
E. Giovanis

Abstract—The purpose of this paper is to present two different approaches of financial distress pre-warning models appropriate for risk supervisors, investors and policy makers. We examine a sample of the financial institutions and electronic companies of Taiwan Security Exchange (TSE) market from 2002 through 2008. We present a binary logistic regression with panel data analysis. With the pooled binary logistic regression we build a model including more variables in the regression than with random effects, while the in-sample and out-sample forecasting performance is higher in random effects estimation than in pooled regression. On the other hand we estimate an Adaptive Neuro-Fuzzy Inference System (ANFIS) with Gaussian and Generalized Bell (Gbell) functions and we find that ANFIS outperforms significant Logit regressions in both in-sample and out-of-sample periods, indicating that ANFIS is a more appropriate tool for financial risk managers and for the economic policy makers in central banks and national statistical services.

Keywords—ANFIS, Binary logistic regression, Financial distress, Panel data

I. INTRODUCTION

In the most cases economists use regression and parametric models in order to investigate the significance of the explanatory variables on the output they would like to estimate and forecast. The definition of a financial distress period or stage is distinguished among the researches and previous studies. The first definition is based on Law criteria [1], while the second is a dynamic stage which is entered to a financial distress stage step by step [2]. In this paper we examine a binary logistic pooled regression as also we apply Hausman test to decide if there are random or fixed effects. We conclude that there are random effects, so we present only the estimated regression results and forecasts of random effects panel logistic regression as fixed effects present poor estimating and forecasting results. Previous studies used various approaches for the financial and bankruptcy modeling formulation.

II. METHODOLOGY

A. Binary Logit Regressions

The logistic distribution is defined as [8]:

\[
Pr(Y = 1 \mid x) = \frac{e^{x'\beta}}{1 + e^{x'\beta}} = \Phi(x' \beta)
\]  (1)

Platt and Platt [3], and Cheng et al. [4] used a Logit model to analyze pre-warning model and to a build financial distress model, while Zhang et al. [5], and O’leary [6] used artificial neural networks. One problem of the logistic regression is that serial correlation might exist in the explanatory variables. Another problem is the inconsistency generated from the errors on construction of the dummy variable indicating the financial stage, distress or stability periods, crisis or no crisis periods, leading to misclassification of time points. With ANFIS approach we do not face these problems, which are very usual in conventional econometric modelling. A significant study was made by Cheng et al. [7]. The authors study a pre-warning financial distress model for the TSE listed companies and they apply a binary logit and a fuzzy regression model with triangular membership function. Their results support fuzzy regression, where the correctly predicted percentage of fuzzy regression is 90.98 percent versus logit regression which predicts correctly the 90.30 percent. In this paper we expand this approach by taking panel data as we have a group of companies. Because we have various companies among time periods we need to examine logistic regressions through panel data analysis and to investigate if random or fixed effects are more appropriate. With this approach we show that the overall percentage, and especially the correct percentage of financial distress periods, of panel Logit model is significant higher to simple binary Logit model examined by Cheng et al. [7]. Additionally, we propose ANFIS because the overall correct classified percentage of financial distress and stability periods is significant superior to Logit and fuzzy regressions.
The marginal partial effects of explanatory variables are given by:

\[
\frac{\partial E(y \mid x)}{\partial x} = \Phi(x' \beta)[1 - \Phi(x' \beta)]\beta
\]  

(2)

Logit model can be written a general form regression as:

\[
y = \alpha + \sum_{i=1}^{n} \beta_i x_i + \varepsilon
\]  

(3)

where variable \( y \) is a binary dummy variable taking value 1 if is a financial distress period and value zero otherwise (financial stability period), \( x_i \) indicates the explanatory variables, \( \alpha \) is the constant, \( \beta_i \) are the regression estimators. Pooled regression can be defined as:

\[
y_{it} = w_i + x_{it}' \beta + \varepsilon_{it}
\]  

(4)

where \( x_{it} \) express the explanatory variables, \( \beta \) are the estimated parameters of the independent variables, \( w_i \) contains the constant term and the set of the firms or companies, \( i \) and \( t \) indicate the firm and time period respectively. Because in our analysis \( w_i \) contains only a constant term, then ordinary least squares method provides consistent and efficient estimates. On the other hand if \( w_i \) is correlated with \( x_{it} \) then the least squares estimations are biased and we have a fixed effect model, where \( w_i \) embodies all the observable effects. Finally if the unobserved firm heterogeneity can be assumed to be uncorrelated with the explanatory variable then we have a random effects model, which can be written as [8]:

\[
y_{it} = a + x_{it}' \beta + u_{it}
\]  

