohan algorithm [4] uses the concept of priority to enhance the resource utilization, they do not allow individual processes to assign priority level to their request. However, our algorithm allows individual processes to assign priority level to their request. This characteristic makes our algorithm suitable for use in soft real time environment. An interesting extension of the work could be making it suitable for hard real time systems, which have more stringent deadlines to meet.

**APPENDIX**

Appendixes, if needed, appear before the acknowledgment.

**A. The pseudo code of the algorithm**

**Code for initialization:**

```
For i = 1 to n
{ state[i] = N; captain[i] = NULL
RS = ids of all other processes except Pi
For j = 1 to n SN[j] = 0; }
```
\[
\begin{align*}
\text{state} &= \text{HI}; \quad RS = \emptyset \\
\text{token.type} &= \text{NULL}; \quad \text{token.queue} = \emptyset \\
\text{token.followers} &= 0
\end{align*}
\]

**Pj request for a forum X with priority Z:**

\(SN[i] = SN[i] + 1\)

\begin{align*}
\text{If} \quad (\text{state} = \text{HI}) \\
&\quad \{ \\
&\quad \quad \text{token.type} = X; \quad \text{state} = \text{EC} \\
&\quad \quad RS = \emptyset; \quad \text{Enter CS}
\}
\end{align*}

\begin{align*}
\text{Else if} \quad (\text{state} = \text{HS}) \\
&\quad \{ \\
&\quad \quad \text{If} \quad (\text{token.type} = 0) \&\& (\text{token.type} = X) \\
&\quad \quad \quad \{ \\
&\quad \quad \quad \quad \text{state} = \text{EC}; \quad \text{Enter CS}
\}
\end{align*}

\begin{align*}
\text{Else} \quad &\quad \text{call Add_request} (i, X, Z) \\
\text{Else} \\
&\quad \{ \\
&\quad \quad \text{state} = \text{R}; \quad \\
&\quad \quad \text{Send request} \quad (i, SN[i], X, Z) \quad \text{to all members of} \quad RS
\}
\end{align*}

**Pj receives request (j, SN[X, Z]):**

\begin{align*}
\text{If SN[j] = SN} & \quad \text{* otherwise old request} \\
&\quad \{ \\
&\quad \quad \text{SN[j]} = \text{SN} \\
&\quad \quad \text{If} \quad (\text{state} = \text{R}) \&\& (j \notin RS) \\
&\quad \quad \quad \{ \\
&\quad \quad \quad \quad \text{Add} \quad j \quad \text{to} \quad RS \\
&\quad \quad \quad \quad \text{Send request} \quad (i, SN[i], Y) \quad \text{to} \quad j
\}
\end{align*}

\begin{align*}
\text{Else if} \quad (\text{state} = \text{EC}) \\
&\quad \{ \\
&\quad \quad \text{If} \quad (\text{token.type} = X) \&\& (\text{token.type} = 0) \\
&\quad \quad \quad \{ \\
&\quad \quad \quad \quad \text{token.followers} = \text{token.followers} + 1 \\
&\quad \quad \quad \quad \text{Send start} \quad (i) \quad \text{to} \quad P_j
\}
\end{align*}

\begin{align*}
\text{Else} \quad &\quad \text{call add_request} (j, X, Z) \\
\text{Else if} \quad (\text{state} = \text{HI}) \\
&\quad \{ \\
&\quad \quad \text{Add} \quad j \quad \text{to} \quad RS; \quad \text{Send token} \quad P_j
\}
\end{align*}

\begin{align*}
\text{Else if} \quad (\text{state} = \text{HS}) \\
&\quad \{ \\
&\quad \quad \text{If} \quad (\text{token.type} = X) \&\& (\text{token.type} = 0) \\
&\quad \quad \quad \{ \\
&\quad \quad \quad \quad \text{token.followers} = \text{token.followers} + 1 \\
&\quad \quad \quad \quad \text{Send start} \quad (i) \quad \text{to} \quad P_j
\}
\end{align*}

\begin{align*}
\text{Else} \quad &\quad \text{call add_request} (j, X, Z) \\
\text{Else} \quad &\quad \text{Add} \quad j \quad \text{to} \quad RS
\}
\end{align*}

**Pj receives start (j):**

\(\text{captain} = j; \quad \text{State} = \text{EF}; \quad \text{Enter CS}
\]

**Pj exits from CS:**

\(\text{If state} = \text{EF}
\]

\[
\begin{align*}
\text{Send complete} \quad (i) \quad \text{to captain}, \quad \\
\text{captain} = \text{NULL}; \quad \text{state} = \text{N}
\end{align*}
\]

