Abstract—This paper presents the use of anti-sway angle control approaches for a two-dimensional gantry crane with disturbances effect in the dynamic system. Delayed feedback signal (DFS) and proportional-derivative (PD)-type fuzzy logic controller are the techniques used in this investigation to actively control the sway angle of the rope of gantry crane system. A nonlinear overhead gantry crane system is considered and the dynamic model of the system is derived using the Euler-Lagrange formulation. A complete analysis of simulation results for each technique is presented in time domain and frequency domain respectively. Performances of both controllers are examined in terms of sway angle suppression and disturbances cancellation. Finally, a comparative assessment of the impact of each controller on the system performance is presented and discussed.

Keywords—Gantry crane, anti-sway control, DFS controller, PD-type Fuzzy Logic Controller.

I. INTRODUCTION

THE main purpose of controlling a gantry crane is transporting the load as fast as possible without causing any excessive sway at the final position. Research on the control methods that will eliminate sway angle of gantry crane systems has found a great deal of interest for many years. Active sway angle control of gantry crane consists of artificially generating sources that absorb the energy caused by the unwanted sway angle of the rope in order to cancel or reduce their effect on the overall system. Lueg in 1930 [1], is among the first who used active vibration control in order to cancel noise vibration.

Various attempts in controlling gantry crane systems based on open loop system were proposed. For example, open loop time optimal strategies were applied to the crane by many researchers such as discussed in [2,3]. They came out with poor results because open loop strategy is sensitive to the system parameters (e.g. rope length) and could not compensate for wind disturbances. Another open loop control strategies is input shaping [4,5,6]. Input shaping is implemented in real time by convolving the command signal with an impulse sequence. An IIR filtering technique related to input shaping has been proposed for controlling suspended payloads [7]. Input shaping has been shown to be effective for controlling oscillation of gantry cranes when the load does not undergo hoisting [8, 9].

On the other hand, feedback control which is well known to be less sensitive to disturbances and parameter variations [11] is also adopted for controlling the gantry crane system. Recent work on gantry crane control system was presented by Omar [1]. The author had proposed proportional-derivative PD controllers for both position and anti-sway controls. Furthermore, a fuzzy-based intelligent gantry crane system has been proposed [12]. The proposed fuzzy logic controllers consist of position as well as anti-sway controllers. However, most of the feedback control system proposed needs sensors for measuring the cart position as well as the load sway angle.

This paper presents investigations of anti-sway angle control approach in order to eliminate the effect of disturbances applied to the gantry crane system. A simulation environment is developed within Simulink and Matlab for evaluation of the control strategies. In this work, the dynamic model of the gantry crane system is derived using the Euler-Lagrange formulation. To demonstrate the effectiveness of the proposed control strategy, the disturbances effect is applied at the hoisting rope of the gantry crane. This is then extended to develop a feedback control strategy for sway angle reduction and disturbances rejection. Two feedback control strategies which are Delayed feedback signal and PD-type fuzzy logic controller are developed in this simulation work. Performances of each controller are examined in terms of sway angle suppression and disturbances rejection. Finally, a comparative assessment of the impact of each controller on the system performance is presented and discussed.

II. GANTRY CRANE SYSTEM

The two-dimensional gantry crane system with its payload considered in this work is shown in Fig. 1, where \( x \) is the horizontal position of the cart, \( l \) is the length of the rope, \( \theta \) is the sway angle of the rope, \( M \) and \( m \) is the mass of the cart and payload respectively. In this simulation, the cart and payload can be considered as point masses and are assumed to move in two-dimensional, x-y plane. The tension force that may cause the hoisting rope elongate is also ignored. In this study the length of the cart, \( l = 1.00 \text{ m} \), \( M = 2.49 \text{ kg} \), \( m = 1.00 \text{ kg} \) and \( g = 9.81 \text{ m/s}^2 \) is considered.
III. DYNAMIC MODELLING OF THE GANTRY CRANE SYSTEM

This section provides a brief description on the modelling of the gantry crane system, as a basis of a simulation environment for development and assessment of the input shaping control techniques. The Euler-Lagrange formulation is considered in characterizing the dynamic behaviour of the crane system incorporating payload.

