A Quantum-Inspired Evolutionary Algorithm for Multiobjective Image Segmentation

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***Abstract*—**In this paper we present a new approach to deal with image segmentation. The fact that a single segmentation result do not generally allow a higher level process to take into account all the elements included in the image has motivated the consideration of image segmentation as a multiobjective optimization problem. The proposed algorithm adopts a split/merge strategy that uses the result of the k-means algorithm as input for a quantum evolutionary algorithm to establish a set of non-dominated solutions. The evaluation is made simultaneously according to two distinct features: intra-region homogeneity and inter-region heterogeneity. The experimentation of the new approach on natural images has proved its efficiency and usefulness.

***Keywords*—**Image segmentation, multiobjective optimization, quantum computing, evolutionary algorithms.

I. INTRODUCTION

IMAGE segmentation is one of the key stages in computer vision processes. It is a low-level image processing task that aims at partitioning an image into homogeneous regions [1]. Its result could be presented as input to higher-level processing tasks such as pattern recognition, object tracking and scene analysis.

Several segmentation algorithms have been developed [2], [3]. Those algorithms can be categorized into four classes [4]:

- Histogram-based approaches in which pixels are classified using the image histogram according to their colour intensity. K-means [5],[6] is the most popular among those approaches.
- Edge-based approaches in which pixels representing marked intensity shifts are extracted and then linked into contours that represent objects boundaries. These approaches offer low computational cost but present on the other hand serious difficulties in setting the appropriate thresholds and producing continuous one-pixel-wide contours [7],[8].
- Region-based approaches that aim to detect regions satisfying a certain homogeneity criterion. This class includes region growing [9]–[11] and pyramidal methods [12] which are powerful but may lead to an oversegmentation.
- Split/merge approaches aim to overcome the problem of oversegmentation by means of a two-phase process. The first phase subdivides the original image into primitive homogeneous regions. The second one tries to get a better segmentation by merging neighbouring regions which are judged similar enough [4],[13],[14].

The image segmentation problem has very often been treated as a mono-objective one. i.e. each decision is based on the evaluation of only one expression which might sum up a set of objectives that are generally conflicting.

In the present work, we are dealing with image segmentation as a multiobjective optimization problem in which the aim is to get a set of non-dominated solutions. The evaluation is made simultaneously according to two distinct features: intra-region homogeneity and inter-region heterogeneity.

Our motivation in considering image segmentation as a multiobjective problem is based on the fact that a given segmentation result, however of good quality it may be, may not allow to a higher-level process to extract and consider all the information included within the image. So, having different segmentation results may allow considering differently the image in the following stages. This will be more valuable if the modern parallel processing possibilities are exploited.

Quantum computing is a new field in computer science which has induced intensive investigations and researches during the last decade. It takes its origins from the foundations of the quantum physics. The parallelism that the quantum computing provides reduces obviously the algorithmic complexity [15]–[17]. Such an ability of parallel processing can be used to solve efficiently optimization problems.

Since there are no powerful quantum machines till today, some ideas such as simulating quantum algorithms on conventional computers or combining them to existing methods have been suggested to get benefit from this new science. In this paper we are using a combination of evolutionary algorithms and quantum computing principles which has already proved its usefulness in solving many problems such as the knapsack problem [18],[19], the traveling salesman problem [20], the N-queens problem [21] and image registration [22],[23].

The proposed approach for image segmentation consists essentially of two phases: A split procedure using the well-known k-means algorithm followed by a merge procedure that uses a quantum-inspired evolutionary algorithm for the establishment of the non-dominated solutions set.

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By consequence, the remaining of the paper is organized as follows. In section 2, basic notions about genetic algorithms, quantum computing and multiobjective optimization are introduced. Section 3 presents the proposed approach for image segmentation. Experimental results and comparisons are given in section 4. Finally, the conclusions drawn from our study are presented in section 5.

II. Basic Concepts

A. Genetic Algorithms

Genetic algorithms derive from the evolution theory. They were introduced in 1975 by *John Holland* and his team as a highly parallel search algorithm. Later, they have been mainly used as an optimization device.

