Phenomenological and Semi-microscopic Analysis for Elastic Scattering of Protons on $^{6,7}$Li

A. Amar, N. Burtebayev, Sh. Hamada, Kerimkulov Zhambul and N. Amangieldy

I. INTRODUCTION

In practice it is required to obtain the potential from the experimental data, and this may be done by systematically varying the parameters of the optical potential to optimize the overall fit to the data, using appropriate computer programs. Optical model analysis of the elastic scattering and polarization data cannot give unique values of the all parameters of the potential. Many such analysis of nucleon scattering have now been made and is found that the potentials are quite similar for all nuclei and vary slowly with the incident energy [1]. The folding model has been used for years to calculate the nucleon-nucleus optical potential and inelastic form factors. It can be seen from the basic folding formulas that this model generates the first-order term of the microscopic optical potential that is derived from Feshbach’s theory of nuclear reactions. The success of this approach in describing the nucleon-nucleus elastic scattering in the same region of energy. The potential used in the single folding were obtained in a semi-microscopic way including one nucleon exchange effects and the density dependence of the nucleon-nucleon interaction. In each case the real potential obtained from the folding model was supplemented by a phenomenological imaginary potential, and during the fitting process the real potential was normalized and the imaginary potential optimized. Normalization factor $N_R$ is calculated in the range between 0.70 and 0.84.

II. SINGLE FOLDING MODEL

The real part of the optical potential for the nucleon–nucleus elastic scattering is given for the single folding model, in the following form:

$$U_R(R) = \int dr_1 \rho(R_1) V(r) ,$$

where $r = R - r_1$, $\rho(r_1)$ is the matter density distribution of the target nucleus, $V(r)$ is the effective NN-interaction. In the present calculation the effective NN-interaction is taken according to [4] in the form of M3Y-interaction

$$V(R) = 2999 \exp(-4R/4R) -2134 \exp(-25R/25R) -276(1-0.0005R)R.$$  

The density of the $^6$Li target nucleus is considered in the form [5]

$$\rho = \frac{6}{8\pi} \left(1 + \frac{a^2}{4} \exp \left( \frac{r^2}{4a^2} \right) - \frac{c^2}{4b^2} \exp \left( -\frac{r^2}{4b^2} \right) \right) ,$$

where $a$, $b$, and $c$ are constants parameters and in the range:

$$a := \sqrt{0.87} \text{ fm} , \quad b := \sqrt{1.7} \text{ fm} \quad \text{and} \quad c := \sqrt{0.205} \text{ fm}.$$  

And for the density of the $^7$Li nucleus is considered in the form [6]

$$\rho(R) = \rho_0 \left(1 + \frac{\alpha}{a} R \right)^2 \exp \left(-\frac{R^2}{a^2} \right) ,$$

where for harmonic oscillator $a=1.77$ fm and $\alpha=0.327$ fm. The analytical form of the real part of the optical potential is obtained by substituting Eqs. (2), (3) and (4) into Eq. (1) and carrying out the required integrations over $r_1$. 

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III. RESULTS AND DISCUSSIONS

Measurements of elastic scattering of protons on $^6\text{Li}$ nuclei in low energy region were carried out with using the extracted beam from the complex recharged UKP-2-1 accelerator of the Institute of Nuclear Physics (National Nuclear Center, Republic of Kazakhstan, Almaty, Kazakhstan) in the angular range 40-170°. The proton energy varied in the range 400 – 1200 KeV. For $^6\text{Li}$ nuclei, the most suitable phenomenological parameters values are $r_0=1.05\text{fm}$, $r_c=1.3\text{fm}$, $r_D=1.92\text{fm}$, $a_0=0.20\text{fm}$ and $r_s=1.20\text{ fm}$. Optical parameters obtained for protons elastic scattering on $^6\text{Li}$ are shown in table 1. As expected the relations between $V_0$, $W_D$ and $E_p$ are linear. The parameters values are $r_0=1.05\text{fm}$, $r_c=1.3\text{fm}$, $r_D=1.92\text{fm}$, $a_0=0.20\text{fm}$ and $r_s=1.20\text{ fm}$. Optical parameters obtained for protons elastic scattering on $^6\text{Li}$ are shown in table 1. As expected the relations between $V_0$, $W_D$ and $E_p$ are linear.