Considering the motion of the gantry crane system on a two-dimensional plane, the kinetic energy of the system can thus be formulated as

\[
T = \frac{1}{2} M \ddot{x}^2 + \frac{1}{2} m (\ddot{x}^2 + \dot{\theta}^2 + 2 \ddot{x} \dot{\theta} \cos \theta + 2 \dot{x} \ddot{\theta} \cos \theta) \tag{1}
\]

The potential energy of the beam can be formulated as

\[
U = -mg \ell \cos \theta \tag{2}
\]

To obtain a closed-form dynamic model of the gantry crane, the energy expressions in (1) and (2) are used to formulate the Lagrangian \( L = T - U \). Let the generalized forces corresponding to the generalized displacements \( \vec{q} = \{x, \theta\} \) be \( F = \{F_x, 0\} \). Using Lagrangian’s equation

\[
\frac{d}{dt} \left( \frac{\partial L}{\partial \dot{q}_j} \right) - \frac{\partial L}{\partial q_j} = F_j \quad j = 1, 2 \tag{3}
\]

the equation of motion is obtained as below,

\[
F_x = (M + m) \ddot{x} + m \dot{\theta} \cos \theta - m \dot{\theta}^2 \sin \theta + 2m \dot{\theta} \dot{x} \cos \theta + 2m \dot{x} \ddot{\theta} \cos \theta \tag{4}
\]

\[
I \ddot{\theta} + 2I \dot{\theta} + \dot{x} \cos \theta + g \sin \theta = 0 \tag{5}
\]

The model of the uncontrolled system can be represented in a state-space form as

\[
x = Ax + Bu
\]

\[
y = Cx
\]

with the vector \( x = [x \quad \theta \quad \dot{x} \quad \dot{\theta}]^T \) and the matrices \( A \) and \( B \) are given by

\[
A = \begin{bmatrix}
0 & 0 & 1 & 0 \\
0 & 0 & 0 & 1 \\
0 & \frac{mg}{M} & 0 & 0 \\
0 & -\frac{(M + m)g}{MI} & 0 & 0
\end{bmatrix}, \quad B = \begin{bmatrix}
0 \\
0 \\
\frac{1}{M} \\
-\frac{1}{MI}
\end{bmatrix} \tag{7}
\]

IV. CONTROLLER DESIGN

In this section, two feedback control strategies (DFS and PD-type fuzzy logic controller) are proposed and described in detail. The main objective of the feedback controller in this study is to suppress the sway angle due to disturbances effect. All the feedback control strategies are incorporated in the closed-loop system in order to eliminate the effect of disturbances.

A. Delayed Feedback Signal Controller

In this section, the control signal is calculated based on the delayed position feedback approach described in (8) and illustrated by the block diagram shown in Fig. 2.

\[
u(t) = k(y(t) - y(t - \tau)) \tag{8}
\]

Substituting Equation (8) into Equation (6) and taking the Laplace transform gives

\[
sIx(s) = Ax(s) - kBC(1 - e^{-s\tau})x(s) \tag{9}
\]

The stability of the system given in (9) depends on the roots of the characteristic equation.
\[ \Delta(s, \tau) = |sI - A + kBC(1 - e^{-s\tau})| = 0 \]  \hspace{1cm} (10)

Equation (10) is transcendental and results in an infinite number of characteristic roots [13]. Several approaches dealing with solving retarded differential equations have been widely explored. In this study, the approach described in [14] will be used on determining the critical values of the time delay \( \tau \) that result in characteristic roots of crossing the imaginary axes. This approach suggests that Equation (10) can be written in the form

\[ \Delta(s, \tau) = P(s) + Q(s)e^{-s\tau} \]  \hspace{1cm} (11)

\( P(s) \) and \( Q(s) \) are polynomials in \( s \) with real coefficients and \( \text{deg}(P(s)) = n > \text{deg}(Q(s)) \) where \( n \) is the order of the system. In order to find the critical time delay \( \tau \) that leads to marginal stability, the characteristic equation is evaluated at \( s = j\omega \). Separating the polynomials \( P(s) \) and \( Q(s) \) into real and imaginary parts and replacing \( e^{j\omega \tau} \) by \( \cos(\omega \tau) - j\sin(\omega \tau) \), Equation (11) can be written as

\[ \Delta(j\omega, \tau) = P_R(\omega) + jP_I(\omega) + (Q_R(\omega) + jQ_I(\omega))(-\sin(\omega \tau) - j\cos(\omega \tau)) \]  \hspace{1cm} (12)