According to the evolution theory, within a population only the individuals well adapted to their environment can survive and transmit some of their characters to their descendants. In genetic algorithms, this principle is traduced into the problem of finding the best individuals represented by chromosomes. So, each chromosome encodes a possible solution for the given problem and, starting from a population of chromosomes, the evolution process performs a parallel search through the solutions' space. The fitness is measured for each individual by a function related to the objective function of the problem to be solved.

Basically, a genetic algorithm consists of three major operations: selection, crossover, and mutation. The selection evaluates each individual and keeps only the fittest ones in the population. In addition to those fittest individuals, some less fit ones could be selected according to a small probability. The others are removed from the current population. The crossover recombines two individuals to have new ones which might be better. The mutation operator induces changes in a small number of chromosomes units. Its purpose is to maintain the population diversified enough during the optimization process.

B. Quantum Computing

In early 80, *Richard Feynman*'s observed that some quantum mechanical effects cannot be simulated efficiently on a computer. His observation led to speculation that computation in general could be done more efficiently if it used this quantum effects. This speculation proved justified in 1994 when *Peter Shor* described a polynomial time quantum algorithm for factoring numbers [16].

In quantum systems, the computational space increases exponentially with the size of the system which enables an exponential parallelism. This parallelism could lead to exponentially faster quantum algorithms than possible classically [17].

The *quantum bit* (*qubit*) is the elementary information unit. Unlike the classical bit, the qubit does not represent only the value 0 or 1 but a superposition of the two. Its state can be given by:

$$\Psi = \alpha\, |0\rangle + \beta |1\rangle \tag{1}$$

where $|0\rangle$ and $|1\rangle$ represent respectively the classical bit values 0 and 1; α and β are complex numbers such that

$$|\alpha|^2 + |\beta|^2 = 1 \tag{2}$$

If a superposition is measured with respect to the basis $\{|0\rangle,|1\rangle\}$, the probability to have the value 0 is $|\alpha|^2$ and the probability to have the value 1 is $|\beta|^2$.

In classical computing, the possible states of an n bits system form a vector space of n dimensions, i.e. we have 2^n possible states. However, in a quantum system of n qubits the resulting state space has 2^n dimensions. It is this exponential growth of the state space with the number of particles that suggests a possible exponential speed-up of computation on quantum computers over classical computers. Each quantum operation will deal with all the states present within the superposition in parallel. The basis of the state space of a quantum system of n qubits is: $\{|00...0\rangle, |00...1\rangle \ldots |11...1\rangle\}$.

The *measurement* of a single qubit projects the quantum state onto one of the basis states associated with the measuring device. The result of a measurement is probabilistic and the process of measurement changes the state to that measured. Multi-qubit measurement can be treated as a series of single-qubit measurements in the standard basis.

The dynamics of a quantum system are governed by *Schrödinger*'s equation. The *quantum gates* that perform transformations must preserve orthogonality. For a complex vector space, linear transformations that preserve orthogonality are unitary transformations, defined as follows. Any linear transformation on a complex vector space can be described by a matrix. A matrix M is unitary if $M.M'=I$. Any unitary transformation of a quantum state space is a legitimate quantum transformation and vice-versa. Rotations constitute one among the unitary transformations types.

One important consequence of the fact that quantum transformations are unitary is that they are reversible. Thus quantum gates, which can be represented by unitary matrices, must be reversible. It has been shown that all classical computations can be done reversibly.

C. Multiobjective Optimization

1. What is a Multiobjective Optimization Problem?

Most problems in nature have several objectives to be considered. A multiobjective optimization problem can be defined as [24] the problem of finding a vector of decision variables which satisfies constraints and optimizes a vector function whose elements represent the objective functions. These functions form a mathematical description of performance criteria which are usually in conflict with each other.

Formally, the general multiobjective optimization problem can be defined as:

Find the vector $x^* = \left[x_n^*, x_n^*, \ldots\ldots, x_n^*\right]^T$ which satisfies the m inequality constraints:

$$g_i(x) \geq 0 \quad i=1,2,\ldots,m \tag{3}$$

the p equality constraints:

$$h_i(x) = 0 \quad i=1,2,\ldots,p \tag{4}$$

and will optimize the vector function:

$$f(x)=[f_1(x), f_2(x), \ldots, f_k(x)]^T \quad (5)$$

2. Pareto Optimality and the Non-Domination Concept

A vector of decision variable $x^* \in F$ is Pareto optimal if there does not exist another vector $x \in F$ *such that* $f_i(x) \leq f_i(x^*)$ for all i=1,2,…,k and $f_j(x) < f_j(x^*)$ for at least one j.

