Determination of Optimum Length of Frames and Number of Vectors to Compress ECG Signals

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Abstract—In this study, to compress ECG signals, KLT (Karhunen-Loeve Transform) method has been used. The purpose of this method is to perform effective ECG coding by a correlation between the length of frames and the number of vectors of ECG signals.

Keywords—ECG Compression, EKG Compression.

I. INTRODUCTION

Today, the basic struggle of signal processing groups is to increase the speed of data transmission for different transmission environments which have limited bandwidth. The other struggle in different data storage environments is to work on how the data is stored in the less space as much as possible. The medical signal processing, the speech coding, the speech recognition, the speech classification, the information security etc. for different aims like methods frequency domain, time domain, transform domain, fuzzy logic and neural networks are used [1,2,3,4,5,6,7].

The reason of why biomedical data processing, storing and transmitting important is to use in medical diagnosis and treatment. Because of the needed very large capacity, data storing, transmitting and reconstructing may cause speed and memory problems particularly in biomedical signals (ECG, EEG, EMG etc.). The mentioned problems are sorted out by compressing the signals in appropriate ratios by keeping the important parts of the signals [8].

The electrocardiogram (abbreviated ECG or EKG) is a graphic representation of the heart's electrical activity. If we wish to store an ECG recording that spans more than a few minutes, some form of data compression is highly desirable.

II. METHOD

$x(n)$ discrete signal which has N length can be defined as follows;

$$x(n) = \sum_{i=1}^{N} x_i \delta(n-i)$$

(1)

In here, $\delta(n)$ is unit sample sequence and $x_i$ is amplitude of the signal. By dividing the signal into equal parts;

$$X_{F_k} = \begin{bmatrix} x_{(k-1)L_F+1} \\ x_{(k-1)L_F+2} \\ \vdots \\ x_{kL_F} \end{bmatrix} \quad k = 1,2 \ldots N_{F_F}$$

(2)

following the result is taken.

$$X_F = \begin{bmatrix} X_{F_1} & X_{F_2} & \cdots & X_{F_{N_{F_F}}} \end{bmatrix}$$

(3)

$X_{F_k}$ is the signal of $k^{th}$ frame, $N_{F_F}$ is the sum of the frames. For each frame, toeplitz matrix is calculated as shown.
below.

\[
r_k(l+1) = \frac{1}{L_F-l} \sum_{j=[(k-l)+1]}^{[(k-l)+1]} X_j X_{j+l}
\]

(3)

\[
l = 0,1,2,\ldots,(L_F-1)
\]

as indicated,

\[
R_{\ell m} = \begin{bmatrix}
    r_1(l) & r_1(2) & r_1(3) & \cdots & r_1(L_F) \\
r_2(l) & r_2(1) & r_2(2) & \cdots & r_2(L_F-1) \\
r_3(l) & r_3(2) & r_3(1) & \cdots & r_3(L_F-2) \\
\vdots & \vdots & \vdots & \ddots & \vdots \\
r_k(l) & r_k(L_F-1) & r_k(L_F-2) & \cdots & r_k(1)
\end{bmatrix}
\]

(4)

\[k\] is the selected frame number. The calculated \(R_{\ell m}\) toeplitz matrix for the \(k^{th}\) frame is a real symmetric and positive matrix as below \(L_F \times L_F\) dimensions.

\[
R_{Fk} V_k = \lambda_k V_k
\]

(5)

\(\lambda_k\) and \(V_k\) define eigenvalue and eigenvector for \(R_{Fk}\) toeplitz matrix. The relation between the toeplitz matrix of the \(X_{Fk}\) series and its eigenvectors can be shown as below;

\[
X_{Fk} = \sum_{i=1}^{L_F} C_i V_k
\]

(6)

\[C_i = (X_{Fk})^T V_k\]

(7)

By putting \(\lambda_{1k}, \lambda_{2k}, \ldots, \lambda_{L_F k}\) eigenvalues in the descending orders, \(V_{1k}, V_{2k}, \ldots, V_{L_F k}\) are found with the eigenvectors matching \(\lambda_{1k} \geq \lambda_{2k} \geq \cdots \geq \lambda_{L_F k}\).