The characteristic equation \( \Delta(s, \tau) = 0 \) has roots on the imaginary axis for some values of \( \tau \geq 0 \) if Equation (12) has positive real roots. A solution of \( \Delta(j\omega, \tau) = 0 \) exists if the magnitude \( |\Delta(j\omega, \tau)| = 0 \). Taking the square of the magnitude of \( \Delta(j\omega, \tau) \) and setting it to zero lead to the following equation

\[ P_R^2 + P_I^2 - (Q_R^2 + Q_I^2) = 0 \]  \hspace{1cm} (13)

By setting the real and imaginary parts of Equation (13) to zero, the equation is rearranged as below

\[
\begin{bmatrix}
Q_R & Q_I \\
Q_I & -Q_R
\end{bmatrix}
\begin{bmatrix}
\cos \beta \\
\sin \beta
\end{bmatrix} =
\begin{bmatrix}
-P_R \\
-P_I
\end{bmatrix},
\]

where \( \beta = \omega \tau \).

Solving for \( \sin \beta \) and \( \cos \beta \) gives

\[ \sin(\beta) = \frac{(-P_RQ_I + P_IQ_R)}{(Q_R^2 + Q_I^2)} \]  and
\[ \cos(\beta) = \frac{(-P_RQ_I - P_IQ_R)}{(Q_R^2 + Q_I^2)} \]

The critical values of time delay can be determined as follows: if a positive root of Equation (13) exists, the corresponding time delay \( \tau \) can be found by

\[ \tau_k = \frac{\beta}{\omega} + \frac{2k\pi}{\omega} \]  \hspace{1cm} (15)

where \( \beta \in [0, 2\pi] \). At these time delays, the root loci of the closed-loop system are crossing the imaginary axis of the \( s \)-plane. This crossing can be from stable to unstable or from unstable to stable. In order to investigate the above method further, the time-delayed feedback controller is applied to the single-link flexible manipulator. Practically, the control signal for the DFS controller requires only one position sensor and uses only the current output of this sensor and the output \( \tau \) second in past. There is only two control parameter: \( k \) and \( \tau \) that needs to be set. Using the stability analysis described in [14], the gain and time-delayed of the system is set at \( k = 168.08 \) and \( \tau = 13.60 \). The control signal of DFS controller can be written as below

\[ u_{DFS}(t) = 168.08(\theta(t) - \theta(t - 13.60)) \]

B. PD-type Fuzzy Logic Controller

A PD-type fuzzy logic controller utilizing sway angle and sway velocity feedback is developed to control the rigid body motion of the system. The hybrid fuzzy control system proposed in this work is shown in Fig. 3, where \( \theta \) and \( \dot{\theta} \) are the sway angle and sway velocity of the hoisting rope, whereas \( k_1, k_2 \) and \( k_3 \) are scaling factors for two inputs and one output of the fuzzy logic controller used with the normalised universe of discourse for the fuzzy membership functions.

![Fig. 3 PD-type Fuzzy Logic Controller structure](image)

In this paper, the sway velocity is measured from the system instead of deriving it with the equation above. Triangular membership functions are chosen for sway angle, sway velocity, and force input with 50% overlap. Normalized universes of discourse are used for both sway angle and velocity and output force. Scaling factors \( k_1 \) and \( k_2 \) are chosen in such a way as to convert the two inputs within the universe of discourse and activate the rule base effectively, whereas \( k_3 \) is selected such that it activates the system to generate the desired output. Initially all these scaling factors are chosen based on trial and error. To construct a rule base, the sway angle, sway velocity, and force input are partitioned into five primary fuzzy sets as
Sway angle $A = \{\text{NM NS ZE PS PM}\}$,
Sway velocity $V = \{\text{NM NS ZE PS PM}\}$,
Force $U = \{\text{NM NS ZE PS PM}\}$,
where $A$, $V$, and $U$ are the universes of discourse for sway angle, sway velocity and force input, respectively. The $n$th rule of the rule base for the FLC, with angle and angular velocity as inputs, is given by