**Else**

\begin{align*}
\text{If} \quad (\text{token.followers} = 0) \&\& (\text{token.queue} = \emptyset) \\
&\quad \{ \\
&\quad \quad \text{state} = \text{HI}; \quad \text{token.type} = \text{NULL}
\}
\end{align*}

\begin{align*}
\text{Else} \quad &\quad \text{If} \quad (\text{token.followers} = 0) \&\& (\text{token.queue} = \emptyset) \\
&\quad \{ \\
&\quad \quad \text{If} \quad (i \text{'s request in token.queue}) \quad \text{state} = \text{N} \quad \text{else} \quad \text{state} = \text{R}
\}
\end{align*}

\begin{align*}
&\quad \quad \text{Increment priority level of all entries in token.queue by one} \\
&\quad \quad \text{Add all processes which are in token.queue and} \\
&\quad \quad \text{which can work as captain to} \quad RS
\end{align*}

\begin{align*}
&\quad \quad \text{Select new captain} \quad P_j \\
&\quad \quad \text{Remove Process} \quad j \quad \text{and its followers from the front of the} \\
&\quad \quad \text{queue (requesting for a session X)}
\end{align*}

\begin{align*}
&\quad \quad \text{token.type} = X \\
&\quad \quad \text{token.followers} = \text{number of follower processes}
\end{align*}

\begin{align*}
&\quad \quad \text{Send token} \quad (\text{token.queue}, \text{token.type}, \text{token.followers}) \quad \text{to} \quad P_j \\
&\quad \quad \text{Send start} \quad (j) \quad \text{to all followers}
\end{align*}

\begin{align*}
\text{Else} \quad &\quad \text{state} = \text{HS}
\end{align*}

**Pj receives complete(j):**

\(\text{token.followers} = \text{token.followers} - 1
\]

\begin{align*}
\text{If} \quad (\text{token.followers} = 0) \&\& (\text{state} = \text{HS}) \\
&\quad \{ \\
&\quad \quad \text{If} \quad (\text{token.queue} = \emptyset) \quad \text{state} = \text{HI}
\}
\end{align*}

\begin{align*}
\text{Else} \quad &\quad \text{If} \quad (\text{token.request} = \emptyset) \quad \text{state} = \text{N} \quad \text{else} \quad \text{state} = \text{R}
\end{align*}

\begin{align*}
&\quad \quad \text{Increment priority level of all entries in token.queue by one} \\
&\quad \quad \text{Add all processes which are in token.queue and can work} \\
&\quad \quad \text{as captain to} \quad RS
\end{align*}

\begin{align*}
&\quad \quad \text{Remove Process} \quad P_j \quad \text{and its followers from the front of the} \\
&\quad \quad \text{token.queue (requesting for X)}
\end{align*}

\begin{align*}
&\quad \quad \text{token.type} = X \\
&\quad \quad \text{token.followers} = \text{number of follower processes}
\end{align*}

\begin{align*}
&\quad \quad \text{Send token} \quad (\text{token.queue}, \text{token.type}, \text{token.followers}) \quad \text{to} \quad P_j \\
&\quad \quad \text{Send start} \quad (j) \quad \text{to all followers}
\end{align*}

**Pj receives token**

\(\text{state} = \text{EC}; \quad \text{Enter CS}; \quad RS = \emptyset
\]

**Procedure Add_request(i, X, Z)**

\begin{align*}
\text{If} \quad (\text{entry for session X already in token.queue}) \\
\quad \text{Add current request also in the list of requests for X}
\end{align*}

\begin{align*}
\text{If} \quad (X.\text{priority} < Z) \\
\quad X.\text{priority} = Z
\end{align*}

\begin{align*}
\text{else} \\
\quad \text{create a new entry in token.queue for session X}
\end{align*}

\begin{align*}
\quad \text{add this request at the rear of the token.queue}
\end{align*}

\begin{align*}
\quad X.\text{priority} = Z
\end{align*}

\begin{align*}
\quad Y = \text{session in entry ahead of} \quad \text{entry of X in token.queue}
\end{align*}

\begin{align*}
\quad \text{While} \quad (Y.\text{priority} < X.\text{priority}) \\
\quad \quad \text{Swap entry corresponding to session X with}
\end{align*}
entry corresponding to session Y
Y = session in entry ahead of entry of X in token.queue

REFERENCES