In other words, x^* is Pareto optimal if there is no feasible vector x which would decrease some criterion without causing a simultaneous increase in at least one other criterion.

Almost always, this concept does not give a single solution, but rather a set of solutions called Pareto Optimal Set. The vectors in that set are called non-dominated and the plot drawn from the Pareto optimal set is called Pareto Front.

III. The Proposed Approach

Our task of multiobjective image segmentation is realized through a split/merge approach (Fig. 1). In the split procedure, we have adopted the k-means algorithm which is relatively faster compared with other algorithms such as fuzzy-c-means [6]. The k-means produces k clusters, where each is composed of a set of pixels whose colours are close together. *Pappas* found in [5] that the value k=4 gives very acceptable results for most images.

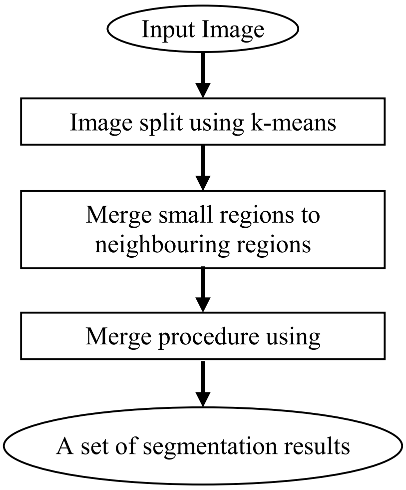

Fig. 1 Broad description of the segmentation algorithm

As the pixels forming one cluster are not necessarily adjacent, we have to use the k-means output to look for the different regions whose number is usually greater than k.

Because of the noise and in order to reduce the search space, the small regions are filtered out and merged to the neighbouring regions that have the closer colorimetric properties. A region will be filtered out if it contains less than 10 pixels [13]. At the end of this stage, we have R regions and N edges.

The resulting image partitioning is considered as input for our multiobjective quantum inspired evolutionary algorithm. The aim of the algorithm is to establish a set of non-dominated segmentations. Each segmentation result is obtained through merging some adjacent regions.

The algorithm uses only 3 quantum chromosomes. Each chromosome is a string of N qubits where each qubit represents the edge separating two among the initial regions.

A chromosome can then be represented by:

$$\begin{pmatrix} \alpha_1 & \alpha_2 & \ldots\ldots & \alpha_N \\ \beta_1 & \beta_2 & \ldots\ldots & \beta_N \end{pmatrix}$$

where the column i reflects the state of the i^{th} qubit.

The basic state $|0\rangle$ signifies that the edge is removed, i.e. the two regions separated initially by that edge will be fused while the basic state $|1\rangle$ signifies that the edge is maintained. An example is illustrated in Fig. 2.

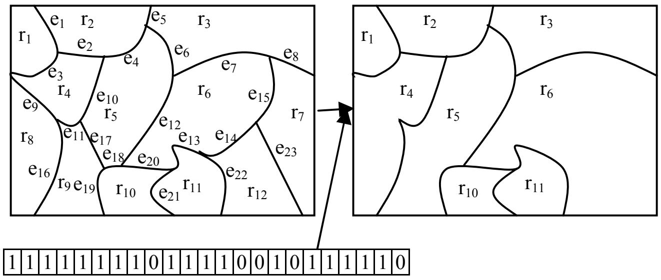

Fig. 2 an example of a solution binary representation

Initially, the chromosomes are generated randomly and the set of non-dominated solutions contains only one solution, a string of N ones. Afterwards, we apply cyclically 4 operations (Fig. 3):