$$R_n: \text{IF}\ (\theta \text{ is } A_i) \text{ AND } (\dot{\theta} \text{ is } V_j) \text{ THEN } (u \text{ is } U_k),$$

where $R_n$, $n=1,2,\ldots,N_{\text{max}}$, is the $n$th fuzzy rule, $A_i$, $V_j$, and $U_k$, for $i,j,k = 1,2,\ldots,5$, are the primary fuzzy sets.

A PD-type fuzzy logic controller was designed with 11 rules as a closed loop component of the control strategy for maintaining suppressing the sway angle due to disturbances effect. The rule base was extracted based on underdamped system response and is shown in Table 1. The control surface is shown in Fig. 4. The three scaling factors, $k_1$, $k_2$, and $k_3$ were chosen heuristically to achieve a satisfactory set of time domain parameters. These values were recorded as, $k_1 = 1.02$, $k_2 = 0.30$ and $k_3 = 1.0$.

<table>
<thead>
<tr>
<th>No.</th>
<th>Rules</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.</td>
<td>If ( $\theta$ is NM) and ( $\dot{\theta}$ is ZE) then (u is PM)</td>
</tr>
<tr>
<td>2.</td>
<td>If ( $\theta$ is NS) and ( $\dot{\theta}$ is ZE) then (u is PS)</td>
</tr>
<tr>
<td>3.</td>
<td>If ( $\theta$ is NS) and ( $\dot{\theta}$ is PS) then (u is ZE)</td>
</tr>
<tr>
<td>4.</td>
<td>If ( $\theta$ is ZE) and ( $\dot{\theta}$ is NM) then (u is PM)</td>
</tr>
<tr>
<td>5.</td>
<td>If ( $\theta$ is ZE) and ( $\dot{\theta}$ is NS) then (u is PS)</td>
</tr>
<tr>
<td>6.</td>
<td>If ( $\theta$ is ZE) and ( $\dot{\theta}$ is ZE) then (u is ZE)</td>
</tr>
<tr>
<td>7.</td>
<td>If ( $\theta$ is ZE) and ( $\dot{\theta}$ is PS) then (u is NS)</td>
</tr>
<tr>
<td>8.</td>
<td>If ( $\theta$ is ZE) and ( $\dot{\theta}$ is PM) then (u is NM)</td>
</tr>
<tr>
<td>9.</td>
<td>If ( $\theta$ is PS) and ( $\dot{\theta}$ is NS) then (u is ZE)</td>
</tr>
<tr>
<td>10.</td>
<td>If ( $\theta$ is PS) and ( $\dot{\theta}$ is ZE) then (u is NS)</td>
</tr>
<tr>
<td>11.</td>
<td>If ( $\theta$ is PM) and ( $\dot{\theta}$ is ZE) then (u is NM)</td>
</tr>
</tbody>
</table>

Table I
LINGUISTIC RULES OF FLC

Fig. 4 Control surface of the Fuzzy Logic Controller

V. IMPLEMENTATION AND RESULTS
In this section, the proposed control schemes are implemented and tested within the simulation environment of the gantry crane system and the corresponding results are presented. The control strategies were designed by undertaking a computer simulation using the fourth-order Runge-Kutta integration method at a sampling frequency of 1 kHz. The system responses namely sway angle of the hoisting rope and its corresponding power spectral density (PSD) are obtained. In all simulations, the initial condition $x_0 = [0 \ 1.5 \ 0 \ 0]^T$ was used. This initial condition is considered as the disturbances applied to the gantry crane system. The first three modes of swaying frequencies of the system are considered, as these dominate the dynamic of the system. Two criteria are used to evaluate the performances of the control strategies:

1. Level of swaying angle reduction at the natural frequencies. This is accomplished by comparing the power spectral density response of the controller and open loop system.
2. Disturbance cancellation. The capability of the controller to achieved zero sway angles.

