Abstract—The paper examines the performance of bit-interleaved parity (BIP) methods in error rate monitoring, and in declaration and clearing of alarms in those transport networks that employ automatic protection switching (APS). The BIP-based error rate monitoring is attractive for its simplicity and ease of implementation. The BIP-based results are compared with exact results and are found to declare the alarms too late, and to clear the alarms too early. It is concluded that the standards development and systems implementation should take into account the fact of early clearing and late declaration of alarms. The window parameters defining the detection and clearing thresholds should be set so as to build sufficient hysteresis into the system to ensure that BIP-based implementations yield acceptable performance results.

Keywords—Automatic protection switching, bit interleaved parity, excessive bit error rate

I. INTRODUCTION

This paper examines the performance of bit-interleaved parity (BIP) methods in error rate monitoring, especially for high bit error rates. The term high here is relative. For example, when the BIP calculation is taken over 801 frames, a bit error rate (BER) greater than $10^{-3}$ is high, in that for values of BER above this level there is noticeable disparity between the BIP-based and exact probabilities of bit error in the BIP word. When the number of frames is 9720 the transition level is at a BER of $2 \times 10^{-3}$, and so on. The reason for this is given by (3) and (4) and depicted in Fig.4. The analysis presented here can be applied in SONET systems employing automatic protection switching (APS). Technical standards exist that specify requirements (and objectives) for declaring and clearing alarms. Details of SONET frame structure and automatic protection switching can be found in telecommunication standards [1-4] and other texts [5-8].

The protection mechanism can be of two types [6], the 1:1 and 1:n protection mechanisms. Fig.1 (a) depicts the 1:1 protection architecture where a protection interface is paired with each working interface. Fig.1 (b) depicts the other the 1:n protection architecture, consisting of a single protection facility for several working interfaces. In either case, when a working interface fails, traffic is automatically switched over to the protection interface. The failed working facility is marked with an x in both halves of Fig.1.

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the window containing the alarm is determined, and then the alarm is located within the window. Section IV discusses the clearing of the alarm. In a manner parallel to the declaration of the alarm, the window containing the alarm clear condition is first determined, then the alarm clear location is found within the window. Section IV presents the results and conclusions. Furthermore, the Appendix provides some information about the SONET STS-1 frame and some pertinent BIP count parameters.

II. BIT ERROR PROBABILITY IN A BIP WORD

Fig. 2 shows the bytes to be included in the computation of the bit interleaved parity (BIP) byte. The bit error for the bit in the second position is registered only if there is a total of an odd number of errors in the bits considered. If there is only one byte to consider, the probability that there is a bit error in the second bit position is $p$. Suppose $N-1$ bytes have been considered so far.

$$P_N = p \left[ 1 - P_{N-1} \right] + (1-p)P_{N-1}$$

These two give rise to the equation

$$P_N = P_{N-1}(1-p) + (1-P_{N-1})p$$

which is quickly rearranged as

$$P_N = (1-2p)P_{N-1} + p$$

with the initial condition that $P_0 = 0$.

It is easily verified that the solution to the above equation is

$$P_N = \frac{1}{2} \left[ 1 - (1 - 2p)^N \right]$$

Here $N$ is the number of bits included in the calculation. The exact bit error probability considers that there are $N$ bits and the probability of bit error is then the complement of the probability of no bit error in any of the second position bits. Accordingly the exact probability that a bit in the BIP word is registered as being in error is given by

$$P_{\text{Exact}} = 1 - \left( 1 - p \right)^N$$

In addition to the $N$ bytes included in (3) and (4) there is further potential for bit errors in the BIP byte of the current frame. This is compared with a calculated version of the BIP byte. Thus there are $N+1$ bytes to included in (3) and (4). These equations are compared in Fig. 3 for some typical values of $N$ listed in Table A1.

For $N=801$, the BIP-based (3) and the exact (4) values begin to diverge at a bit error rate of $2 \times 10^{-4}$, with the BIP-based value settling at 0.5, and the exact one at 1.0. This trend is seen for the other values of $N$. Indeed the point of separation of the two values gets smaller as $N$ increases, but for all the cases, the exact traces go to 1.0 whereas the BIP-based trace goes to 0.5. These can be derived from the expressions already given by letting $p$ tend to 1 in (3) and (4), respectively.

It is also observed that as $N$ increases, the curves in Fig. 3 shift towards the left, the BIP-based traces approaching a limit of 0.5, and the "exact" traces approach unity. Despite the above disparity of (3) and (4), it is possible to obtain working bit error rate monitoring methods based on BIP calculations using a window of an appropriate size and two thresholds one for declaring alarm, and the other for clearing.

