Prediction of Load Capacity of Reinforced Concrete Corbels Strengthened with CFRP Sheets

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Abstract—Analytical procedure was carried out in this paper to calculate the ultimate load capacity of reinforced concrete corbels strengthened or repaired externally with CFRP sheets. Strut and tie method and shear friction method proposed earlier for analyzing reinforced concrete corbels were modified to incorporate the effect of external CFRP sheets bonded to the corbel. The points of weakness of any method that lead to an inaccuracy, especially when overestimating test results were checked and discussed. Comparison of prediction with the test data indicates that the ratio of test / calculated ultimate load is 0.82 and 1.17 using strut and tie method and shear friction method, respectively. If the limits of maximum shear stress is followed, the calculated ultimate load capacity using shear friction method was found to underestimate test data considerably.

Keywords—Corbel, Strengthening, Strut and Tie Model, Shear Friction

NOMENCLATURE

- \( h \) : corbel height.
- \( k \) : height of the slope portion of the corbel.
- \( \rho_f \) : ratio of CFRP layers perpendicular to shear plane.
- \( \rho_{fi} \) : ratio of inclined CFRP layers provided to shear plane.
- \( \Delta_{si} \) : ratio of change in shear stress due to strengthening.
- \( T_{sf} \) : horizontal tensile force carried by steel reinforcement and CFRP strip in flexural zone bonded to concrete at both sides of the corbel.
- \( T_s \) : tensile force resisted by stirrups.
- \( T_{fs} \) : total tensile force resisted by the horizontal CFRP strip in shear zone.
- \( T_{fl} \) : total tensile force carried by inclined strips of CFRP, all provided to both sides of the corbel.
- \( f_{yt} \) : yield stress of steel reinforcement in shear zone.
- \( f_{ft} \) : yield stress of steel reinforcement in shear zone.
- \( f_{fr} \) : fracture stress of CFRP strips- epoxy composite in flexural zone.
- \( f_{fr} \) : fracture stress of CFRP strips- epoxy composite in shear zone.
- \( V_{n} \) : maximum shear force capacity of the corbel.
- \( V_{nc} \) : shear stress capacity of the corbel.

I. INTRODUCTION

ADVANCED polymer products were used extensively in concrete structures to elongate their lifetime. FRP layers are successful to control crack extension and propagation in concrete. Indeed FRP application has an important role in the case of those concrete members undergo cracks concentration like in the case of corbels. Test results [1] indicate that strengthening reinforced concrete corbels with CFRP sheets able to enhance a load capacity by 28.3%. Results also showed that the benefit of the provided CFRP layers for strengthening increased by reducing the amount of flexural and/or shear reinforcement and reducing the concrete compressive strength. However the benefit is more important in repairing damaged corbels occurred by preloading especially in the case of corbels of lower concrete strength.

In this paper an attempt was made to calculate the load capacity of RC corbels strengthened externally with CFRP sheets. First, the strut and tie model was used and adjusted to incorporate the action of bonded CFRP layers. Later, the shear friction method was used and equations were adjusted for the case of corbels strengthened with CFRP sheets. The accuracy of each method was checked by making a comparison with the previous test data. The suitability of each method was discussed to use the better one in the case of strengthening and repairing of reinforced concrete corbels.
II. LOAD CAPACITY PREDICTION

A. Strut and Tie Model

Fig. 1 shows the force acting on the reinforced concrete corbel externally bonded with CFRP strips. Compressive and tensile forces shown in the figure are given by the following equations:

$$C = a_f f_c' b c$$  \hspace{1cm} (1)

$$T_{sf} = A_f f_{sf} + \phi A_f f_f$$  \hspace{1cm} (2)

$$T_s = A_s f_{sv}$$  \hspace{1cm} (3)

$$T_{fs} = \phi A_f f_f$$  \hspace{1cm} (4)

$$T_{fs} = \phi A_f f_f$$  \hspace{1cm} (5)