(5)

The fixed effects model is:

\[
y_{it}' = d_{it} + x_{it}' \beta + \varepsilon_{it}
\]  

(6)

for \( i = 1,2,\ldots,n \) and \( t = 1, 2, \ldots, T \)

where \( d_{it} \) is a dummy variable which takes value one for \( i \) firm and zero for otherwise. Parameter \( a_i \) is the constant, The random effects model model is:

\[
y_{it}' = x_{it}' \beta + v_{it} + u_{it}
\]  

(7)

for \( i = 1,2,\ldots,n \) and \( t = 1, 2, \ldots, T \)

where \( u_{it} \) is the unobserved company or individual specific heterogeneity and the assumption that \( u_{it} \) is unrelated to \( x_{it} \) produces the random effects models. The Logit fixed effects model can be defined as:

\[
Pr(\{ y_{it} = 1 \mid x_{it} \}) = \frac{e^{\alpha_i + x_{it}' \beta}}{1 + e^{\alpha_i + x_{it}' \beta}}
\]  

(8)

Random effects Logit model can be defined in a similarly way. In order to test for orthogonality of the random effects and the regressors we apply the specification Hausman test [8]. The required covariance matrix for Hausman test is defined as:

\[
Var[b - \hat{\beta}] = Var[b] + Var[\hat{\beta}]
\]  

(9)

The chi-squared test which is based on Wald criterion is:

\[
W = \chi^2[k-1] = [b - \hat{\beta}]Var[b - \hat{\beta}][b - \hat{\beta}]
\]  

(10)

Under the null hypothesis \( W \) follows a chi-squared distribution where \( k \) is the number of regressors. More specifically under the null hypothesis we assume that the individual effects are uncorrelated with the other regressors and so if we accept the null hypothesis random effects model is appropriate.

**B Adaptive Neuro-Fuzzy Inference System (ANFIS)**

In this section we describe the neuro-fuzzy modelling and the genetic algorithms optimization training procedure. First we take two inputs in our system which are the cash flow and the returns on assets from Table I, exactly the same used in the study of Cheng et al. [7] for comparison purposes, and the output is the classification dummy variable. The reason why we propose neuro-fuzzy logic is that the traditional classification of one and zero can be misleading. We incorporate three linguistic terms {low, medium, high}. More linguistic terms can be introduced, as very low and very high, but the forecasting performance is almost the same, indicating that we can simplify the procedure by taking less linguistic terms and less rules and reducing with this way the computation time. The rules are 9 because we have two inputs with three linguistic terms and it is 3 * 3 = 9. These rules are

If cash flow is low AND returns on assets are low THEN

\[
f_1 = p_1 x_1 + q_1 x_2 + r_1
\]

If cash flow is low AND returns on assets are medium THEN

\[
f_2 = p_2 x_1 + q_2 x_2 + r_2
\]
IF cash flow is low AND returns on assets are high THEN
\[ f_1 = p_1 x_1 + q_1 x_2 + r_1 \]

IF cash flow is medium AND returns on assets are low THEN
\[ f_2 = p_2 x_1 + q_2 x_2 + r_2 \]

IF cash flow is medium AND returns on assets are medium THEN
\[ f_3 = p_3 x_1 + q_3 x_2 + r_3 \]

IF cash flow is medium AND returns on assets are high THEN
\[ f_4 = p_4 x_1 + q_4 x_2 + r_4 \]

IF cash flow is high AND returns on assets are low THEN
\[ f_5 = p_5 x_1 + q_5 x_2 + r_5 \]

IF cash flow is high AND returns on assets are medium THEN
\[ f_6 = p_6 x_1 + q_6 x_2 + r_6 \]

IF cash flow is high AND returns on assets are high THEN
\[ f_7 = p_7 x_1 + q_7 x_2 + r_7 \]

We choose the AND operator so we will take the product instead to min operator to avoid monotonic result. Also each rule has 2 parameters plus the constant hence there will be 3*9=27 parameters. Jang [9] and Jang and Sun [10] introduced the adaptive network-based fuzzy inference system (ANFIS). This system makes use of a hybrid learning rule to optimize the fuzzy system parameters of a first order Sugeno system. Because we have five rules and two inputs in the case we examine the steps for ANFIS system computation are:

In the first layer we generate the membership grades

\[ O_i^1 = \mu_{A_i}(x_1), \mu_{B_i}(x_2) \]  

where \( x_1 \) and \( x_2 \) are the inputs. In layer 2 we generate the firing strengths or weights

\[ O_i^2 = w_i = \prod_{j=1}^{m} (\mu_{A_i}(x_1), \mu_{B_i}(x_2)) = \text{andmethod} (\mu_{A_i}(x_1), \mu_{B_i}(x_2)) \]

\[ = \text{product} (\mu_{A_i}(x_1), \mu_{B_i}(x_2)) \]  

In layer 2 we use the AND relation, as it was mentioned previously, so we take the product operator. In layer 3 we normalize the firing strengths. Because we have five rules will be:

\[ O_i^3 = \overline{w}_i = \frac{w_i}{w_1 + w_2 + w_3 + \ldots + w_8 + w_9} \]

where \( i \) is for \( i=1,2,\ldots,9 \). In layer 4 we calculate rule outputs based on the consequent parameters.

\[ O_i^4 = y_i = \overline{w}_i f_i = \overline{w}_i (p_1 x_1 + q_1 x_2 + r_1) \]  

In layer 5 we take

\[ O_i^5 = \sum_{i} w_i f_i = \sum_{i} \overline{w}_i f_i \]

In the last layer the consequent parameters can be solved for using a least square algorithm as:

\[ Y = X \cdot \theta \]

\[ X = [w_1 x + w_1 x + w_2 x + w_2 + \ldots + w_8 x + w_9] \]

\[ \theta = [p_1, q_1, r_1, p_2, q_2, r_2, \ldots, p_9, q_9, r_9]^T \]

The singular values in \( S \) are positive and arranged in decreasing order. Their magnitude is related to the information content of the columns of \( U \)-principle components- that span \( X \). Therefore, to remove the noise effects on the solution of the weight matrix, we simply remove the columns of \( U \) that correspond to small diagonal values in \( S \). The weight matrix is then solved for using the following:

\[ \theta = Y V S^{-1} U^T \]

For the first layer and relation (11) we use the Gaussian and the Generalized Bell membership functions. The symmetrical Gaussian membership function is defined as:

\[ \mu_y(x_j; c_y, \sigma_y) = \exp \left( -\frac{(x_j - c_y)^2}{2 \sigma_y^2} \right) \]

\[ = \text{product} (\mu_{A_i}(x_1), \mu_{B_i}(x_2)) \]

The second membership function we examine is the Generalized Bell function in a specific form such as:

\[ \mu_y(x_j; a_y, b_y, c_y) = \exp \left( -\left( \frac{x_j - c_y}{a_y} \right)^{2b_y} \right) \]
where $c$ locates the center of the curve and parameters $a$ and $b$ vary the width of the curve. In order to find the optimized antecedent parameters we use the backpropagation algorithm with the simple steepest descent method [14]-[16]. The update equation is:

$$c_{ij}(n+1) = c_{ij}(n) - \eta_e \frac{\partial E}{\partial c_{ij}}$$ (23)

where $\eta_e$ is the learning rate for the parameter $c_{ij}$, $p$ is the number of patterns and $E$ is the error function which is:

$$E = \frac{1}{2} (y - y')^2$$ (24)

where $y'$ is the target-actual and $y$ is ANFIS output variable.

The chain rule used in order to calculate the derivatives and update the membership parameters are [14]-[16]:

$$\frac{\partial E}{\partial a_{ij}} = \frac{\partial E}{\partial y} \cdot \frac{\partial y}{\partial y_{ij}} \cdot \frac{\partial y_{ij}}{\partial w_{ij}} \cdot \frac{\partial w_{ij}}{\partial \mu_{ij}} \cdot \frac{\partial \mu_{ij}}{\partial a_{ij}}$$ (25)

After some partial derivatives computations, the update equations for $c_{ij}$ are $\sigma_{ij}$ of Gaussian membership function are respectively:

$$c_{ij}(n+1) = c_{ij}(n) - \eta_e \cdot e^{(px+r)} \cdot \frac{1}{\sigma_{ij}^2} \left( x_{ij} - c_{ij} \right) \mu_j(x_j)$$ (26)

$$\sigma_{ij}(n+1) = \sigma_{ij}(n) - \eta_e \cdot e^{(px+r)} \cdot \frac{1}{\sigma_{ij}^2} \left( x_{ij} - c_{ij} \right) \mu_j(x_j)$$ (27)