\[ V_0 = 56.1 - 0.61E_p, \quad W_D = -0.66 + 0.46E_p \]  

(5)

<table>
<thead>
<tr>
<th>$E_p$ (MeV)</th>
<th>$V_0$ (MeV)</th>
<th>$r_0$ (fm)</th>
<th>$a_0$ (fm)</th>
<th>$W_D$ (MeV)</th>
<th>$r_D$ (fm)</th>
<th>$a_D$ (fm)</th>
<th>$V_s$ (MeV)</th>
<th>$r_s$ (fm)</th>
<th>$a_s$ (fm)</th>
<th>$J_R$ (MeV.fm$^3$)</th>
<th>$J_W$ (MeV.fm$^3$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.4</td>
<td>59.0</td>
<td>1.05</td>
<td>0.85</td>
<td>0.30</td>
<td>1.92</td>
<td>0.57</td>
<td>0.93</td>
<td>1.08</td>
<td>0.66</td>
<td>490</td>
<td>20.5</td>
</tr>
<tr>
<td>0.74</td>
<td>57.2</td>
<td>1.05</td>
<td>0.67</td>
<td>0.35</td>
<td>1.92</td>
<td>0.65</td>
<td>0.93</td>
<td>1.02</td>
<td>0.20</td>
<td>475</td>
<td>22.19</td>
</tr>
<tr>
<td>0.97</td>
<td>54.0</td>
<td>1.05</td>
<td>0.52</td>
<td>0.35</td>
<td>1.92</td>
<td>0.57</td>
<td>0.93</td>
<td>1.02</td>
<td>0.20</td>
<td>454</td>
<td>55.19</td>
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<td>3.0</td>
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<td>1.92</td>
<td>0.80</td>
<td>0.93</td>
<td>1.02</td>
<td>0.20</td>
<td>437</td>
<td>55.72</td>
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<td>50.0</td>
<td>1.05</td>
<td>0.50</td>
<td>1.18</td>
<td>1.92</td>
<td>0.57</td>
<td>1.56</td>
<td>1.02</td>
<td>0.20</td>
<td>407</td>
<td>75.58</td>
</tr>
<tr>
<td>0.0</td>
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<td>1.05</td>
<td>0.65</td>
<td>2.78</td>
<td>1.92</td>
<td>0.40</td>
<td>12.2</td>
<td>0.02</td>
<td>0.20</td>
<td>391</td>
<td>148.3</td>
</tr>
<tr>
<td>4.0</td>
<td>46.5</td>
<td>1.05</td>
<td>0.5</td>
<td>6.72</td>
<td>1.92</td>
<td>0.42</td>
<td>9.86</td>
<td>0.10</td>
<td>0.20</td>
<td>378</td>
<td>304.0</td>
</tr>
<tr>
<td>6.0</td>
<td>39.0</td>
<td>1.05</td>
<td>0.5</td>
<td>2.80</td>
<td>1.92</td>
<td>0.80</td>
<td>5.57</td>
<td>0.12</td>
<td>0.20</td>
<td>270</td>
<td>309.0</td>
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<td>1.05</td>
<td>0.5</td>
<td>2.93</td>
<td>1.92</td>
<td>0.37</td>
<td>0.12</td>
<td>0.20</td>
<td>149</td>
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</tr>
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<td>2.93</td>
<td>1.92</td>
<td>0.37</td>
<td>0.37</td>
<td>0.20</td>
<td>142</td>
<td>111.0</td>
<td></td>
</tr>
<tr>
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<td>30.0</td>
<td>1.05</td>
<td>0.65</td>
<td>2.63</td>
<td>1.92</td>
<td>0.80</td>
<td>2.33</td>
<td>0.12</td>
<td>0.20</td>
<td>122</td>
<td>100.0</td>
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<tr>
<td>19.0</td>
<td>26.0</td>
<td>1.05</td>
<td>0.65</td>
<td>1.69</td>
<td>1.93</td>
<td>0.80</td>
<td>1.69</td>
<td>0.12</td>
<td>0.20</td>
<td>64.0</td>
<td>106.0</td>
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</table>

