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Analytical Mathematical Expression for the Channel Capacity of a Power and Rate Simultaneous Adaptive Cellular DS/FFH-CDMA System in a Rayleigh Fading Channel

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***Abstract*—In this paper, an accurate theoretical analysis for the achievable average channel capacity (in the Shannon sense) per user of a hybrid cellular direct-sequence/fast frequency hopping code-division multiple-access (DS/FFH-CDMA) system operating in a Rayleigh fading environment is presented. The analysis covers the downlink operation and leads to the derivation of an exact mathematical expression between the normalized average channel capacity available to each system's user, under simultaneous optimal power and rate adaptation and the system's parameters, as the number of hops per bit, the processing gain applied, the number of users per cell and the received signal-to-noise power ratio over the signal bandwidth. Finally, numerical results are presented to illustrate the proposed mathematical analysis.

***Keywords*—Shannon capacity, adaptive systems, code-division multiple access, fading channels.

I. Introduction

THE Shannon's capacity of a single-user time-invariant channel is defined as the maximum mutual information between the channel input and output. In a mobile radio environment, physical channels exhibit randomly time-varying characteristics that result in signal fading and substantial channel capacity degradation.

The maximum rate at which data can be transmitted in a fading environment with arbitrarily small bit-error-rate (BER) is obtained by finding the best distribution of the transmitted signal power in terms of the instantaneous signal-to-noise power ratio (SNR) and, then, averaging over the SNR distribution, [1].

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Channel capacity can also be estimated in an average sense, while its degradation is anticipated to a certain degree by utilizing some kind of diversity reception technique, [2]. This average channel capacity formula, with optimal rate adaptation to channel fading and a constant transmit power, would indeed provide the true channel capacity, if channel side information (CSI) were available only at the receiver, [3].

In this work, following the method and the hybrid system firstly described in [4] and described again here only for presentation reasons, we evaluate the achievable channel capacity (in the Shannon sense) per user, on the condition that this is estimated in an average sense, but now, under simultaneous optimal power and rate adaptation. The final mathematical expression, theoretically derived, to the author's best knowledge, is the first time such expression has been exposed for a hybrid DS/FFH-CDMA cellular system. It must be noticed that, in the followed analysis, a fixed number of simultaneously transmitting users is assumed. Although a dynamic user population is a reasonable assumption, the results derived in the paper, can be applied directly in a DS/FFH-CDMA cellular system with a variable number of users, considering that the number of users per cell K, represents the mean value of users per cell in a birth-death model describing the variable allocation of users, [5]. However, in a future work a simulation process must be described in order to compare with the theoretical mathematical results derived here.

II. Operation In An AWGN Non-Fading Channel

While in a cellular DS-CDMA system multiple-access interference (MAI) is typically the dominant source of interference, in cellular FFH-CDMA systems the users within each cell can be approximately orthogonalized so that they do not interfere with one another, [6]. Under this assumption, in a cellular DS/FFH-CDMA system the original transmitted signal is only corrupted by additive white gaussian noise (AWGN) and co-channel interference (CCI). During each frequency hop, a DS signal is transmitted in the form of a spread signal with bandwidth $W_{ds}=G_p \cdot W_s$, where W_s is the signal bandwidth and G_p is the processing gain. Bandwidth spreading is accomplished by multiplying the information data by a pseudo-random sequence. Hopping from one frequency to another is also determined by a pseudo-random sequence, while at the receiver, the signal is de-hopped by a frequency synthesizer controlled by an identical pseudo-random sequence. Assuming that q is the available number of different hopping frequencies assigned to each cell for frequency hopping, the totally allocated system's bandwidth W_t is equal to:

$$W_t=q \cdot W_{ds}=q \cdot \mathrm{G_p} \cdot W_s \tag{1}$$

We consider only the twelve co-channel cells, in the first tier, of a cellular DS/FFH-CDMA system operating in a non-fading AWGN environment. All base stations' and mobile units' antennas are assumed omnidirectionals. The channel capacity available to all 12K users of the cellular DS/FFH-CDMA system is limited only by CCI power, since, the users of each cell are assumed mutually orthogonal. Clearly, transmission of each user signal with arbitrarily small BER depends on CCI level. Thus, the channel capacity $C_{i,DS/FFH}$ required for errorless transmission of the spread signal of bandwidth W_{ds} will be given by the Shannon-Hartley theorem, [7]:

$$C_{i,\mathrm{DS/FFH}} = W_{ds} \cdot \log_2(1+S_{i,DS/FFH}) \tag{2}$$

where $S_{i,,DS/FFH}$, i=[1,..,12K], is the spread signal-to interference plus noise ratio (SINR) received at the i-th user as it reaches the boundary of a cell, reflecting the lowered signal power spectral density due to spreading.