As a result of using the eigenvectors indicating the signal with the maximum energy for each frame can be calculated with the equation (6).

### III. RECONSTRUCTION OF ECG SIGNAL

According to (6) and (7) equations, \(X_{Fk}\) can be found easily from the eigenvectors for each \(N_{Fk}\) frame. Starting from the first frame of sample ECG signal, the steps below can be performed sequentially. Fig. 5 illustrates the flow chart of KLT for ECG signal. Compression ratio changes in line with the length of frame.

1. **Step:** To divide the ECG signal to appropriate length of frame (4 - 48).
2. **Step:** To determine the Toeplitz matrix for each frame.
3. **Step:** To determine the eigenvalues and eigenvectors for each Toeplitz matrix.
4. **Step:** To determine the \(C_i\) coefficient which shows the linear relation between the ECG signal and eigenvectors.
5. **Step:** To arrange in decaying sequence the eigenvalues with using KLT method.
6. **Step:** To specify the maximum eigenvectors (having the maximum energy) equal to maximum eigenvalues.
7. **Step:** To reconstruct the ECG signal by multiplying the eigenvectors which are having the maximum energy and the coefficient.

\[
X_R = C_i V_i
\]

(8)

### IV. RESULTS

In order to code ECG signal in effective way optimum length of frame and number of vectors are found. 1:3 ratio which is the number of vectors per length of frame may be sufficient for medical diagnosis and treatment.

In this method, firstly for any selected ECG signal toeplitz matrix is found for each frame. And then, the eigenvectors are selected which have maximum energy. Finally, the coding of the ECG signal is performed by using these selected eigenvectors.

Table 2 shows the mean square errors according to the different length of frame and number of vectors.

\[
MSE = \left\| X_{Fk} - C_i X_R \right\|^2 = \sum_{i=1}^{L_F} (X_{i,Fk} - C_1 X_{i,Fk})^2
\]

(9)

Figures 1-4 show that the ratios of 1:1, 1:2, 1:3, 1:4 which are between the length of frame and number of vectors. The results of figures and tables search are that to have optimum ECG coding, the length of frame should be as much as minimised, the number of vectors should be as much as maximised.

### REFERENCES


### Table I
**SUMMARY OF ECG DATA COMPRESSION SCHEMES**

<table>
<thead>
<tr>
<th>Method</th>
<th>Compression Ratio</th>
<th>Sampling Rate (No.of bits)</th>
<th>Percent RMS Difference</th>
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<td>10,0</td>
<td>500(12)</td>
<td>%28,0</td>
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<td>TP</td>
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<td>CORTES</td>
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<td>Fan/SAPA</td>
<td>3,0</td>
<td>250</td>
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### Table II
**MEAN SQUARE ERRORS ACCORDING TO THE DIFFERENT LENGTH OF FRAMES AND THE NUMBER OF VECTORS**

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</table>
Fig. 1 Mean square errors to the number of vectors / the length of frames = 1 / 1

Fig. 2 Mean square errors to the number of vectors / the length of frames = 1 / 2

Fig. 3 Mean square errors to the number of vectors / the length of frames = 1 / 3

Fig. 4 Mean square errors to the number of vectors / the length of frames = 1 / 4
1. Segment
Choose Frame Length

Determine Toeplitz Matrix

Determine Eigen Values & Eigen Vectors

Arrange decaying order Eigen Values & Eigen Vectors

Specify max. Eigen Vectors

Reconstruct The ECG Signal for Used Vectors/Frame Length Ratio=1/4

Reconstruct The ECG Signal for Used Vectors/Frame Length Ratio=2/4

Reconstruct The ECG Signal for Used Vectors/Frame Length Ratio=3/4

Reconstruct The ECG Signal for Used Vectors/Frame Length Ratio=4/4

Fig. 5 Flow chart of KLT for ECG signal