A frame is considered "errored" when it has more than one bit error in the BIP byte. The probability $P_{FE}$ that a frame is errored is then

$$P_{FE} = \sum_{m=2}^{N} \frac{N!}{m!(N-m)!} \left( 1 - P_{N+1} \right) P_{N+1}^m$$

A window of size $M$ frames is used and an alarm is declared if there are $N_1$ or more errored frames in the window. Conversely, once an alarm is declared the alarm is cleared when there are $N_2$ or more non-errored frames in a window of size $M$ frames. Alarm declaration and clearing is cyclic
process which consists of four stages as shown in Fig.5. The analysis uses a sliding window in monitoring the BIP bit errors. A sliding window has many advantages over a jumping window. It is therefore widely used in many standard network protocols, and several authors[9-12] have provided recent analysis of the performance protocols based on sliding windows.

For the present objective, a sliding window will have the advantage over a jumping window in that the spread of the errors may traverse the boundary of a window, and a jumping window will fail to catch some of those error patterns that lie on the window boundaries. A sliding window, on the other hand will not miss such error patterns.

Returning to Fig.4, it is evident that first there is a hunt for the window containing the alarm, then the alarm is declared within the window. For clearing the alarm, there is a hunt for the window containing the alarm clear, followed by locating the alarm clear condition within the window. This cyclic process is repeated for as long as the system runs, being restarted only when the bit error estimates dictate that the window parameters M, N_1 and N_2 should be changed. The detailed descriptions of the components of Fig.3 are given in the sections that follow.

III. ALARM DECLARATION

The unconditional event that a window contains an alarm is equivalent to the event that there are at least N1 errored frames in a window of size M. This gives the probability P_{Decl} of alarm declaration as

$$P_{Decl} = \sum_{m=N_1}^{M} \binom{M}{m} P_{FE}^m (1 - P_{FE})^{M-m}$$  \hspace{1cm} (6)

To locate, declare and clear alarms, the implementation of the bit error rate monitoring uses a sliding window as depicted in Fig.4.

The process involves first hunting for the window containing the alarm, followed by locating the alarm within the window. The first block after the summation point represents the fact that the system waits for the occurrence of an “errored” frame.

The length K_0 of time in frames up to and including the errored frame is a geometrically distributed random variable satisfying the distribution

$$\text{Prob} \{ K_D = k \} = P_{FE}(1 - P_{FE})^{k-1} \hspace{1cm} k \geq 1$$  \hspace{1cm} (7)

whose moment generating function P_{KD}(z) is

$$P_{KD}(z) = \frac{zP_{FE}}{1 - z(1 - P_{FE})}$$  \hspace{1cm} (8)

Once the first errored frame is found, the system examines the subsequent frames keeping two counts, the number of frames examined, and the accumulated number of errored frames so far. If the number of frames reaches the window size before the number of errored frames reaches the threshold N_1, the search begins afresh. This is the event that there are fewer than N_1-1 errored frames in a window of size M-1, where appropriate allowance has been made for the fact that one errored frame occurs at the beginning of the target window. Thus the probability Q that the search begins afresh is then given by

$$Q = \sum_{m=0}^{N_1-2} \binom{M-1}{m} (1 - P_{FE})^{M-1-m} P_{FE}^m$$  \hspace{1cm} (9)

If the search results in locating the window with alarm, then point at which the alarm is declared has to be determined. There is no need to examine the whole window if the required number of errored frames has been reached. Thus the length of time to the declaration of the alarm is another random variable.

A. Waiting Time to Reach Window Containing Alarm

At this point it is reasonable to obtain an expression for the waiting time to the window containing the alarm. This is done via its moment generating function T_{WD}(z). The feedback diagram of Fig.4 can be used to give

$$T_{WD}(z) = \frac{z(1 - Q)P_{FE}}{1 - z(1 - P_{FE}) - z^2M^2P_{FE}}$$  \hspace{1cm} (10)

It is noted that for z = 1, the denominator of this expression is zero for Q = 1. The quantity in (10) being a moment generating function is required to be analytic inside and on the unit disc \{ z: \text{abs}(z) \leq 1 \}. This requirement will be violated for Q = 1. Indeed, the mean time to the alarm window which is obtained as

$$\frac{d}{dz} T_{WD}(z) \bigg|_{z=1} = \frac{1 + (M-1)QP_{FE}}{(1 - Q)P_{FE}}$$  \hspace{1cm} (11)

reveals that for values of Q close to 1, the system waiting time on average will be unacceptably large. Thus, the window parameters M and N_1 of the system must be chosen to ensure that Q is far enough away from 1. The observations made on (10) and (11) are a consequence of the sliding window
mechanism; it remains to be seen if the BIP calculations are sensitive to this fact. This is deferred to the results presented later.

