Unsteady Free Convection Flow Over a Three-Dimensional Stagnation Point With Internal Heat Generation or Absorption

Mohd Ariff Admon, Abdul Rahman Mohd Kasim, and Sharidan Shafie

Abstract—This paper considers the effect of heat generation proportional to \((T - T_c)^p\), where \(T\) is the local temperature and \(T_c\) is the ambient temperature, in unsteady free convection flow near the stagnation point region of a three-dimensional body. The fluid is considered in an ambient fluid under the assumption of a step change in the surface temperature of the body. The non-linear coupled partial differential equations governing the free convection flow are solved numerically using an implicit finite-difference method for different values of the governing parameters entering these equations. The results for the flow and heat characteristics when \(p \leq 2\) show that the transition from the initial unsteady-state flow to the final steady-state flow takes place smoothly. The behavior of the flow is seen strongly depend on the exponent \(p\).

Keywords—Free convection, Boundary layer flow, Stagnation point, Heat generation

I. INTRODUCTION

A large number of physical phenomena involve natural convection driven by heat generation. The study of heat generation in moving fluids is important in several physical problems dealing with chemical reactions and those concerned with dissociating fluids. Possible heat generation effects may alter the temperature distribution and therefore, the particle deposition rate. In addition, understanding of the effects of internal heat generation also significant in numerous applications that include reactor safety analysis, metal waste, spent nuclear fuel, fire and combustion studies and strength of radioactive materials (Postelnicu, [1]). Foraboschi and Federico [2] investigated steady state temperature profiles for linear parabolic and piston flow in circular tubes. They determined that the volumetric rate of heat generation, \(q\) \([W/m^3]\) varies linearly with \(Q_0(T - T_c)\), where \(Q_0\) is the heat generation constant. The relation explained above is valid as an approximation of the rate of some exothermic process, having \(T_c\) as the free stream temperature.

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Vajravelu and Hadjinicolau [3] studied the heat transfer characteristics in the laminar boundary layer of a viscous fluid over a linearly stretching continuous surface with viscous dissipation or frictional heating and internal heat generation. Chamkha and Camille [4] solved hydromagnetic flow with heat and mass transfer over a flat plate in the presence of heat generation or absorption and thermophoresis. Mendez and Trevino [5] analyzed the effects of the conjugate conduction-natural convection heat transfer along a thin vertical plate with non-uniform heat generation. Continuing the work of Vajravelu and Hadjinicolau [3], natural convection with heat generation along a uniformly heat vertical wavy surface have demonstrated by Molla et al. [6]. Besides that, Mohammadein and Gorla [7], Rahman et al. [8] and Magyari and Chamkha [9] take into account the effect of heat generation to investigate the characteristics of heat and mass transfer in a micropolar fluid flow. Natural convection flows in a porous medium also have received much attention in recent time due to its wide application in such fields as geothermal energy utilization and oil reservoir modeling. Many researchers interested with the problem of plate that is embedded in a uniform porous medium with internal heat generation such as Mohamed [10], Jawdat and Hashim [11], and Ferdousi and Alim [12]. Mohamed [10] studied the effects of first-order homogeneous chemical reaction on the unsteady magnetohydrodynamic (MHD) double-diffusive free convection fluid flow past a vertical porous plate in the presence of heat generation and soret effects. The effects of uniform internal heat generation on chaotic behavior in thermal convection in a fluid-saturated porous layer subject to gravity and heated from below for low Prandtl number was investigated by Jawdat and Hashim [11]. Then, Ferdousi and Alim [12] considered the effect of heat generation on natural convection flow from a porous vertical plate. In 2006, Veena et al. [13] have worked on heat transfer characteristics in the laminar boundary layer flow of a viscoelastic fluid over a linearly stretching continuous surface with variable wall temperature subjected to suction or blowing. Molla et al. [14] examined the natural convection flow of a viscous incompressible fluid past an isothermal horizontal circular cylinder considering the temperature dependent internal heat generation. Mahdy [15] considered the effects of chemical reaction and heat generation on double-diffusive natural convection heat and mass transfer near a vertical truncated cone in porous media. Afterward, Siddiqa et al. [16] studied natural convection flow of a viscous incompressible fluid over a semi-infinite flat plate with the effects of exponentially varying temperature dependent viscosity and the internal heat
Consider the unsteady free convection flow near the stagnation point of a heated three-dimensional body placed in a viscous and incompressible fluid of uniform temperature $T_\infty$. It is assumed that the uniform temperature of the body is suddenly changed from $T_\infty$ to $T_e$, where $T_e > T_\infty$. A locally Cartesian orthogonal system $(x, y, z)$ is chosen with the origin $N$ at the nodal stagnation point as shown in Figure 1, where the $x$- and $y$-coordinates are measured along the body surface, while the $z$-coordinate is measured normal to the body surface. Under these assumptions, the boundary layer equations governing unsteady free convection flow are,