In order to simplify the mathematical solution, we approximate all hexagon cells of the considered systems by circular regions of radius R with the same area. Assuming that in the downlink all mobile units of a certain cell will receive equal average signal power from their cell site when appropriate power control scheme is applied, then, for a fourth power law path loss, the average received signal power P_{av} at the cell boundary by the i-th user, will be:

$$P_{av}=\alpha \cdot R^{-4} \tag{3}$$

where α is a constant factor. Therefore, for a DS/FFH-CDMA cellular system with a number $M=q$ hops per transmitted bit, the SINR received at each one of the total $12K$ users as it reaches the boundary of a cell, $S_{i,\mathrm{DS/FFH}}$, can readily be determined by considering the average CCI power resulting from the eleven co-channel cells of the first dominant tier of interfering cells, i.e., from 11K interfering users, and neglecting all inter-cell interference, i.e.:

$$S_{i,DS/FFH}= \tag{4}$$

$$=\frac{P'_{av}}{n_0 \cdot W_{ds}+P_h \cdot \left[2 \cdot \mathrm{K}\alpha \mathrm{R}^{-4}+3 \cdot \mathrm{K}\alpha(2\mathrm{R})^4+6 \cdot \mathrm{K}\alpha(2.633\mathrm{R})^4\right] \cdot \frac{1}{\mathrm{M}}}=$$

$$=\frac{\frac{P_{av}}{\mathrm{M}}}{n_0 \cdot W_{ds}+P_h \cdot (2.3123\mathrm{K}) \cdot \frac{P_{av}}{\mathrm{M}}}$$

since, for a FFH system, the CCI power, as seen by a desired signal, originates on the average from $1/M$ of the co-channel users, [8]. In addition, in eq.(4), n_0 is the power spectral density of the additive white Gaussian noise, P_h is the probability of hit i.e., the probability that another user (other than the desired user) is transmitting at the same hopped frequency band and P'_{av} is the user's average received signal power in each of the M frequencies being equal to:

$$P'_{av} = \frac{P_{av}}{\mathrm{M}} \tag{5}$$

assuming that, in the FFH case, the totally transmitted signal power is equally shared, by hopping, among the M carrier frequencies. In eq.(4), the probability of hit P_h for the FFH case, can be approximated by:

$$P_h \cong \frac{1}{\mathrm{M}} \tag{6}$$

Furthermore, CCI is considered as gaussian distributed interference even for small values of the number of system's users, [9]. Then, eq.(4) can be rewritten in the form:

$$S_{i,DS/FFH} = \frac{S}{\mathrm{G_p} \cdot \mathrm{M} + \frac{1}{\mathrm{M}} \cdot (2.3123 \cdot \mathrm{K}) \cdot S} \tag{7}$$

where, in eq.(7), S=(P_{av}/N) is the received SNR over the signal bandwidth W_s. Following eq.(2), the channel capacity for the twelve cells of the cellular DS/FFH-CDMA system under consideration, that is, the total channel capacity available to all 12K users, will be given by the sum of the individual rates:

$$C_{DS/FFH} = \sum_{i=1}^{12\mathrm{K}} C_{i,DS/FFH} = W_{ds} \cdot \sum_{i=1}^{12\mathrm{K}} \log_2(1+S_{i,DS/FFH}) \tag{8}$$

where $S_{i,DS/FFH}$ is given by eq.(7). Moreover, since in practice, $S_{i,DS/FFH}$, i=[1,..,12K], is well below unity (in linear scale), [9], eq.(8) can be approximated by:

$$C_{DS/FFH} = W_{ds} \cdot \log_2(1+12\mathrm{K} \cdot S_{i,DS/FFH}) \tag{9}$$

III. Operation In A Rayleigh Fading Channel

We assume that the physical channel of bandwidth W_{ds} is greater than the coherence bandwidth W_{coh} of the Rayleigh fading channel, [10]. The radio channel is modeled as a slowly fading, time-invariant and discrete multipath channel and, thus, it appears to be frequency-selective to the transmitted DS signals. The maximum number M_{ds} of uncorrelated resolvable paths will then be given by, [11],:

$$\mathrm{M_{ds}} = [W_{ds} \cdot \Delta] + 1 = \left[\frac{\Delta}{\mathrm{T_{ds}}}\right] + 1 \tag{10}$$