**B. Locating the Alarm Within Window**

At this point the window containing the alarm has been found. An accumulated count is kept within the window the errored frames, and the alarm is declared as soon as the number of errored frames reaches the threshold $N_2$; there is no need to reach the end of the window. This is done in the block containing the moment generating function $T_{P}(z)$ in Fig.4. The probability $P_{\text{Decl}(j)}$ of declaring an alarm after $j$ frames is the event that in the preceding $j-1$ frames there are $N_1 - 2$ errored frames, followed by an errored frame. The probability of this event is then

$$P_{\text{Decl}(j)} = \binom{j-1}{N_1-2} (1-P_{FE})^{j-N_1+1}$$

(12)

To obtain the probability of an alarm declaration the expression in (12) can be summed for $j = N_1 - 1$ to $M - 1$, and incorporating a multiplying factor to cater for the fact that (12) is a conditional probability. This is a much longer method than the expression given in (6), which considers that there at least $N_1$ errored frames in the window for the alarm to be declared.

Given that the window containing the alarm has been located, the remaining waiting time $T_D$ to alarm declaration is given via its moment generating function $T_{D}(z)$ as

$$T_{D}(z) = \sum_{j=1}^{M-1} \binom{j-1}{N_1-2} (1-P_{FE})^{j-N_1+1} z^j$$

(13)

The random variables $T_{WD}$ and $T_{D}$ referred to in (10) and (13), respectively, are statistically independent. The total time $T_1$ to declare the alarm is the sum of these two.

$$T_1 = \frac{d}{dz} T_{WD}(z) \bigg|_{z=1} + \frac{d}{dz} T_{D}(z) \bigg|_{z=1}$$

(14)

Substituting (10) and (13) in (14) gives the mean alarm declaration time as

$$T_1 = \frac{(1-P_{FE})^M}{(1-P_{FE})^N_1} + \sum_{j=N_1-1}^{M-1} \binom{j}{N_1-1} \binom{N_1-1}{j} P_{FE}^j (1-P_{FE})^{N_1-j-1}$$

(15)

where $B(N_1-1, j, P_{FE})$ is the binomial probability of $N_1-1$ successes in $j$ Bernoulli trials, and $P_{FE}$ is the probability of success. This quantity along with the corresponding one for clearing of the alarm, is evaluated for different system parameters.

**IV. ALARM CLEARING**

The unconditional probability that a window of size $M$ contains an alarm clear condition is equivalent to the event that there are at least $N_2$ non-errored frames in a window of size $M$. In an analogous manner to (6), this gives the probability of alarm clearing as

$$P_{\text{Clear}} = \sum_{m=0}^{M-1} \binom{M-1}{m} P_{FE}^m (1-P_{FE})^{M-m}$$

(16)

The same sliding window above is used in the bit error rate monitoring to locate the condition to clear the alarm. This section is very similar to the preceding one, the difference being that whereas the hunt for the alarm counts the errored frames, here it is the non-errored frames that are counted, and the parameter $N_1$ is now replaced by $N_2$. With reference to Fig.4, the parameter $P_{FE}$ is replaced by $1-P_{FE}$, and $Q$ is replaced by $P$, defined similar to (9) as

$$P = \sum_{m=0}^{N_2-2} \binom{M-1}{m} P_{FE}^m (1-P_{FE})^{M-m}$$

(17)

and is the probability that there are fewer than $N_2 - 1$ non-errored frames in the window of size $M-1$. As before the length $K_C$ of time in frames up to and including the terminating non-errored frame is a geometrically distributed random variable satisfying the distribution

$$P_{\text{Clear}}(j) = \frac{z(1-P_{FE})}{1-zP_{FE}}$$

(19)

with the corresponding one for clearing of the alarm, is evaluated for different system parameters.