$$\frac{\partial u}{\partial t} + u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} + w \frac{\partial u}{\partial z} = v \frac{\partial^2 u}{\partial z^2} + g \beta ax(T - T_e)$$

subject to the initial and boundary conditions

$$t < 0: \quad u = v = w = 0, \quad T = T_e \quad \text{for any} \quad x, y, z$$

$$t \geq 0: \quad u = v = 0, \quad T = T_e \quad \text{on} \quad z = 0, x \geq 0, y \geq 0$$

$$u = v = 0, \quad T = T_e \quad \text{on} \quad x = 0, y \geq 0, z > 0$$

$$u = v = 0, \quad T = T_e \quad \text{on} \quad y = 0, x \geq 0, z > 0$$

$$u = v = 0, \quad T = T_e \quad \text{as} \quad z \to \infty, x \geq 0, y \geq 0$$
Here \( u, v, w \) are the velocity components along \( x, y, z \) axes, \( t \) is the time, \( T \) is the fluid temperature, \( g \) is the magnitude of the gravity acceleration, \( \alpha \) is the coefficient of thermal diffusivity, \( \nu \) is the kinematic viscosity, \( \beta \) is the volumetric coefficient of thermal expansion, \( \rho \) if the fluid density, \( c_p \) is the specific heat at constant pressure and \( a \) and \( b \) are the parameters of the principal curvatures at \( N \), of the body measured in the planes \( y \) and \( x \), respectively.

The term \( Q_0(T - T_\infty)^n \) is assumed to be amount of heat generated or absorbed per unit volume, which \( Q_0 \) may take on either positive or negatives values. Further, if \( Q_0 > 0 \) then it represents heat generation and on the other hand when \( Q_0 < 0 \) it represents heat absorption.

There is no loss of generality in requiring that \( |a| \geq |b| \) with \( a > 0 \). Clearly \( b = 0 \) corresponds to the plane stagnation flow case, while \( b = a \) is the axisymmetric case. We assume here that \( a \) and \( b \) are positive so that solutions of the resulting equations lead to stagnation points which are nodal points of attachment, i.e. 0 equations lead to stagnation points which are nodal points of the form, see Shesadri et al.

A little inspection shows that equations (1) to (4) along with the boundary conditions (5) admit a semi-similar solution of the form, see Shesadri et al. [31],

\[
\begin{align*}
\eta & = Gr^{1/4}a^{1/2}z, \\
u & = va^2xGr^{1/2}f'(\xi, \eta) \\
\nu & = va^2cyGr^{1/2}h'(\xi, \eta), \\
\theta(\xi, \eta) & = \left(\frac{T - T_\infty}{T_a - T_\infty}\right), \\
\tau & = va^2Gr^{1/2}t
\end{align*}
\]

where \( Gr = \frac{g\beta(T_a - T_\infty)}{(a^2\nu^2)} \) is the Grashof number and primes denote partial differentiation with respect to \( \eta \).

