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The Relations between the Fractal Properties of the River Networks and the River Flow Time Series

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***Abstract*—**All the geophysical phenomena including river networks and flow time series are fractal events inherently and fractal patterns can be investigated through their behaviors. A non-linear system like a river basin can well be analyzed by a non-linear measure such as the fractal analysis. A bilateral study is held on the fractal properties of the river network and the river flow time series. A moving window technique is utilized to scan the fractal properties of them. Results depict both events follow the same strategy regarding to the fractal properties. Both the river network and the time series fractal dimension tend to saturate in a distinct value.

***Keywords*—**river flow time series, fractal, river networks

I. Introduction

FRACTAL geometry is rooted in the works of late 19th and early 20th century mathematicians who found their fancy in generating complex geometrical structures from simple objects like a line, a triangle, a square, or a cube (the initiator) by applying a simple rule of transformation (the generator) in an infinite number of iterative steps. The complex structure that resulted from this iterative process proved equally rich in detail at every scale of observation, and when their pieces were compared to larger pieces or to those of the whole, they proved similar to each other [1].The concept of a fractal dimension to describe structures, which look the same at all length scales, was first proposed by Mandelbrot [2]. Although in strict terms, this is a purely mathematical concept, there are many examples in nature that closely approximate a fractal object, though only over particular ranges of scale. Such objects are usually referred to as self-similar to indicate their scale-invariant structure. In simple terms, the common characteristic of such fractal objects is that their length (if the object is a curve, otherwise it could be the area or volume) depends on the length scale used to measure it, and the fractal dimension tells us the precise nature of this dependence [3]. A river network consists of a main river accompanied by a hierarchy of side streams of decreasing lengths and flow capacities. Ignoring the ground absorption and evaporation, the network drains out the whole amount of rain water dropped uniformly on every small piece of land in the river basin and therefore necessarily spans the whole drainage area [4].

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A classic problem in hydrologic science is the prediction of river flow properties according to given knowledge of rainfall and drainage basin properties. Although many approaches have been used for flow prediction, recent significant advances have been made using the framework of scaling invariance. It is by now well known that both river networks, representing the primary mechanism for transport of water over the surface of large basins, and rainfall exhibit certain forms of scaling invariance[5]-[6]. Given that both rainfall and the network obey certain scaling laws, an important ongoing research problem is to understand what type of structure this induces on the resulting flow through the network, both spatially and in time [7].

A fractal river network is a striking example of self-organized criticality. The physics of river network evolution arises from interplay of the structured landscape governing the water flow with the erosion effects of the water feeding back into further sculpting of the landscape. Extensive studies of the fractal characteristics of real river networks have been carried out [4]-[7]. Hack [8] has studied the relationship between the length of a river l and the area of a drainage basin s. s is a measure of the total area of the land covered by the principal stream and its tributaries that feed into the network. Hack's measurements indicate that for basin areas s ranging over almost five decades (up to 375 square miles), $s \sim l^{\varphi}$ with the exponent $1/\varphi \sim 0.57$. Other measurements of the distribution of drainage basin areas suggest a power law scaling of the form $P(s) \sim s^{-\tau}$ with $\tau = 1.45 \pm 0.03$ [9].

II. Methods of Fractal Dimension Calculation

A. Box counting dimension

The concept of capacity dimension was developed to estimate D_{SS} for real, non exact, statistical fractals. D_{cap} is a generalization of D_{SS} and is calculated as follows. It uses 'balls' whose dimension equals E of the space in which the object is embedded. For E = 1 the ball is a line segment of length 2r, for E = 2 it is a circle with radius r, and for E = 3 it is a sphere with radius r. The object is to be covered by balls so that its every point is enclosed within at least one ball. It is important to note that overlapping balls are allowed. The minimum number of balls of size r needed to cover the object, N(r), is found, then r is decreased and N(r) is found again. D_{cap}

tells how the number of balls needed to cover the object changes as the size of the ball is decreased (Bassingthwaighte J, 1994).