where T_{ds}=1/W_{ds} is the "chip" duration of the DS spreading sequence, Δ is the maximum delay spread of the fading channel and [.] returns the largest integer less than, or equal to, its argument. In general, the multipath-intensity profile (MIP) in an urban Rayleigh fading environment is exponential, but, in this work, the MIP in a Rayleigh fading environment is assumed discrete and constant, so that the "resolvable" path model can be considered to have equal path strengths on the average. In a maximal-ratio combining (MRC) RAKE receiver, the output's decision variable is identical to the decision variable which corresponds to the output of a M-branch space diversity MRC technique, with $M=M_{ds}$, [10,11]. Consequently, the MRC reception of DS-CDMA spread signals, achieved by the considered RAKE receiver, is equivalent to a M_{ds}-branch space diversity MRC technique. Therefore, the probability density function (p.d.f.) of the combined instantaneous SINR γ of the spread signal over the bandwidth W_{ds}, with no correlation among the M_{ds} branches, will follow the distribution, [11],:

$$p_\gamma(\gamma) = \frac{1}{(M_{ds}-1)!}\cdot\frac{\gamma^{M_{ds}-1}}{\langle\gamma\rangle^{M_{ds}}}\cdot\exp\left(-\frac{\gamma}{\langle\gamma\rangle}\right) \quad (11)$$

where $\langle\gamma\rangle$ is the average received spread SINR value in the m-th, m=[1,…,M_{ds}], diversity branch and M_{ds} is obtained from eq.(10). The statistics of each interfering signal in eq.(11) need not considered separetely since, either the total interference power at the RAKE receiver output, or the CCI from the co-channel interfering users prior despreading, even for a small number of users, tends to be Gaussian, [9], and thus it can directly be incorporatedin the Shannon formula regardless the interference statistics. However, the performance of the coherent MRC RAKE receiver depends on the number of the employed taps and the fading channel estimation. If the number of taps is less than the resolvable paths' number, the receiver performance will substantially be degraded because the power of the remaining "branches" will appear at the receiver output as self-noise power. In this work, we consider the optimum operation of the coherent MRC RAKE receiver where the number of taps employed is equal to the number M_{ds} of resolvable paths as given by eq.(10).

If path-diversity reception, provided by a MRC RAKE receiver, is also applied to the DS/FFH-CDMA system, then diversity will be achieved. Hence, the SINR after path-diversity applied, in each of the M frequencies, $S_{i,pt,DS/FFH}$, will be given by, [12]:

$$S_{i,pt,DS/FFH} = M_{ds}\cdot S_{i,DS/FFH} = M_{ds}\cdot\frac{\Gamma}{G_pM+\frac{1}{M}\cdot(2.3123K)\cdot S} \quad (12)$$

where the new suffice 'pt' refers to the path-diversity reception applied. Given an average transmit power constraint, the average channel capacity of a fading channel with bandwidth W_t and received $SINR$ p.d.f. $p_\gamma(\gamma)$ under simultaneous optimal power and rate adaptation is given by, [3], :

$$\langle C_i\rangle_{opra} = \frac{W_t}{\ln 2}\cdot\int_{\gamma_0}^{\infty}\ln\left(\frac{\gamma}{\gamma_0}\right)\cdot p_\gamma(\gamma)\,d\gamma \quad (13)$$

where the suffice 'opra' refers to the optimal power and rate adaptation and γ_0 is the optimal cutoff SINR level below which data rate transmission is suspended.

IV. The Distribution Of SINR

In this section, we seek an expression for the cumulative probability distribution function of the signal- to interference plus noise ratio γ as:

$$p_{\gamma=SINR}(\gamma) = -\frac{d}{d\gamma}P_r(SINR > \gamma) \quad (14)$$

Then, eq.(13) is rewritten as:

$$\langle C_i\rangle_{opra} = -\frac{W_t}{\ln 2}\cdot\int_{\gamma_0}^{\infty}\frac{1}{\gamma}\cdot\left[1-P_r(SINR<\gamma)\right]d\gamma \quad (15)$$

which is obtained by applying the rules of the integration by parts. For M=2 and M=3, the exacts mathematicals expressions for $P_r(SINR<\gamma)$ are given respectively by, [13],:

$$P_{r(M=2)}(SINR<\gamma) = 1-e^{-\frac{\gamma}{<\gamma>}}\cdot\left(\frac{\gamma}{<\gamma>}+1\right) \quad (16)$$

$$P_{r(M=3)}(SINR<\gamma) = 1-e^{-\frac{\gamma}{<\gamma>}}\cdot\left(\frac{\gamma}{<\gamma>}+\frac{\gamma^2}{2\cdot<\gamma>^2}+1\right) \quad (17)$$

where the average received spread SINR value <γ> for M=2 and M=3, is given respectively by:

$$\langle\gamma\rangle_{(M=2)} = \frac{2\cdot M_{ds}\cdot S}{4\cdot G_p+2.3123\cdot K\cdot S} \quad (18)$$

and

$$\langle\gamma\rangle_{(M=3)} = \frac{3\cdot M_{ds}\cdot S}{9\cdot G_p+2.3123K\cdot S} \quad (19)$$