Substitution of (6) into (2) to (4) gives

\[
f'' + (1 - \xi)\frac{\eta}{2}f'' + \xi\left[(f + ch)f'' - f'^2\right] \\
+ \xi\theta = \xi(1 - \xi)\frac{\partial f'}{\partial \xi} \\
h'' + (1 - \xi)\frac{\eta}{2}h'' + \xi\left[(f + ch)h'' - ch'^2\right] \\
+ \xi\theta = \xi(1 - \xi)\frac{\partial h'}{\partial \xi} \\
\frac{1}{Pr} \theta'' + (1 - \xi)\frac{\eta}{2}\theta'' + \xi(f + ch)\theta'' \\
+ \xi Q \theta' = \xi(1 - \xi)\frac{\partial \theta}{\partial \xi}
\]

while the boundary conditions (5) become

\[
f(\xi, 0) = f'(\xi, 0) = 0, \\
h(\xi, 0) = h'(\xi, 0) = 0, \\
\theta(\xi, 0) = 1 \\
f'' \rightarrow 0, \\
h'' \rightarrow 0, \\
\theta \rightarrow 0 \text{ as } \eta \rightarrow \infty
\]

for \( 0 \leq \xi \leq 1 \) where \( Q = (T_a - T_\infty)^{n-1} Q_0 / (c_p a^2 \mu Gr^{1/2}) \) is the dimensionless heat generation or absorption coefficient. Here \( Pr \) is the Prandtl number and primes denote partial differentiation with respect to \( \eta \).

The physical quantities of practical interest in this problem are the skin friction coefficients in the \( x \) and \( y \) directions, \( C_f \) and \( C_h \) and the Nusselt number, \( Nu \), that are defined as

\[
C_f = \mu \left(\frac{\partial u}{\partial \xi}\right)_{\eta=0} / (\rho v^2 a^3 x),
\]

\[
C_h = \mu \left(\frac{\partial h}{\partial \xi}\right)_{\eta=0} / (\rho v^2 a^3 y),
\]

\[
Nu = a^{-1} \left(\frac{\partial T}{\partial \xi}\right)_{\eta=0} / (T_w - T_\infty)
\]

where \( \rho \) and \( \mu \) is the density and dynamic viscosity, respectively. In terms of the non-dimensional variables (6), we have

\[
C_f \xi^{1/2} / Gr^{3/4} = f'(\xi, 0),
\]

\[
C_h \xi^{1/2} / Gr^{3/4} = h'(\xi, 0),
\]

\[
Nu \xi^{1/2} / Gr^{3/4} = -\theta'(\xi, 0)
\]

For the unsteady-initial flow case, where \( \xi = 0 \), equations (7) to (9) reduce to the following form

\[
f'' + \frac{\eta}{2} f'' = 0, \\
h'' + \frac{\eta}{2} h'' = 0, \\
\theta'' + Pr \frac{\eta}{2} \theta'' = 0
\]

subject to the boundary conditions

\[
f(0) = f'(0) = 0, \\
h(0) = h'(0) = 0, \\
\theta'(0) = 1
\]

\[
f'(-\infty) = 0, \\
h'(-\infty) = 0, \\
\theta(-\infty) = 0
\]

The solution of Eqs. (14) subject to (15) is given by

\[
f = h = \theta(\eta) = \text{erfc}\left(\sqrt{\frac{Pr}{2}} \eta\right)
\]

where \( \text{erfc}(\eta/\sqrt{Pr/2}) \) is the complimentary error function. For the final steady-state flow case, where \( \xi = 1 \), equations (7) to (9) reduce to the following similar form

\[
f'' + (f + ch)f'' - f'^2 + \theta = 0
\]

\[
h'' + (f + ch)h'' - ch'^2 + \theta = 0
\]

\[
\theta'' + Pr(f + c h)\theta' + Pr Q \theta'' = 0
\]
subject to the boundary conditions
\[ f(0) = f'(0) = 0, \quad h(0) = h'(0) = 0, \quad \theta(0) = 1 \]
\[ f'(\xi) = 0, \quad h'(\xi) = 0, \quad \theta(\xi) = 0 \] (19)

These equations are identical with those first found by Poots [20], Banks [21] and Sharidan et al. [22].