In which $T_{sf}$ is the horizontal tensile force carried by steel reinforcement and CFRP strip in flexural zone bonded to concrete at both sides of the corbel. $T_s$ is the tensile force resisted by stirrups. $T_{fs}$ is the total tensile force resisted by the horizontal CFRP strip in shear zone. $T_{fs}$ is the total tensile force carried by inclined strips of CFRP, all provided to both sides of the corbel. $f_f$ is the bond reduction factor between CFRP strip and concrete, taken as 0.75\[2\]. $\alpha$ is the angle of inclination of CFRP strip from the horizontal. $A_f$ and $A_s$ are the total area of steel reinforcement and CFRP layers provided to the flexural zone, respectively. $A_s$ and $A_{fs}$ are the total area of steel reinforcement and CFRP strips provided to the shear zone, respectively. $A_{fs}$ is the total area of inclined CFRP strips. $f_{sf}$ and $f_{sv}$ are the yield stress of steel reinforcement in flexural and shear zones, respectively, and $f_{fs}$, $f_f$ and $f_{fs}$ are the fracture stress of CFRP strips- epoxy composite in flexural zone, shear zone and inclined strips, respectively.

$\alpha_f$ is the compressive stress distribution parameter given by\[3\]

$$\alpha_f = 0.85 - 0.0015 f_c'$$

The horizontal component of the compressive force $C$ is given by

$$C \sin \beta = T_{sf} + T_s + T_{fs} + T_{fs} \cos \alpha$$  \hspace{1cm} (6)

The vertical component of $C$ is given by

$$C \cos \beta + T_{fs} \sin \alpha = V_n$$  \hspace{1cm} (7)

Substituting Eq.(1) into Eq.(6) and rearranging yields

$$c \sin \beta = \frac{T_{sf} + T_s + T_{fs} + T_{fs} \cos \alpha}{a_f f_c' b}$$  \hspace{1cm} (8)

Substituting Eq.(1) into Eq.(7) and rearranging yields

$$V_n = a_f f_c' b c \sin \beta \cot \beta + T_{fs} \sin \alpha$$  \hspace{1cm} (9)

Combining Eqs.(8) and (9) and simplifying yields

$$V_n = (T_{sf} + T_s + T_{fs} + T_{fs} \cos \alpha) \cot \beta + T_{fs} \sin \alpha$$  \hspace{1cm} (10)

$cot \beta$ must be determined to calculate $V_n$ and can be obtained by equating the external moment caused by the vertical force $V_n$ and the internal moments resisted by the corbel materials. Equilibrium of moment acting on the corbel about point $a$ [Fig. 1] yields

$$V_n (a + c \cos \beta) = T_s (d_f \frac{c \sin \beta}{2}) + T_s (d_f \frac{-c \sin \beta}{2}) + T_s (d_f \frac{-c \sin \beta}{2}) + T_s \cos \alpha \frac{c \cos \beta}{2} d_f$$  \hspace{1cm} (11)

$d_f$ is the distance between the center of the inclined strip crossing the shear plane and the inclined surface-column junction point. Substituting Eqs.(8) and (10) into (11) and rearranging yields the following equation for calculating $cot \beta$

$$cot \beta = \frac{a_f f_c' b}{T_s + T_s + T_{fs} + T_{fs} \cos \alpha \sqrt{(a + c \sin \alpha)/(2a_f f_c')}}$$  \hspace{1cm} (12)
In which

$$\varphi = a' - \frac{2}{a' f' b} \left( T_e + T_s + T_i \cos \alpha \right) + T_i \sin \alpha - \left( T_{d} + T_{d} + T_{d} + T_{d} \cos \alpha \right)$$

(Later, the value of cot $\beta$ is substituted into Eq.(10) and vertical shear force, $V_n$, can be calculated as follows)

$$V_n = a' f'b \left[ \sqrt{\varphi - a} \right] + T_i \sin \alpha$$

For those corbels containing no inclined strips of CFRP

$$V_n = a' f'b \left[ \sqrt{\varphi - a} \right] + T_i \sin \alpha$$

Fig. 2 shows the necessary parameter used for calculating the value of $\cos \alpha$ which can be obtained from the geometry of the corbel. If the inclined strips of CFRP are provided in a manner that cover the whole height of the corbel-column junction, the equivalent depth used for calculating $V_n$ in Eq.(13) is equal to $h/2$. $A_b$ is equal to the distance covered by the direct shear plane multiplied by the CFRP thickness. The thickness of CFRP sheet – epoxy composite in addition to the direct shear plane multiplied by the CFRP thickness. The shear capacity of the corbel – epoxy composite in addition to the fracture stress should be taken from tensile measurements obtained from test results.

$$V_n = \mu A_f f_{yv} + \phi A_{fi} f_{fi} \cos \alpha$$

For concrete placed monolithically like the case of the tested specimens the value of $\mu$ is 1.44 and $\lambda$ is equal to 1.0 for normal weight concrete, as recommended by the ACI 318 Code. $f_{yv}$ is the yield stress of the stirrups. $A_b$ is the area and fracture stress of the horizontal CFRP strip, respectively, provided in the shear zone. $A_{fi}$ is the area for the inclined strips, and $f_{fi}$ is the fracture stress for the inclined strips. $f_{fi}$ is not necessary to be equal to $f_{fi}$, because more than one layer of CFRP strip can be provided in each direction.