$$N = R^{D_{cap}} \text{ Where } R = 1/r \tag{1}$$

and the radius of the ball covering the whole object is 1. Equation (1) gives a precise mathematical definition of D_{cap}, but it does not allow for an effective calculation of it. D_{box} is a practical implementation of the algorithm of D_{cap}. It uses an overlapping grid of boxes instead of overlapping balls, which greatly simplifies the procedure and makes the calculation of D applicable to traced 2D or 3D objects feasible. The essence of this method is to cover the image or structure by this grid of overlapping boxes of various edge lengths, L = 1/R, and to determine the number of boxes covering any part of the object N. N is determined for progressively smaller box sizes. D_{box} is given as the slope of a linear regression fit to data pairs on a log–log plot of N as a function of R

$$N = R^{D_{box}} \text{ Which yields } D_{box} = \log N / \log R \tag{2}$$

and the edge length of the box containing the whole object is 1. The fractal dimension for a line segment, square and cube equals the Euclidean and` the topological dimension and are 1, 2, and 3, respectively. For 2D and 3D fractals, D falls in between these landmark values and gives a good characterization of the space-filling properties of the structure. The more D differs from DT and is closer to E, the more the structure invades the Euclidean space. At the beginning, Mandelbrot emphasized the fractional value of D. He also noted, however, that fractals could be found with integer D like the Peano curve whose D = 2 [1].

B. *Power spectral density analysis (PSD)*

A time series can be represented as a sum of cosine wave components of different frequencies

$$x_i = \sum_{n=0}^{N/2} A_n \cos\left[\omega_n t_i + \varphi_n\right] = \sum_{n=0}^{N/2} A_n \cos\left[\frac{2\pi n}{N} + \varphi_n\right] \tag{3}$$

where A_n is the amplitude and ϕ_n is the phase of the cosine-component with ω_n angular (i.e. $f_n = \omega_n/2\pi$ cyclic) frequency. The $A_n(f_n)$, $\varphi_n(f_n)$ and $A^2_n(f_n)$ functions are termed as amplitude, phase and power spectrum of the signal, respectively. These spectra can be determined by an effective computational technique, the fast Fourier transform (FFT). The power spectrum (periodogram, power spectral density or PSD) of a fractal process is a power law relationship

$$A_n^2 =_d p\omega_n^{-\beta} \text{ which yields } \log A_n^2 =_d \log p - \beta \log f_n \tag{4}$$

where β is termed spectral index. Signals with this form of power spectrum are referred to as $1/f$ noise. The power law relationship expresses the idea that as one doubles the frequency the power changes by the same fraction ($2^{-\beta}$) regardless of the chosen frequency, i.e. the ratio is independent of where one is on the frequency scale. The signal has to be preprocessed before applying the FFT, which means subtracting the mean, multiplying with a parabolic window (windowing) and bridge de-trending (end-matching). After calculating the power spectrum using a FFT, the high-frequency part of the spectrum should be excluded before fitting the regression line [1].

The value of β can be related to Hurst coefficient using the following equations

$$H = \frac{\beta+1}{2} \text{ when } -1 < \beta < +1$$
$$H = \frac{\beta-1}{2} \text{ when } 1 < \beta < 3 \tag{5}$$

C. *Hurst's rescaled range analysis (R/S)*

The first method for assessing *H* was invented by Hurst [10] when confronting the question of how high the Aswan dam had to be built so that it would contain the greatly varying levels of the Nile within a given window of observation, *n*. The logic he followed was governed by the three criteria of an ideal reservoir: (1) the outflow is uniform, (2) the level is the same at the beginning and at the end of the observation window, (3) the reservoir never overflows [11]. He looked at retrospectives records of water levels, x_i, which are proportional to the velocity of water inflow to the dam. Hurst assumed a uniform outflow, which can be calculated as the mean of the varying inflow. The time series of the increase in water volume in the container is given by the summed difference of inflow and outflow $y_i = \sum_{n=1}^{j} (x_i - \bar{x}_i)\Delta t$ The range of y_j, $R = y_{max} - y_{min}$, determines how high the dam should be built. In addition, Hurst divided the range by the standard deviation of inflow fluctuations, *S*, and found much to the surprise of contemporary statisticians that R/S_n showed a power law scaling relationship with the length of observation *n*

$$(R/S)_n - pn^H \text{ which yields } \log(R/S)_n =_d \log p + H \log n \tag{6}$$

where *p* is a pre-factor. This method is known as Hurst's rescaled range (*R/S*) analysis. His work led Mandelbrot and Wallis (1969) to discover the widespread occurrence of self-similar fluctuations in natural phenomena [1].