The update equations for Generalized Bell membership function can be obtained in a similar way. The next step is to define the initial values for antecedent parameters. In all cases we get as initial values for center and bases parameters the mean and standard deviation. To be specific we get one sample where the returns on assets are negative and one sample where the returns are positive. The same procedure is followed for cash flow. So for center parameters of Gaussian and Generalized Bell function respectively we take the average for negative and positive samples. In the case of Gaussian we take the standard deviation as the initial value for parameters $\sigma$, while for parameter $a$ of Generalized bell function we take the following formula [15]:

$$\alpha = \frac{\text{range}}{\text{mfs} * 2}$$ (28)

, where the \text{range} is the well known statistical measure, while mfs is the number of membership functions. For the parameter $b$ we take as initial value 1.1. The learning rates for all the parameters of ANFIS with both membership functions we examine are set up at 0.5. The number of maximum epochs is set up at 100.

### III. RESEARCH DESIGN

In this section we present the variables which are used in the analysis [7]. We should mention that in logistic regressions we include some of the variables presented on Table I, while with ANFIS we get the same variables with those in the study of Cheng et al. [7], as we mentioned above which are the cash flow and the returns on assets. The dependent dummy binary variable expresses the financial stage, where takes the value 1 if the specific company in the certain time period is on financial distress and value 0 if is characterized by financial stability. To define whether a company is in financial distress or stability stage we follow the approach followed by Gentry et al. [17], where a company which reduce dividends typically encounter financial distress.

<table>
<thead>
<tr>
<th>Category</th>
<th>Financial variables</th>
<th>Symbol</th>
</tr>
</thead>
<tbody>
<tr>
<td>Financial structure</td>
<td>Shareholders' equity to total assets ratio (%)</td>
<td>$x_1$</td>
</tr>
<tr>
<td></td>
<td>Debt to total assets ratio (%)</td>
<td>$x_2$</td>
</tr>
<tr>
<td></td>
<td>Permanent capital to fixed assets ratio (%)</td>
<td>$x_3$</td>
</tr>
<tr>
<td>Liquidity</td>
<td>Current assets (%)</td>
<td>$x_4$</td>
</tr>
<tr>
<td>Cash</td>
<td>Cash flow ratio (%)</td>
<td>$x_5$</td>
</tr>
<tr>
<td>Asset utilization</td>
<td>Accounts receivable turnover</td>
<td>$x_6$</td>
</tr>
<tr>
<td></td>
<td>Fixed asset turnover</td>
<td>$x_7$</td>
</tr>
<tr>
<td></td>
<td>Total asset turnover</td>
<td>$x_8$</td>
</tr>
<tr>
<td>Profitability</td>
<td>Returns on assets (%)</td>
<td>$x_9$</td>
</tr>
<tr>
<td></td>
<td>Return on common equity (%)</td>
<td>$x_{10}$</td>
</tr>
<tr>
<td></td>
<td>Pre-tax profit to capital (%)</td>
<td>$x_{11}$</td>
</tr>
<tr>
<td></td>
<td>Earnings per share</td>
<td>$x_{12}$</td>
</tr>
</tbody>
</table>

### IV. DATA

We use data from a sample of electronic companies and financial institutions listed in TSE Securities and Futures Institute Network from 2002 through 2008. We should mention that we obtained a sample of these companies and not all of them, so we expect that random effects will be probably more appropriate, concerning the binary Logit regression. Specifically our estimation sample is constituted by 179
V. EMPIRICAL RESULTS

Concerning the binary logistic model and panned data models we present only the estimation of random effects, because we selected it based on Hausman test, as because the estimation results of the fixed effects regression are poor and the correctly classification percentage is low. In Table II we present the estimation results of the binary pooled Logit regression. All the estimated parameters are statistically significant and have the expected and correct sign. More specifically there is positive relation between \( \text{debt to total assets ratio} \) and \( \text{permanent capital to fixed assets ratio} \) to the probability of being in the financial distress regime. So if the \( \text{debt to total assets ratio} \) and \( \text{permanent capital to fixed assets ratio} \) increased respectively by 1% then the risk of financial distress occurrence is increased by 1.021 and 1.00 respectively. We computed these values as \( e^{0.02089} \) and \( e^{0.000012} \). The exactly same computations are applied for the remaining coefficients in both pooled and random effects Logit regression. On the other hand if the \( \text{cash flow ratio, the Accounts receivable turnover, the fixed asset turnover, the Return on common equity} \) and \( \text{earnings per share} \) are increased by 1%, then the risk of financial distress occurrence is decreased by 0.998, 0.987, 0.998, 0.998 and 0.968 respectively. We observe that we reject the existence of ARCH effects but we conclude that there is autocorrelation. Based on Log-Likelihood chi-square we accept that pooled Logit regression is statistically significant.