2. Maximum Shear Capacity of the Strengthened Section

According to the ACI 318 Code, the direct shear capacity of the concrete section should be taken as smaller than $0.2 f'c$ and 5.5 MPa. For compressive strength larger than 27.6 MPa, the 5.5 MPa governs the shear strength of the section and the use of high strength concrete instead of lower strength concrete in corbel design becomes useless. For this purpose, some attempts were made for deriving equations for calculating the shear strength capacity of reinforced concrete section made from high strength concrete. The following equation was obtained by Hassan and Mohammed[6] and used here beside the limits of ACI 318 Code for calculating the maximum shear strength of the section

$$\nu_n = 5.77 + 0.88 f_{cy} f'c \sqrt{\rho_{ci} f_{cy}}$$

$\nu_n$ is the nominal shear stress, $f'c$ is the compressive strength of concrete, and $\rho_{ci} f_{cy}$ is the clamping stress or shear reinforcement index.

Test results obtained by Zanganah[4] are used here for making a justification on the above limits of shear capacity. Such justification is necessary for calculating shear capacity of concrete strengthened with CFRP limits. Table (1) shows the results of the direct shear strength of concrete strengthened with CFRP strips obtained from Reference[4]. The ultimate shear capacity is represented by the percentage increase over...
that of plain concrete. It is assumed here that the percentage increase in direct shear is not affected by the existence of reinforcement in the section. Value of $\alpha$ usually taken from test results. The nonlinear equation of the following form was proposed for the percentage increase in the nominal shear stress

$$\Delta V_n = a(\rho_f f_f + \rho_r f_r \cos \alpha)^b$$  

(17)

$\rho_f$ is the ratio of CFRP layer in the concrete section perpendicular to the shear plane. $\rho_r$ is the CFRP ratio of the inclined strips provided to the section and $\alpha$ is the angle of inclined strips measured from shear plane. Regression analysis carried out on the data of Table 1 shows that the constant $a$ is equal to 0.069, and the constant $b$ is equal to 1.177. Accordingly, the value of $\Delta V_n$ becomes

<table>
<thead>
<tr>
<th>Angle between strip and shear planes</th>
<th>No. of strips (at both sides)</th>
<th>Width of strips (mm)</th>
<th>Thickness of strips (mm)</th>
<th>$A_{f_s}$ (or $A_f$)</th>
<th>$\rho_f = \frac{A_h}{bd}$</th>
<th>$\rho_n = \frac{A_h}{bd}$</th>
<th>$f_f$ (MPa)</th>
<th>$\rho_f f_f$</th>
<th>$\Delta V_n$</th>
</tr>
</thead>
<tbody>
<tr>
<td>90°</td>
<td>8</td>
<td>20</td>
<td>1.4 (one layer)</td>
<td>224</td>
<td>0.00644</td>
<td>-</td>
<td>686</td>
<td>4.421</td>
<td>0.32</td>
</tr>
<tr>
<td>90°</td>
<td>8</td>
<td>20</td>
<td>2 (two layers)</td>
<td>320</td>
<td>0.00921</td>
<td>-</td>
<td>590</td>
<td>5.432</td>
<td>0.52</td>
</tr>
<tr>
<td>90°</td>
<td>8</td>
<td>20</td>
<td>2.7 (three layers)</td>
<td>432</td>
<td>0.01243</td>
<td>-</td>
<td>520</td>
<td>6.463</td>
<td>0.60</td>
</tr>
<tr>
<td>45°</td>
<td>8</td>
<td>20</td>
<td>1.4 (one layer)</td>
<td>224</td>
<td>-</td>
<td>0.00644</td>
<td>686</td>
<td>3.125</td>
<td>0.25</td>
</tr>
<tr>
<td>45°</td>
<td>8</td>
<td>20</td>
<td>2 (two layers)</td>
<td>320</td>
<td>-</td>
<td>0.00921</td>
<td>590</td>
<td>3.840</td>
<td>0.38</td>
</tr>
<tr>
<td>45°</td>
<td>8</td>
<td>20</td>
<td>2.7 (three layers)</td>
<td>432</td>
<td>-</td>
<td>0.01243</td>
<td>520</td>
<td>4.569</td>
<td>0.50</td>
</tr>
</tbody>
</table>