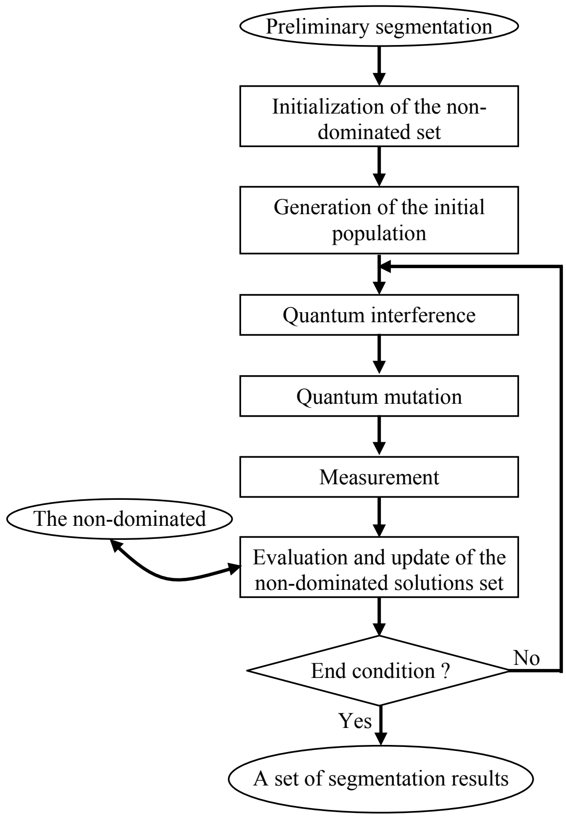

Fig. 3 The QEA for multiobjective image segmentation

The first operation is a quantum interference which allows a shift of each qubit toward the corresponding bit value in one of the non-dominated solutions. It is the solution that has the minimal Euclidian distance from the chromosome's derived solution. The interference is performed by applying a unitary quantum operator which achieves a rotation (Fig. 4) whose angle is function of α_i , β_i and the value of the corresponding bit in the selected non-dominated solution (Table I).

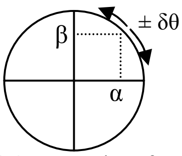

Fig. 4 Quantum interference

$\delta\theta$ has been chosen experimentally equal to $\pi/8$ (different values had been tested and this value gave a good search space exploration).

TABLE I
LOOKUP TABLE OF THE ROTATION ANGLE

α	β	Reference bit value	Rotation angle
>0	>0	1	+δθ
>0	>0	0	-δθ
>0	<0	1	-δθ
>0	<0	0	+δθ
<0	>0	1	-δθ
<0	>0	0	+δθ
<0	<0	1	+δθ
<0	<0	0	-δθ

The second operation consists of a quantum mutation which performs for some qubits, according to the mutation rate, a permutation between their α_i and β_i values. That will reverse the probabilities of having the values 0 and 1 when applying a measurement. An example is given in Fig. 5.

$$\begin{pmatrix} 0.7446 & -0.6833 & 0.1338 & \mathbf{0.3705} & -0.0272 & 0.6831 \\ -0.6675 & 0.7301 & 0.9910 & \mathbf{-0.9288} & -0.9996 & 0.7303 \end{pmatrix}$$

$$\Downarrow$$

$$\begin{pmatrix} 0.7446 & -0.6833 & 0.1338 & \mathbf{-0.9288} & -0.0272 & 0.6831 \\ -0.6675 & 0.7301 & 0.9910 & \mathbf{0.3705} & -0.9996 & 0.7303 \end{pmatrix}$$

Fig. 5 Quantum mutation

In the third phase, we apply a measurement on each chromosome to have from it one solution among all those present in superposition. But unlike pure quantum systems, the measurement here does not destroy the states' superposition. Since our algorithm operates on a conventional computer and does not require the presence of a quantum machine, it is possible and in our interest to keep all the possible solutions in the superposition for the next iterations. At the end of this operation, we will have 3 binary solutions issued from the 3 quantum chromosomes.

The fourth operation is the non-dominated solutions set update. Each measured solution is evaluated and then compared to all the existing non-dominated solutions. Each existing solution that is dominated by a new solution is removed from the non-dominated solutions set. And if a new solution is not dominated by any other solution, it will be integrated into the set of the non-dominated solutions.

The evaluation is based on two criteria: the intra-region homogeneity and the inter-region heterogeneity.