In Table III we provide Hausman tests for the choice between random and fixed effects. In the case of Logit model we reject the null hypothesis in \( \alpha = 0.01 \) and \( \alpha = 0.05 \) statically significance level, but we accept it in \( \alpha = 0.10 \). The question is whether we should estimate with random or fixed effects and in which estimation we could rely on. The answer is that we prefer the random instead to fixed effects estimation, where the coefficients are statistically significant, as also the forecasting performance is superior. For these reasons and after experimental estimations we select random effects.

In Table IV we provide the random effects Logit regression results, where once again all the estimated coefficients are statistically significant and present the correct and the expected sign. In this case if the \( \text{permanent capital to fixed assets ratio} \) increased by 1% then the risk of financial distress occurrence is increased by 1.00, so there is a one to one relation as in the pooled Logit regression results. If \( \text{fixed asset turnover} \) and \( \text{earnings per share} \) increased by 1%, then the risk of financial distress occurrence is decreased by 0.995 and 0.0142. Furthermore random effects Logit regression presents ARCH effects, while we reject autocorrelation in \( \alpha = 0.05 \) and \( \alpha = 0.01 \) levels.
period, while the respective percentages for pooled regression are only 80.41 and 81.74. In the out-of-sample the correct classified percentages for financial distress and stability periods with random effects model are reduced at 88.88 and 93.43 per cent respectively, and the respective percentages with the pooled regression are 77.77 and 83.78 respectively. Generally, we conclude that random effects model outperforms significant the pooled regression in both samples we examine. Furthermore our findings with random effects model higher to those of Chen et al. [7], who found that the correctly predicted percentages of fuzzy and Logit regressions are respectively 90.98 and 90.30, lower to the overall percentage of the random effects model, which is 94.32 and 92.65 in the in-sample and out-of-sample periods respectively. On the other hand Chen et al. [7] with fuzzy regression predict the financial distress periods at 91.59 in the out-of-sample period, while the respective predicted value with the random effects Logit model is 88.88 per cent.

In tables IX and X the in-sample and out-of-sample forecasts, for ANFIS with Gaussian membership function, are reported. We observe that in the in-sample period ANFIS predicts correct at 98.96 and 93.31 per cent the distress and stability periods respectively. On the other hand with ANFIS we predict the financial distress periods at 100.00 in the out-of-sample period, which is significant higher to other models. Additionally, we predict the stability periods correct at 87.25 per cent in relation to random effects model, which is 93.43 or the fuzzy regression in the paper of Chen et al. [7], who predict the stability periods at 89.77. So a first conclusion is that with ANFIS and Gaussian function we predict perfect the financial distress periods and therefore the last model is more appropriate for financial distress periods predictions than binary Logit, random effects Logit model or fuzzy regression are.

Finally, in Tables XI and XII we present the in-sample and out-of-sample forecasts respectively for ANFIS with Generalized Bell function. From the results we observe that Gbell function presents almost the same forecasting performance in the in-sample periods with Gaussian function, outperforming significant the Logit panel models. On the other hand in the out-of-sample period Gbell presents the highest forecasting performance. Specifically, we predict the financial distress period at 100.00 per cent, while the correct classification percentage for financial stability periods is 95.36 with an overall percentage of 96.16 significant superior to Logit models, to the findings of Chen et al. [7], as also superior to ANFIS with Gaussian function. ANFIS is proposed as an alternative superior model, where we can predict very successfully both distress and stability periods.
VI. CONCLUSIONS

In this paper we examined Logit pooled and random effects models and we compared the results with ANFIS and with other studies. With random Logit effects model we get higher forecasting performance in relation to the simple binary logistic regression without panel data analysis. On the other hand with ANFIS and Gaussian function we predict at 100.00 per cent the financial distress periods, while with generalized bell function we get very strong forecasts in both distress and stability periods. For this reason we propose ANFIS technology for application and use by the financial risk managers, the national central banks and national statistical services, as well as the introduction in the economic university departments around the world as basic course. Additionally, we used only two symmetrical fuzzy membership functions, while more functions can be used as the triangle or the sigmoid. Moreover, genetic algorithms can be applied instead to error backpropagation algorithm for the training process. Finally, more inputs can be taken or similar systems can be built in order to predict bankruptcies, global or national economic crises, food crisis, or even war predictions.

REFERENCES