III. Results And Discussion

A. *Ghareh-Aghaj Basin*

For the present study Ghareh-Aghaj river basin with available 36 years data and over 3,000 sq km area has been used. The basin is located in Fars province of Iran. This river comes from (Bon-Rood Hights) hillside village of Tabask

Zanganeh (about 30 km north-east of Kazeroun city) in Doshman-Ziari rural district and then passes through a valley in the northwest of Shiraz, and leaves Mamasani district in the vicinity of the Chehel-Cheshmeh. Ghareh-Aghaj River in its path, along with several rivers and springs, after passing through farm lands of Khafr, Kavar, Jahrom, Ghir–o-Karzyn enters the Persian Gulf through the Boushehr province entitled as Mond River. The flow of this river is permanent and the width of it ranges from 20 meters in Mountainous regions to 400 meters in plains. Ghareh-Aghaj River is one of the most important rivers of Fars Province and its water is now used for drinking and agriculture. Construction of Salman Farsi dam and the studies for construction of Kavar dam on the river is a sign of the importance of this river in the Fars province.

The average annual discharge of the river is 18 cubic meters per second at the Tang-e-Karzin station. Minimum flow rate at this station is 3.5 cubic meters per second and the maximum is 43 cubic meters per second. Also statistics show recorded flood discharge of 6,000 cubic meters per second.

Fig. 1 shows the location of Ghare-Aghaj River basin.

B. River Network Calculated Dimensions

For calculation of the river network fractal dimension several windows with increasing dimensions were considered to define the desired area of study. The selection of windows was started from the top corner of upstream zone which is located at the North West of the basin area, and considering this point as the fixed top left corner of all the windows, the dimensions of successive selection windows were increased such that the last window covered the entire area of the basin.The fractal dimension of that part of river network located within each selection window was calculated using regular Box-Counting method.

Table (I) shows the results of the calculation of fractal dimension for Ghareh-Aghaj River network.

TABLE I
RESULT OF THE CALCULATION OF FRACTAL DIMENSION FOR GHAREH-AGHAJ RIVER NETWORK

window no.	*Fractal dimension*	*C*
1	1.285	0.084
2	1.510	0.158
3	1.602	0.187
4	1.593	0.172
5	1.632	0.191
6	1.622	0.181
7	1.630	0.194
8	1.618	0.189
9	1.618	0.191
10	1.626	0.201
11	1.596	0.172
12	1.592	0.178
13	1.595	0.178
14	1.608	0.181
15	1.622	0.186
16	1.619	0.182

As it is shown in table (1) the fractal dimension calculated for each window increases with the size of window and eventually tends to a constant value around 1.62.

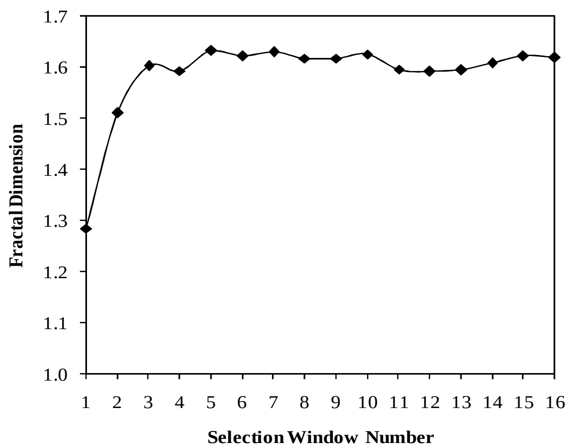

Fig. 2 Fractal Dimension of Ghare-Aghaj River network in different selection windows

C. River Time Series Fractal Dimensions calculated using PSD Method

A similar procedure was used to find the fractal dimension of river flow time series. The available data was sampled using windows starting from the latest recorded data available; the data with three different intervals including daily, weekly and monthly records were windowed such that the selection period increases progressively to finally include the whole available data series. In each selected period the fractal dimension of the time series was calculated using a predefined computer program. Tables (II) and (III) show the results of the calculation of fractal dimension for Ghareh-Aghaj River flow time series by PSD method for daily and weekly data respectively. Fig (3) depicts the results of the windowing technique scanning the fractal dimension of the time series using PSD method. It is clear that the fractal dimension tends to saturate to 1.18 after 400 weeks and keep the same procedure even up to 2000 weeks.