Using eqs (13) and (18), the average channel capacity normalized over the total system's bandwidth W_t and under simultaneous optimal power and rate adaptation, is plotted in Fig.1 as a function of the SNR S (expressed in dB) for M=2 and where, in addition, the following values are assumed: (i) totally constant allocated system's bandwidth: W_t=10MHz, (ii) signal bandwidth: W_s=30KHz, (iii) total multipath spread of the Rayleigh fading channel: Δ=3μsec, (iv) number of users per cell K=10, (v) γ_0=0.5. In addition, in Fig.2, the average channel capacity normalized over signal bandwidth W_t and under simultaneous optimal power and rate adaptation, is plotted in Fig.2 as a function of the SNR S (expressed in dB) for M=3 and assuming the same values for the other system's parameters as in Fig.1.

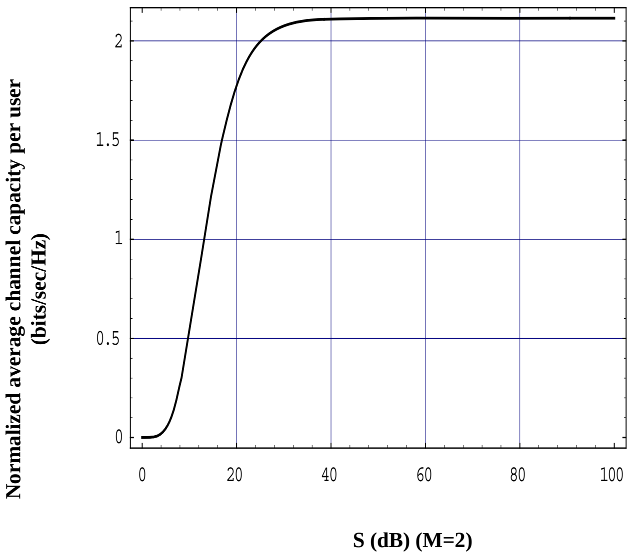

Fig. 1 Normalized average channel capacity (in bits/sec/Hz) per user for a DS/FFH-CDMA cellular system under power and rate adaptation against the the signal-to-noise ratio (SNR) Γ (expressed in dB) in a Rayleigh fading environment for M=2.

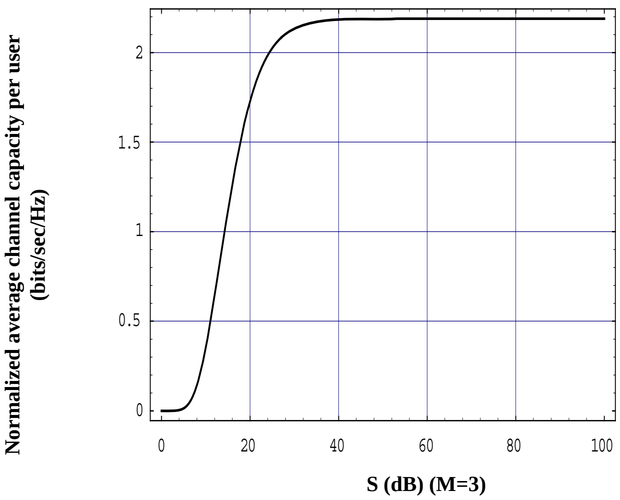

Fig. 2 Normalized average channel capacity (in bits/sec/Hz) per user for a DS/FFH-CDMA cellular system under power and rate adaptation against the the signal-to-noise ratio (SNR) Γ (expressed in dB) in a Rayleigh fading environment for M=3.

As it can be concluded directly from Fig.1 and 2, in a DS/FFH-CDMA cellular system operating in a Rayleigh fading environment, there is no way to increase the normalized average channel capacity per user by increasing the SNR S i.e., by increasing the transmitted signal power, although simultaneous power and rate adaptation is applied in the system.

V. CONCLUSION

In the presented paper, the average channel capacity per user, achieved by a cellular DS/FFH-CDMA system, in the downlink transmission, when operating in a Rayleigh fading environment, is examined. The channel capacity per system's user is evaluated in terms of the Shannon sense channel capacity. Hence, a novel exact mathematical expression for a number of hops per bit M=2 and M=3 and respective numerical results are derived, assuming simultaneous optimal power and rate adaptation to channel fading. Furthermore, the relation between the system's parameters and the achieved normalised average channel capacity per user is revealed. As it can be concluded, the normalised average channel capacity per user is limited, also in this case of transmission, by CCI power and, in addition, there is no way to increase the average channel capacity per user by increasing the transmitted signal power although the considered adaptative scheme is applied. However, a simulation process and the future extension of this work for the multi-cell case are needed in order to compare with the theoretical results derived here and for the application of these results to a practical scenario.

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