TABLE II

RESULTS OF TEST AND CALCULATED ULTIMATE LOAD USING DIFFERENT METHODS

<table>
<thead>
<tr>
<th>Corbel</th>
<th>$V_{n,test}$ (kN)</th>
<th>$V_{n,ST}$ (kN)</th>
<th>$V_{n,test}$/$V_{n,ST}$</th>
<th>$V_{n,ST}$/$V_{n,Eq.(15)}$</th>
<th>$V_{n,test}$/$V_{n,Eq.(20)}$</th>
<th>$V_{n,test}$/$V_{n,Eq.(21)}$</th>
<th>$V_{n,test}$/$V_{n,Eq.(22)}$</th>
<th>$\Delta V_n$</th>
</tr>
</thead>
<tbody>
<tr>
<td>C1</td>
<td>478.0</td>
<td>510.19</td>
<td>0.937</td>
<td>342.05</td>
<td>1.39</td>
<td>0.91</td>
<td>287.1</td>
<td>1.66</td>
</tr>
<tr>
<td>C2</td>
<td>462.8</td>
<td>506.33</td>
<td>0.914</td>
<td>342.05</td>
<td>1.35</td>
<td>0.88</td>
<td>287.1</td>
<td>1.61</td>
</tr>
<tr>
<td>C3</td>
<td>408.65</td>
<td>450.96</td>
<td>0.906</td>
<td>342.05</td>
<td>1.19</td>
<td>0.8</td>
<td>287.1</td>
<td>1.42</td>
</tr>
<tr>
<td>C4</td>
<td>494.55</td>
<td>372.59</td>
<td>1.327</td>
<td>342.05</td>
<td>1.44</td>
<td>0.94</td>
<td>287.1</td>
<td>1.72</td>
</tr>
<tr>
<td>C5</td>
<td>520.0</td>
<td>592.14</td>
<td>0.878</td>
<td>342.05</td>
<td>1.52</td>
<td>0.99</td>
<td>287.1</td>
<td>1.81</td>
</tr>
<tr>
<td>C6</td>
<td>410.0</td>
<td>523.50</td>
<td>0.783</td>
<td>342.05</td>
<td>1.19</td>
<td>0.80</td>
<td>287.1</td>
<td>1.43</td>
</tr>
<tr>
<td>C7</td>
<td>548.15</td>
<td>671.17</td>
<td>0.817</td>
<td>486.13</td>
<td>1.13</td>
<td>0.79</td>
<td>378.9</td>
<td>1.45</td>
</tr>
<tr>
<td>C8</td>
<td>553.1</td>
<td>687.75</td>
<td>0.804</td>
<td>486.13</td>
<td>1.14</td>
<td>0.79</td>
<td>378.9</td>
<td>1.46</td>
</tr>
<tr>
<td>C9</td>
<td>491.8</td>
<td>645.47</td>
<td>0.762</td>
<td>372.11</td>
<td>1.52</td>
<td>0.77</td>
<td>378.9</td>
<td>1.29</td>
</tr>
<tr>
<td>C10</td>
<td>469.3</td>
<td>681.63</td>
<td>0.688</td>
<td>486.13</td>
<td>0.96</td>
<td>0.68</td>
<td>378.9</td>
<td>1.24</td>
</tr>
<tr>
<td>C11</td>
<td>516.7</td>
<td>683.63</td>
<td>0.756</td>
<td>486.13</td>
<td>1.06</td>
<td>0.74</td>
<td>378.9</td>
<td>1.36</td>
</tr>
<tr>
<td>C12</td>
<td>583.85</td>
<td>827.74</td>
<td>0.751</td>
<td>551.45</td>
<td>1.06</td>
<td>0.74</td>
<td>429.65</td>
<td>1.36</td>
</tr>
<tr>
<td>C13</td>
<td>603.35</td>
<td>865.13</td>
<td>0.719</td>
<td>551.45</td>
<td>1.09</td>
<td>0.76</td>
<td>429.65</td>
<td>1.40</td>
</tr>
<tr>
<td>C14</td>
<td>511.3</td>
<td>703.92</td>
<td>0.768</td>
<td>551.45</td>
<td>0.93</td>
<td>0.65</td>
<td>429.65</td>
<td>1.19</td>
</tr>
<tr>
<td>C15</td>
<td>430.0</td>
<td>706.15</td>
<td>0.646</td>
<td>551.45</td>
<td>0.78</td>
<td>0.59</td>
<td>429.65</td>
<td>1.00</td>
</tr>
</tbody>
</table>