Let R be the number of regions, m_i the mean value of the pixels belonging to the region i, var_i the variance inside the region i, ne the number of the maintained edges and nr the number of the resulting regions.

The intra-region homogeneity is given by:

$$Hom = -\frac{1}{nr}\sum_{i=1}^{nr} var_i \tag{6}$$

The inter-region heterogeneity is given by:

$$Het = \frac{1}{ne}\sum_{i,j}^{nr}(m_i - m_j)^2 \qquad i, j \text{ adjacent} \tag{7}$$

IV. EXPERIMENTAL RESULTS

The proposed algorithm has been tested successfully on different natural images. In this section we present the obtained result from the application of our algorithm on a 256x256 pixels image (Fig. 6). We have, in parallel, implemented a classical multiobjective genetic algorithm (MOGA) [25] with the same encoding method but with a population of 25 chromosomes and with a crossover operation (the omission of the crossover operation leads to a bad performance). The mutation rate for the MOGA has been fixed at 0.1.

Fig. 6 Image "cameraman" of size 256x256

The two algorithms have been executed for 2500 iterations on an Intel P4 2.8GHz computer. Fig. 7 shows the evolution of the size of the non-dominated solutions set.

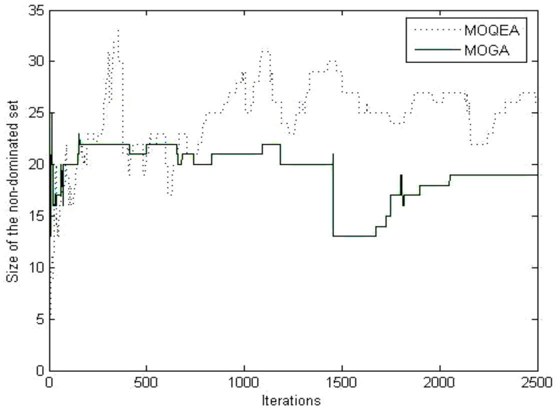

Fig. 7 The evolution of the size of the non-dominated solutions set

We notice that our algorithm (MOQEA), in spite of its small population size (3 quantum chromosomes only), gives more non-dominated solutions and is more dynamic then MOGA.

Table II illustrates the non-dominated solutions set update dynamics of the two algorithms.

TABLE II
SUMMARY ABOUT THE NON-DOMINATED SET DYNAMICS

	Number of iterations inserting non-dominated solutions	Number of iterations removing solutions from the non-dominated set	Total number of insertions	Total number of deletions
MOQEA	209	111	235	210
MOGA	67	41	140	121

It is obvious that our algorithm is more successful in exploring the search space. 3 quantum chromosomes find new non-dominated solutions about three times more than do the MOGA's 25 chromosomes.

In Fig. 8, we present a qualitative comparison between the two algorithms.

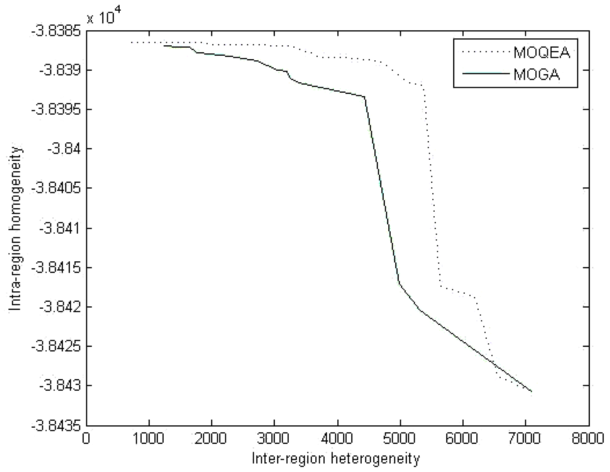

Fig. 8 The fitness function values for the non-dominated solutions

We notice that almost all the solutions found by MOGA are dominated by solutions found by our MOQEA. On the other side, no solution found by the MOQEA is dominated by a solution found by MOGA (even the solution that is under the MOGA's curve is not dominated, since the line above it is only to link two solutions that none of them dominates a MOQEA's solution). So, it is clear that the proposed algorithm is much more powerful than the classical genetic and evolutionary existing algorithms.