### III. RESULT AND DISCUSSION

Equations (8) to (10) subject to boundary conditions (11) has been solved numerically using an implicit finite difference scheme, known as the Keller-box method developed by Keller [32]. This method has been found to be very suitable in dealing with nonlinear parabolic problems. Details of the method may be found in many recent publications, examples Hussain and Hossain [33], Sharidan et al. [34] and Cebeci and Bradshaw [35].

Results are obtained for \( p = 1 \) and \( 2, \) \( Pr = 0.015 \) (mercury), 0.7 and 0.72 (air), 4 (R-12 refrigerant), 7 (water at 20°C) and 100 (engine oil) and \( c = 0 \) (plane stagnation point), 0.25, 0.5, 0.75 and 1.0 (axi-symmetric stagnation point). To access the accuracy of the solutions, the present results for the reduced skin friction coefficients \( f(0) \) and \( h(0) \), and heat transfer from the surface of the body, \( \theta(0) \) are compared with those calculated by Banks [21] and Sharidan et al. [22] for natural convection heat transfer from a three-dimensional body with constant wall temperature in Newtonian fluids with \( Pr = 0.72, c = 0 \) and values of \( c \) between 0 and 1, as shown in Table 1. The present results agree well with the solutions presented by Banks [21] and Sharidan et al. [22].

**TABLE I**

<table>
<thead>
<tr>
<th>( Pr = 0.72, Q = 0 ) AND DIFFERENT VALUES OF ( c )</th>
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<tr>
<td>Banks [21]</td>
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<td>( c )</td>
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<td>( f(0) )</td>
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<td>( h(0) )</td>
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<td>( -\theta(0) )</td>
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A comparison of the present results for these quantities with those obtained by Sharidan et al. [22] is shown also in Figure 2. It is seen that all these results are in excellent agreement. Therefore, this favorable comparison lends confidence in the numerical results obtained in this paper.

Fig. 2 Comparison of the skin friction coefficients \( f'(0) \) and \( h'(0) \), and heat flux from the surface of the body \( \theta(0) \) for steady flow case \( (\xi = 1) \) when \( Pr = 0.72 \) and \( Q = 0 \).

The variation of the velocity \( f(\xi, \eta) \), \( h(\xi, \eta) \) and temperature \( \theta(\xi, \eta) \) profiles with \( \eta \) for some values of \( \xi \) are shown in Figures 4 and 5 for \( Pr = 0.72, c = 0, 1, \) and \( p = 1 \) and 2. It has been seen from Figures 3 and 4 that as \( \xi \) increases, the velocity \( f(\xi, \eta) \), \( h(\xi, \eta) \) and temperature \( \theta(\xi, \eta) \) profiles increase. The changes of velocity profiles, \( f(\xi, \eta) \) and \( h(\xi, \eta) \) in the \( \eta \) direction reveals the typical velocity profiles for natural convection boundary layer flow, i.e., the velocity is zero at the boundary wall then the velocity increases to the peak values as \( \eta \) increases and finally the velocity approaches to zero (the asymptotic value). The changes of temperature profiles, \( \theta(\xi, \eta) \) in the \( \eta \) direction also shows the typical temperature profile for natural convection boundary layer flow that is the value of temperature profile is one at the boundary wall then the temperature profile decreases gradually along \( \eta \) direction for the values of \( \xi \) from 0 to 1 to the asymptotic value. Regarding to the changes in values of \( p \), the heat generation term \( \theta' \) can be dominant, leading to a breakdown in the solution with a thermal runaway. This happens for moderate values of \( p \). For even larger values of \( p \), the temperature profiles remain bounded throughout, \( \theta' \) remains small for \( p \) large and eventually the initial heat input decays away.
Unsteady natural convection flow on a three-dimensional body in the presence of heat generation near the stagnation point has been investigated in this paper for different values of relevant physical parameters including Prandtl number, Pr and heat generation parameter, Q. From the present investigation, the following conclusions may be drawn:

- The steady-state flow for the reduced skin friction and heat transfer coefficients has a similar structure to what was studied by Banks [21] and Sharidan et al. [22].
- There is a smooth transition from unsteady-state flow ($\xi = 0$) to the steady-state flow ($\xi = 1$) for all velocity and temperature profiles.

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