The open loop responses of the free end of the sway angle of the hoisting rope were considered as the system response with disturbances effect and will be used to evaluate the performance of feedback control strategies. It is noted that, in open loop configuration, the sway angle start to oscillate between ±1.5 rad and the sway frequencies of the hoisting rope under disturbances effect were obtained as 0.3925 Hz, 1.276 Hz and 2.159 Hz for the first three modes of swaying frequencies.

The system responses of the gantry crane system with the delayed feedback signal controller (DFS) are shown in Fig. 5 and 6. The overall result demonstrates that, the DFS controller can handle the effect of disturbances in the system by compensating the value of gain and time delayed in order to achieve zero radian steady state conditions. This is evidenced in sway angle of hoisting rope response as shown in Fig. 5 whereas the amplitudes of sway angle were reduced in a very fast response as compared to the open loop response. The sway angle settled down at 0.875 s with maximum overshoot of -0.1346 rad. The suppression of sway angle can be clearly demonstrated in frequency domain results as the magnitudes of the PSD at the natural frequencies were significantly reduced.

Fig. 7 and 8 show the closed loop system responses of the gantry crane system under PD-type fuzzy logic controller for the sway angle of the hoisting rope and its PSD results respectively. The results demonstrated that the sway amplitude in the sway angle responses was reduced as compared to the open loop response. The sway angle response also shows a similar pattern as the case of DFS controller with the maximum overshoot of -0.0420 rad and settled down at 0.771 s. It is noted that the PD-type fuzzy logic controller can eliminate the impact of disturbances in a faster response with minimum overshoot as compared to the DFS controller. The
PSD result shows that the magnitudes of sway angle were significantly reduced especially for the first three modes of swaying frequencies.

For comparative assessment, the levels of sway reduction of the hoisting rope using DFS and PD-type fuzzy logic controller are shown with the bar graphs in Fig. 9. The result shows that the DFS controller achieved higher level of swaying angle reduction than the PD-type fuzzy logic controller with the value of 60.06 dB and 57.08 dB for the first mode of swaying frequencies respectively. However, in the second mode, the level of reduction for PD-type fuzzy logic controller is higher than the DFS controller with the value of 15.85 dB and 11.43 dB respectively. For the third mode, both controller shows almost similar impact in level of swaying angle reduction with the value of 5.5 dB. Therefore, it can be concluded that overall the DFS and PD-type fuzzy logic controller provide better performance in level of swaying angle reduction.

VI. CONCLUSION

Investigations into anti-sway techniques of a gantry crane system with disturbances effect using the DFS and PD-type fuzzy logic controller have been presented. Performances of the controller are examined in terms of sway angle suppression and disturbances cancellation. The results demonstrated that the effect of the disturbances in the system...
can successfully be handled by DFS and PD-type fuzzy logic controller. A significant reduction in the system swaying for three modes of swaying frequencies has been achieved with both controllers.

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REFERENCES


M.A. Ahmad is born at Colorado, United State of America in 1983. He received his first degree in B.Eng. Electrical Mechatronics in 2006 from University of Technology Malaysia (UTM) in Johor, Malaysia. In 2008, he received a Master degree in M.Eng. Mechatronics and Automatic Control from University of Technology Malaysia (UTM). He has an experience as a research assistant at Robotic Laboratory in University of Technology Malaysia (UTM) in 2005. As a research assistant, he has developed an experimental investigation for vibration control of flexible robot manipulator. Currently, he is a lecturer in Faculty of Electrical and Electronics Engineering, Universiti Malaysia Pahang (UMP). He has currently published more than 15 paper work in various International Conference and has already published three Journals in the field of vibration control. The latest publications are: Vibration and input tracking control of a flexible manipulator using hybrid fuzzy logic controller (Kagawa, Japan: IEEE International Conference of Mechatronics and Automation, 2008) Hybrid input shaping and feedback control schemes of a flexible robot manipulator (Seoul, Korea: 17th World Congress The International Federation of Automatic Control, 2008). His current research interests are vibration control, input shaping, gantry crane and flexible robot manipulator. Mr. Ahmad became a member of Board of Engineer Malaysia (BEM) starting from 2006 and a member of Institution of Engineer Malaysia (IEM) starting from 2008.