Using SPI GENOA Code [7] we could calculate a set of optical parameters as the volume integral per nucleon pair for the real and imaginary potential, $J_R$ and $J_W$. Figure 1 shows the comparison between calculated using optical model and experimental angular distributions of protons scattered from $^6\text{Li}$.

\begin{figure}[h]
\centering
\includegraphics[width=0.5\textwidth]{fig1.png}
\caption{Fig.1 shows the comparison between calculated and experimental angular distribution of protons scattered from $^6\text{Li}$ at low energies.}
\end{figure}

This agreement might be improved by adjusting the normalization factor and imaginary potential parameters. With comparison of other works for the real part of the optical potential we can introduce the following table which contains a comparison between present and other works. I’d like to mention that when we used another form of nuclear density with the same potential form we obtained $V_0=46\text{MeV}$ and the same $a_0=1.58\text{fm}$ and $r_0=0.56\text{fm}$, the form of the density and potential NN-interaction are very important.

\begin{table}[h]
\centering
\begin{tabular}{cccccccccc}
\hline
$E_p$ (MeV) & $V_0$ (MeV) & $r_0$ (fm) & $a_0$ (fm) & $W_D$ (MeV) & $r_D$ (fm) & $a_D$ (fm) & $V_s$ (MeV) & $r_s$ (fm) & $a_s$ (fm) \\
\hline
0.4 & 39.0 & 1.74 & 0.74 & 3.03 & 1.92 & 0.91 & 7.61 & 1.45 & 0.25 & 420.24 & 0.70 \\
0.74 & 39.0 & 1.74 & 0.74 & 5.78 & 1.92 & 0.92 & 8.47 & 1.28 & 0.60 & 436.31 & 0.70 \\
0.975 & 39.0 & 1.74 & 0.74 & 5.78 & 1.92 & 0.92 & 8.47 & 1.28 & 0.60 & 436.31 & 0.70 \\
\hline
\end{tabular}
\caption{The parameters obtained using single folding model for $^6\text{Li}$}
\end{table}

\begin{figure}[h]
\centering
\includegraphics[width=0.5\textwidth]{fig2.png}
\caption{Fig. 2 shows the measured differential cross sections of protons elastically scattering on $^6\text{Li}$ at 1 MeV, where dots represent}
\end{figure}
experimental data and lines represent the calculated values using folding model.

### TABLE III

**COMPARISON BETWEEN REAL PART OF OPTICAL POTENTIAL OBTAINED FOR p+^6Li USING SINGLE FOLDING**

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Present work</th>
<th>Wiringa</th>
<th>Electron scattering</th>
</tr>
</thead>
<tbody>
<tr>
<td>V₀</td>
<td>39.98</td>
<td>79</td>
<td>34</td>
</tr>
<tr>
<td>r₀</td>
<td>1.74</td>
<td>1.727</td>
<td>1.81</td>
</tr>
<tr>
<td>a₀</td>
<td>0.74</td>
<td>0.613</td>
<td>0.478</td>
</tr>
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</table>

And for ^7Li we made the same calculations using Ecis88, SPI-GENOA and DEPOT codes in our calculations. Table IV contains optical parameters for protons scattering on ^7Li nuclei (phenomenological).