**A. Waiting Time to Reach Window To Clear Alarm**

Exploiting the similarity with the preceding development, the waiting time $T_{WC}$ in frames required to reach the window is defined via its moment generating function $T_{WC}(z)$ as

$$T_{WC}(z) = \frac{z(1-P_{FE})}{1-zP_{FE}-(M-1)(1-P_{FE})}$$

(18)

The mean time to the clear window is obtained as

$$\frac{d}{dz} T_{WC}(z) \bigg|_{z=1} = \frac{1+(M-1)(1-P_{FE})}{1-(1-P_{FE})}$$

(19)

As before, it also holds here that for values of $P$ close to 1, the system waiting time on average will be unacceptably large, which underscores once more the fact that the window parameters $M$ and $N_2$ must be chosen to ensure that $P$ is far enough away from 1.

**B. Locating the Alarm Clear Within Window**

The window containing the alarm clear condition having been located, the alarm clear is indicated as soon as the number of non-errored frames reaches the threshold $N_2$; there is no need to reach the end of the window. The probability $P_{\text{Clear}(j)}$ of clearing an alarm after $j$ frames is the event that in the preceding $j-1$ frames there are $N_2 - 2$ non-errored frames, followed by a non-errored frame. The probability of this event is then

$$P_{\text{Clear}(j)} = \binom{j-1}{N_2-2} P_{FE}^{N_2-2} (1-P_{FE})^{N_2-1}$$

(20)

The probability that an alarm is cleared within the window can be obtained by summing the above for $j = N_2 - 1$ to $M-1$. That is

$$P_{\text{Clear}} = \sum_{j=N_2-1}^{M-1} P_{\text{Clear}(j)}$$

(21)

The expressions in (16) and (21) are equivalent since they
refer to the same events. The results provided here are obtained based on \((16)\). In practice the parameters \(M\), \(N_1\) and \(N_2\) are chosen to ensure that the declaration and clearing probabilities meet certain requirements established by applicable telecommunication standards such as [1] and [3].

Having located the window containing the alarm clear, the next task is to locate the alarm clear within the window. The remaining waiting time \(T_C\) to alarm clearing is then given via its moment generating \(T(z)\) as

\[
T_C(z) = \sum_{j=1}^{M-1} \left[ \frac{(N_2 - 1)(N_2 - 2)}{N_2} P_{FE}^{j-1} (1 - P_{FE})^{N_2 - j} z^j \right]
\]  

(22)

The total time to clear the alarm \(T_2\) is the sum of these two

\[
T_2 = \frac{d}{dz}(T(z))_{z=1} + \frac{d}{dz}(T_C(z))_{z=1}
\]  

(23)

Substituting (18) and (22) in (23) gives the mean time in successes in \(j\) trials, and \(1-P_{FE}\) is the probability of success.

This quantity along with the corresponding one for the declaration of the alarm can be evaluated for different system parameters and the results compared for both the BIP-based and exact methods. Rather than clutter the presentation with too many results, only those corresponding to \(N=801\) are given here.

V. RESULTS AND CONCLUSION

From Fig.4 it has already been observed that the two expressions (3) and (4) for the bit error probability in the BIP byte deviate as the prevailing bit error rate increases, giving the first indication that the BIP-based results may differ from the actual situation. When the bit error rate is low, the deviation is small. The onset of deviation depends on, and decreases with, the number \(N\) of frames used in the BIP calculation. The sliding window parameters \(N = 801\), \(M = 64\), \(N_1 = 49\), and \(N_2 = 13\) were used to generate the results presented here. Table I and Fig.5 give the results for the declaration times in seconds for sliding window parameters indicated.

**TABLE I**

<table>
<thead>
<tr>
<th>BER</th>
<th>Alarm Declaration Times [s]</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>BIP-Based</td>
</tr>
<tr>
<td>1.0E-04</td>
<td>1.0101E+17</td>
</tr>
<tr>
<td>1.78E-04</td>
<td>7.5248E+13</td>
</tr>
<tr>
<td>3.16E-04</td>
<td>6.6734E+02</td>
</tr>
<tr>
<td>5.62E-04</td>
<td>1.4902E-02</td>
</tr>
<tr>
<td>1.0E-03</td>
<td>6.1415E-03</td>
</tr>
<tr>
<td>1.78E-03</td>
<td>6.1315E-03</td>
</tr>
<tr>
<td>3.16E-03</td>
<td>6.1297E-03</td>
</tr>
<tr>
<td>5.62E-03</td>
<td>6.1296E-03</td>
</tr>
<tr>
<td>1.0E-02</td>
<td>6.1296E-03</td>
</tr>
</tbody>
</table>

The declaration times while compliant with the requirements of the telecommunication standards, show a deviation. For example when \(BER = 5.62 \times 10^{-4}\) the declaration times are 14.502 ms (BIP) and 6.17 ms (exact). This indicates that the BIP-based system declares the alarm later than should be case.