TABLE II
RESULT OF THE CALCULATION OF FRACTAL DIMENSION FOR GHAREH-AGHAJ RIVER DAILY TIME SERIES BY PSD METHOD

Data range	*Hurst Coefficient*	*Fractal dimension*
1-2000	0.997	1.003
1-4000	0.962	1.038
1-6000	0.947	1.053
1-8000	0.948	1.052
1-10000	0.953	1.047
1-12000	0.949	1.051
1-14000	0.924	1.076

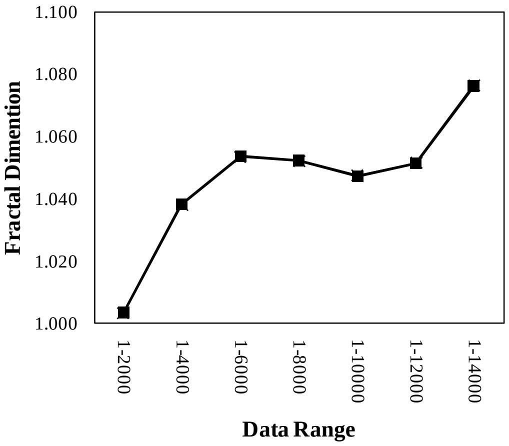

Fig. 3 Fractal dimension of Ghareh-Aghaj River daily time series in different selection windows by R/S Method

TABLE III
RESULT OF THE CALCULATION OF FRACTAL DIMENSION FOR GHAREH-AGHAJ RIVER WEEKLY TIME SERIES BY PSD METHOD

Data range	*Hurst Coefficient*	*Fractal dimension*
1-200	0.929	1.071
1-400	0.856	1.144
1-600	0.814	1.186
1-800	0.818	1.182
1-1000	0.811	1.189
1-1200	0.813	1.187
1-1400	0.822	1.178
1-1600	0.821	1.179
1-1800	0.817	1.183
1-2000	0.822	1.178

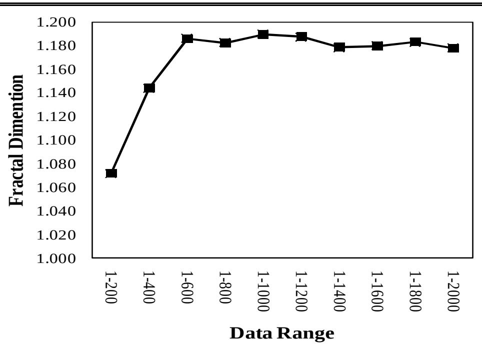

Fig. 4 Fractal dimension of Ghareh-Aghaj River weekly time series in different selection windows by PSD Method

D.River Time Series Fractal Dimensions calculated using R/S Method

As shown in Table (4) and Table (5) the fractal dimension of weekly and monthly time series of the river are calculated using R/S method. Results obviously show that the variation trend of the fractal dimension tends to a constant value (1.42) as the moving window proceed scanning the data range. Fig (4) and Fig (5) and (6) show how the fractal properties change as the window goes ahead in daily and weekly and monthly scale.

For the daily data a non-flatting behavior is detected. This irregularity mostly come from the no compatible method of fractal analysis chosen regarding to the class/length of daily time series [12].

TABLE IV
RESULT OF THE CALCULATION OF FRACTAL DIMENSION FOR GHAREH-AGHAJ RIVER WEEKLY TIME SERIES BY R/S METHOD

Data range	*Hurst Coefficient*	*Fractal dimension*
1-200	0.771	1.229
1-400	0.572	1.428
1-600	0.515	1.485
1-800	0.537	1.463
1-1000	0.579	1.421
1-1200	0.590	1.410
1-1400	0.588	1.412
1-1600	0.592	1.408
1-1800	0.585	1.415
1-2000	0.577	1.423

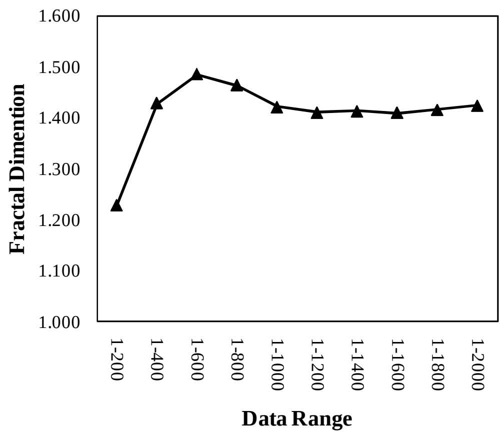

Fig. 5 Fractal dimension of Ghareh-Aghaj River weekly time series in different selection windows by R/S Method

TABLE V
RESULT OF THE CALCULATION OF FRACTAL DIMENSION FOR GHAREH-AGHAJ RIVER MONTHLY TIME SERIES BY R/S METHOD

Data range	*Hurst Coefficient*	*Fractal dimension*
1-50	0.614	1.386
1-100	0.584	1.416
1-150	0.539	1.461
1-200	0.529	1.471
1-250	0.478	1.522
1-300	0.448	1.552
1-350	0.446	1.554
1-400	0.440	1.560
1-450	0.430	1.570