### TABLE IV

**OPTICAL PARAMETERS FOR PROTONS SCATTERING ON ^7Li NUCLEI (PHENOMENOLOGICAL)**

<table>
<thead>
<tr>
<th>E₀ (MeV)</th>
<th>V₀ (MeV)</th>
<th>r₀ (fm)</th>
<th>a₀</th>
<th>W₀ (MeV)</th>
<th>t₀ (fm)</th>
<th>a₀</th>
<th>Vₛ (MeV)</th>
<th>rₛ (fm)</th>
<th>aₛ</th>
<th>J₀</th>
<th>r_D (fm)</th>
<th>a_D (fm)</th>
<th>Vₛ =7.5MeV</th>
<th>rₛ =1.25 r_D =0.65 fm</th>
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<tbody>
<tr>
<td>0.364</td>
<td>56.0</td>
<td>1.17</td>
<td>0.65</td>
<td>0.70</td>
<td>1.80</td>
<td>0.504</td>
<td>12.48</td>
<td>1.17</td>
<td>0.50</td>
<td>228.3</td>
<td>4.10</td>
<td>11.0</td>
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<td></td>
</tr>
<tr>
<td>0.441</td>
<td>62.0</td>
<td>1.17</td>
<td>0.60</td>
<td>0.30</td>
<td>1.80</td>
<td>0.504</td>
<td>12.48</td>
<td>1.17</td>
<td>0.50</td>
<td>220.5</td>
<td>4.10</td>
<td>11.0</td>
<td>4.10</td>
<td></td>
</tr>
<tr>
<td>0.991</td>
<td>55.0</td>
<td>1.17</td>
<td>1.04</td>
<td>0.93</td>
<td>1.80</td>
<td>0.87</td>
<td>18.86</td>
<td>1.17</td>
<td>0.74</td>
<td>535.9</td>
<td>27.25</td>
<td>32.2</td>
<td>27.25</td>
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<tr>
<td>0.03</td>
<td>19.43</td>
<td>1.17</td>
<td>0.91</td>
<td>0.14</td>
<td>1.80</td>
<td>0.34</td>
<td>11.1</td>
<td>1.17</td>
<td>0.77</td>
<td>367.3</td>
<td>19.84</td>
<td>18.40</td>
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<tr>
<td>0.02</td>
<td>19.46</td>
<td>1.17</td>
<td>0.94</td>
<td>0.14</td>
<td>1.80</td>
<td>0.36</td>
<td>11.1</td>
<td>1.17</td>
<td>0.77</td>
<td>367.3</td>
<td>19.84</td>
<td>18.40</td>
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<tr>
<td>0.37</td>
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<td>0.53</td>
<td>0.55</td>
<td>1.80</td>
<td>0.545</td>
<td>12.86</td>
<td>1.17</td>
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<td>80.8</td>
<td>10.20</td>
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<tr>
<td>0.75</td>
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<td>0.46</td>
<td>4.28</td>
<td>1.80</td>
<td>0.785</td>
<td>11.26</td>
<td>1.17</td>
<td>0.75</td>
<td>80.8</td>
<td>10.20</td>
<td>80.8</td>
<td>10.20</td>
<td></td>
</tr>
</tbody>
</table>

The parameters calculated for ^7Li is good agreement with experimental values.

As shown in figures 3 the differential cross sections calculated using optical parameters and experimental values are close to each other in spite of this situation is not completely true at low energies.

![Graph](image1)

**Fig. 3** Measured differential cross sections of protons elastically scattering on ^7Li where dots represent experimental data and lines represent the calculated values using folding model.

![Graph](image2)

**Fig. 4** shows the measured differential cross sections of protons elastically scattering on ^7Li at 1 MeV, where dots represent experimental data and lines represent the calculated values using single folding model.

### IV. CONCLUSION

From the above results and discussions we can say that the single folding model gives a reasonable fit to the experimental data for the differential cross section at low energies.

The optical model calculations from Ecis88 give the most suitable fits among these programs used. The present analysis shows that the optical model can give a good description of the general features of nucleon scattering from light nuclei and as our work concentrated at low energies where we can see normal description in case of ^6,^7Li.

This evidenced by the fact that, to a fair degree, a single set of energy–dependent parameters is able to reproduce the differential cross sections of elastically scattering protons from 1p-shell nuclei. There is no evidence that our parameters obtained in this analysis are the best.

### REFERENCES

[2] Dao T. Khoa, Elias Khan, Gianluca Col, and Nguyen Van Giai, Accepted for publication in Nuclear Physics A.