Mean: - - 0.82 - 1.17 - 0.79 - 1.426 - 0.703

$\Delta V_n = 0.069(\rho_f f_f + \rho_r f_r \cos \alpha)^{1.177}$  

(18)
The value of correlation coefficient ($r$) for the above equation is equal to 0.90.

The ultimate shear capacity for the composite section, $v_{nc}$, can be written as follows

$$v_{nc} = v_{n0} + (\Delta v_{n0} + 1)$$

Eq. (19)

Therefore, the final form of the shear strength of the composite section using equations (16) and (19) becomes

$$v_{nc} = [5.77 + 0.88(\bar{f}_c)^{0.5} \sqrt{\rho_{p} f_{y}}][1 + 0.069(\rho_{f} f_{y} + \rho_{p} f_{y})^{1.177}]$$

Eq. (20)

The shear stress capacity of the composite section using the limits of ACI 318 Code is the smaller value of the following

$$v_{nc} = 5.5[1 + 0.069(\rho_{f} f_{y} + \rho_{p} f_{y})^{1.177}]$$

Eq. (21)

$$v_{nc} = 0.2\bar{f}_c [1 + 0.069(\rho_{f} f_{y} + \rho_{p} f_{y})^{1.177}]$$

Eq. (22)

III. VALIDITY OF THE PREDICTIONS

Table II contain results of calculated ultimate load capacity of corbels using different methods: strut and tie model, shear friction method and the limits of maximum shear force capacity in addition to the test ultimate load (taking from Reference[1]) for the comparison sake. For obtaining the best view of comparison between the test and calculated values, Figs.3 to 7 were drawn. The values of calculated ultimate load using strut and tie model are larger than the test ultimate load and accordingly the ratio of (tested/calculated) is smaller than unity for all corbels with a mean value equal to the 0.82.

Therefore, the strut and tie model is not accurate and not safe for calculating the ultimate load capacity of corbels strengthened with CFRP sheets. The reason of overestimating the ultimate load using strut and tie model is due to neglecting the effect of local stress concentration at the critical nodes, especially in that node near the corbel column junction point. Such effect was not included in the analysis. The effect of compressive stress concentration has a particular importance because other zones are far from failure as a result of strengthening with CFRP sheets. It was observed from test results[1] that the compression zone is the source of failure nearly for all the corbels due to crushing as a result of high stress concentration.

On the contrary, the shear friction method offers the calculated ultimate load smaller than the test ultimate load for all corbels except for corbel $C_{10}$, $C_{15}$, $C_{16}$ but the mean value was found to be 1.17 as shown in Table 2. Therefore using shear friction method for analyzing reinforced concrete corbels strengthened with CFRP is safe and accurate. According to Eq.(15), the shear force capacity not depends on the concrete compressive strength and accordingly the shear force depends on the shear reinforcement properties, because the constant value of $\mu$ was used which is 1.4.

IV. CONCLUSIONS

From the theoretical work presented in this paper the following conclusions can be drawn

1- Strut and tie model of its basic form is not accurate for calculating the load capacity of strengthened or repaired RC corbel with CFRP sheets due to neglecting the effect of stress concentration in critical zones. Better prediction can be obtained using modified shear friction theory of average test/calculate ultimate load equal to 1.17.

2- Using maximum shear stress limits suggested by ACI 318 in strengthened corbel considerably underestimates the predicted ultimate load capacity, especially for those corbels made of high strength concrete. If other limits suitable for the case of higher concrete strength is used the shear friction prediction will be accurate.
Fig. 3 Test versus calculated ultimate load using Strut and tie model

Fig. 4 Test versus calculated ultimate load using Shear friction model

Fig. 5 Test versus calculated ultimate load using Eq.20

Fig. 6 Test versus calculated ultimate load using Eq.21

Fig. 7 Test versus calculated ultimate load using Eq.22

REFERENCES


[5] ACI 318M-08, Building Code Requirements for Reinforced Concrete, American Concrete Institute, 2008.