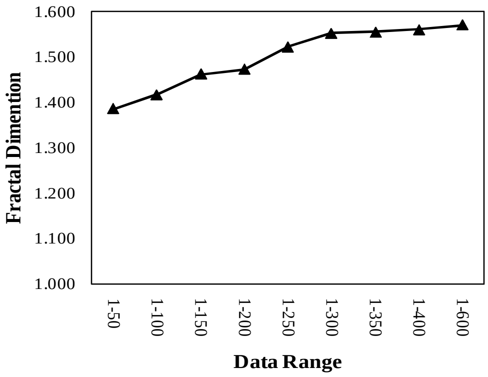

Fig. 6 Fractal dimension of Ghareh-Aghaj River monthly time series in different selection windows by R/S Method

E. River Time Series Fractal Dimensions calculated using Variation Method

Table (6) and (7) are the fractal dimension calculated for Ghareh-Aghaj River using the Variation method in weekly and monthly time scale. The pre discussed trend is noticeably detectable and can be easily distinguished in Fig (7) and (8) where the saturation dimension for weekly data seems to be 1.42 (which depict close similarity to R/S method) and for monthly data 1.6. According to results obtained and the pre researches [12]-[14] it seems that the class of time series (fBm/fGn) and the length of the data series are important factors when working with fractal analyzing methods.Variation and PSD methods gave the best answers for all the time scales and R/S was pretty good for weekly and monthly but seems to miscalculate the fractal dimension of the daily time series as the daily time series are fGn and R/S method is an fBm analyzer inherently.

TABLE VI
RESULT OF THE CALCULATION OF FRACTAL DIMENSION FOR GHAREH-AGHAJ RIVER WEEKLY TIME SERIES BY VARIATION METHOD

Data range	*Fractal dimension*
1-200	1.185
1-400	1.413
1-600	1.379
1-800	1.422
1-1000	1.422
1-1200	1.453
1-1400	1.452
1-1600	1.457
1-1800	1.462
1-2000	1.454

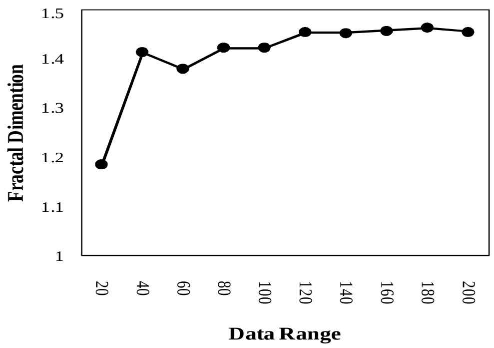

Fig. 7 Fractal dimension of Ghareh-Aghaj River weekly time series in different selection windows by Variation Method

TABLE VII
RESULT OF THE CALCULATION OF FRACTAL DIMENSION FOR GHAREH-AGHAJ RIVER MONTHLY TIME SERIES BY VARIATION METHOD

Data range	*Fractal dimension*
1-50	1.480
1-100	1.550
1-150	1.631
1-200	1.643
1-250	1.623
1-300	1.659
1-350	1.657
1-400	1.649
1-450	1.662

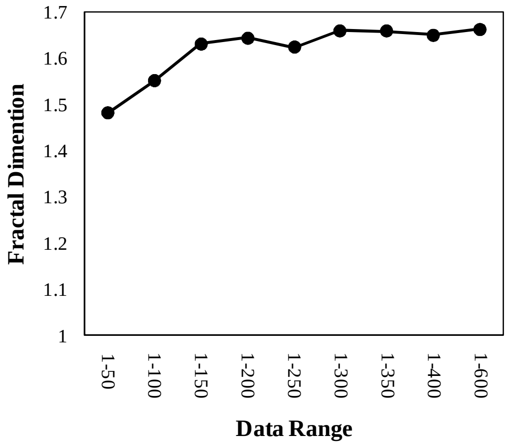

Fig. 8 Fractal dimension of Ghareh-Aghaj River monthly time series in different selection windows by Variation Method

IV. CONCLUSION

Non-linear systems are complex and affected by multiple factors. The fractal patterns of a non-linear system is the resultant of various effective events working simultaneously on a same phenomenon. The main idea behind the present research was to detect whether the related non-linear systems obey the same rules regarding to their fractal patterns or not. As discussed in the results section the variation pattern of the fractal dimension in both the river network as the first non-linear geophysical system and the river flow time series in different time scales as the second non-linear geophysical event depict the same trend. Both tend to saturate in a distinct value regarding their fractal dimension when analyze through a windowing technique that scan their fractal properties.

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