Some of the 26 non-dominated solutions for the image segmentation problem are given in Fig. 9.

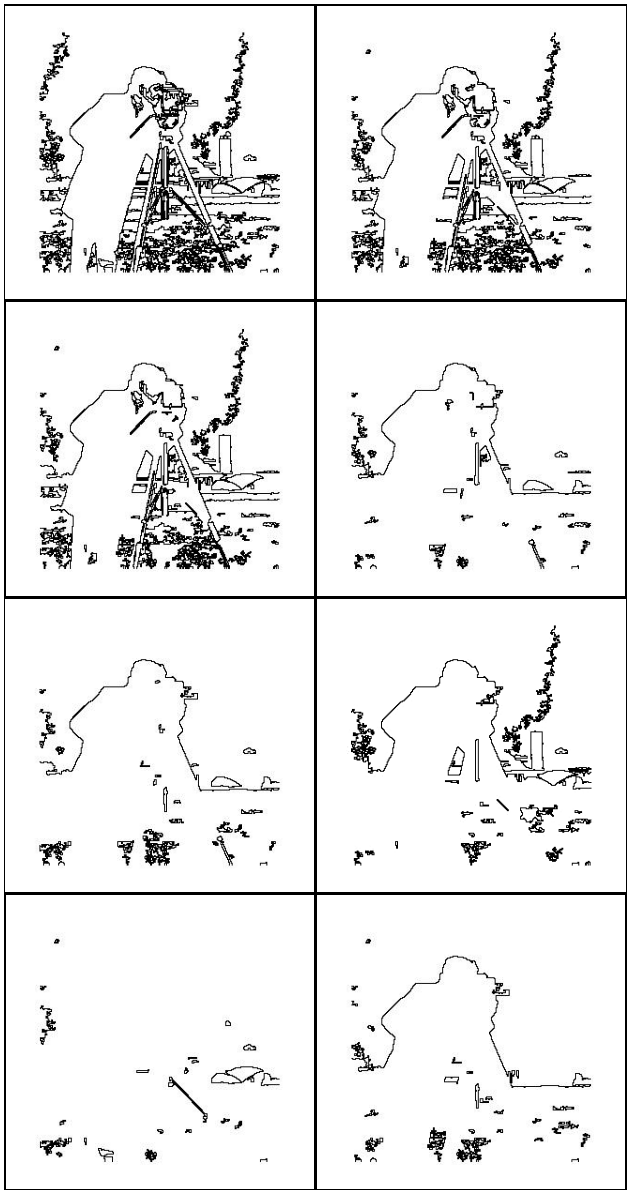

Fig. 9 Some non-dominated solutions resulting from MOQEA

For 2500 iterations, the execution time was 77*s* for MOQEA and 375*s* for MOGA. Thus, MOQEA is obviously more efficient than MOGA; it is nearly 5 times faster. This is due essentially to the small population size and to additional time required for the crossover and selection operations in MOGA.

V. Conclusion

In this paper, we have presented a new approach to deal with image segmentation. We have used a quantum-inspired evolutionary algorithm that used the output of the k-means algorithm to establish a set of non-dominated solutions. The obtained results show that the proposed algorithm is much more powerful and efficient than its classical counterpart. There are two main reasons for this. The first reason is that the quantum encoding of potential solutions reduces considerably the required number of chromosomes that guarantees good search diversity. So, each single chromosome represents at the same time all the possible solutions. What changes is the probability of getting one solution or another when applying a measurement operation. The second reason is that the use of the quantum interference offers a powerful tool to reinforce the search stability. And then, it provides in some way a guide for the population individuals and allows therefore a good exploitation of the current solutions neighbourhood to find Pareto-optimal solutions.

It is possible theoretically to use only one chromosome, but in practice, this leads usually to the reinforcement of local optima. Thus, we need slightly more chromosomes to diversify enough the search. Three quantum chromosomes were sufficient for the studied problem.

As future work, we will try to integrate other objective functions that would ameliorate the obtained segmentation results.

To conclude, we can say that the quantum-inspired evolutionary techniques are well situated to be among the best alternatives in dealing with multiobjective optimization problems.

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