This poses a challenge to the standards developers to ensure that the alarm declaration thresholds are set so that even though the alarm are set later the results can still be used to guarantee acceptable network performance.

**Fig. 5 Alarm declaration times for \(N = 801\)**

Table II and Fig.6 give the alarm clearing times. Evident from these results is the fact that the clearing times higher values of the bit error rate are much lower for the BIP-based sliding window that those obtained by the exact calculations. Indeed for \(BER = 1.78 \times 10^{-3}\) the clearing times are 186.424 seconds (BIP) and 7894.36 seconds (exact). This indicates that the BIP-based system clears the alarm too soon. Whereas the exact calculation indicates a clearing time of over 2 hours, the BIP-based version clears the alarm in just over 3 minutes.

**TABLE II**

<table>
<thead>
<tr>
<th>BER</th>
<th>Clearing Times [s]</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>BIP-Based</td>
</tr>
<tr>
<td>1.0E-04</td>
<td>6.13184E-03</td>
</tr>
<tr>
<td>1.78E-04</td>
<td>1.77282E-03</td>
</tr>
<tr>
<td>3.16E-04</td>
<td>2.47037E-04</td>
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<td>5.62E-04</td>
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<td>2.65636E-01</td>
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<td>1.78E-03</td>
<td>1.86424E+02</td>
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<td>3.16E-03</td>
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<td>6.35641E+03</td>
</tr>
<tr>
<td>1.0E-02</td>
<td>6.40483E+03</td>
</tr>
</tbody>
</table>

The scenario presented here is that the BIP-based result indicates an alarm declaration later and clears the alarm too soon. The sliding window algorithm to be established to take into account the fact that there may be delayed declaration and false clearing of the alarms. Fortunately, the standards developers have built some hysteresis into the requirements [1] to guard against false clearing.

The benefits of the analysis are to the standards developers
who must set the requirements to ensure that when BIP-based error monitoring is employed there will be disparities with the exact results. For the implementers of the network elements, the analysis presented here will be useful in setting the sliding window system parameters, to ensure that acceptable performance is achieved.

Fig. 6 Alarm clearing times for N=801

APPENDIX

Fig. A1 shows the SONET STS-1 frame indicating the overhead bytes and their functions. Table A1 shows the values of the number of frames used in the BIP calculations. The data in this table can be used together with (3) and (4) to obtain the probability that a bit in the BIP word is indicated as being in error.

TABLE A1.

<table>
<thead>
<tr>
<th>BIP Type</th>
<th>Signal</th>
<th>N</th>
</tr>
</thead>
<tbody>
<tr>
<td>B1</td>
<td>STM-0 / STS-1</td>
<td>810</td>
</tr>
<tr>
<td>B1</td>
<td>STM-1 / STS-3</td>
<td>2,430</td>
</tr>
<tr>
<td>B1</td>
<td>STM-4 / STS-12</td>
<td>9,720</td>
</tr>
<tr>
<td>B1</td>
<td>STM-12 / STS-48</td>
<td>29,160</td>
</tr>
<tr>
<td>B1</td>
<td>STM-48 / STS-192</td>
<td>87,480</td>
</tr>
<tr>
<td>B2</td>
<td>All signals</td>
<td>801</td>
</tr>
<tr>
<td>B3</td>
<td>VC-3 / STS-1</td>
<td>783</td>
</tr>
<tr>
<td>B3</td>
<td>VC-4 / STS-3c</td>
<td>2,349</td>
</tr>
</tbody>
</table>

REFERENCES

[2] ANSI T1.105: SONET - Basic Description including Multiplex Structure, Rates and Formats
[3] ANSI T1.105.01: SONET - Automatic Protection Switching

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He was a research associate and teaching assistant while a graduate student at McGill University. He joined MPB Technologies, Inc. in 1989, where he participated in a variety of projects, including meteor burst communication systems, satellite on-board processing, low probability of intercept radio, among others. In 1994 he joined INTELSAT where he initiated research and development work on the integration of terrestrial wireless and satellite systems. After working at COMSAT Labs. (1996-1997) on VSAT networks, and TranSwitch Corp.(1998-2002) on product definition and architecture, he returned to Kenya, where since 2003 he has been with Department of Electrical and Information Engineering, University of Nairobi.

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