Risk Management Analysis: An Empirical Study Using Bivariate GARCH

Chin Wen Cheong

Abstract—This study employs a bivariate asymmetric GARCH model to reveal the hidden dynamics price changes and volatility among the emerging markets of Thailand and Malaysia after the Asian financial crisis from January 2001 to December 2008. Our results indicated that the equity markets are sharing the common information (shock) that transmitted among each others. These empirical findings are used to demonstrate the importance of shock and volatility dynamic transmissions in the cross-market hedging and market risk.

Keywords—multivariate ARCH, structural change, value at risk.

I. INTRODUCTION

THAILAND and Malaysian are among the Southeast Asian emerging markets that received great attentions from researchers and investors across the regional and global financial markets especially during the Asian financial crisis periods. This included [1] who examined six Asian emerging markets (including Thailand and Malaysia) volatility spillover under their liberalization periods using a dynamic integrated capital asset pricing model. [2] on the other hand studied the time-varying volatilities among Malaysia, China, South Korea and Thailand bond markets and indicated strong long and short relationships. In another study, [3] suggested that the contagion effect (among Malaysia, Indonesia, South Korea, Taiwan, Thailand, Singapore and Hong Kong) takes place during the crisis and that herding behaviour dominates the post-crisis period. Due to close geographical proximity and similar economic structure, it is worth to investigate their markets interdependences in term of shocks and volatility.

As parts of the members of Association of Southeast Asian Nation (ASEAN), Thailand and Malaysian are always attempt to achieve greater financial integration among the ASEAN members with various policy and regulation such as ASEAN Free Trade Area (AFTA), ASEAN Investment Area (AIA), ASEAN Industrial Cooperation (AICO) Scheme and ASEAN Economic Community (AEC). Most importantly they have all encountered serious financial crisis which initiated by a massive devaluation of Thai baht and later rapidly spread to Indonesian rupiah as well as Malaysian ringgit in year 1997. They are among the Asian emerging markets that have output depreciation¹ with 57%, 39% and 82% respectively after the hit of Asian financial crisis in year 1997. The deprecations in term of exchange rate (per unit USD) from June 1997 to July 1998 are 39.0% and 83.2% respectively. Both the countries happened to indicate strong economic growth (averagely 8-10%) and low inflation over a decade before the crisis and enjoyed the title of ‘Asian Tigers’ after the ‘Asian Dragons’ for Taiwan, Hong Kong, Singapore and Korea in 1960s.

To the best of our knowledge, there is still lack of volatility transmission studies that have been simultaneously undertaken Thailand and Malaysian markets especially after the financial crisis. This study focuses on the volatility spillover between the price changes and volatility among the three major economic barometers of Malaysia and Indonesia. The available econometric methodologies in the volatility transmission analysis are included multivariate diagonal VEC [4], constant conditional correlation model [5], factor autoregressive conditional heteroscedasticity model [6], BEKK model [7],[8] and Dynamic Conditional Correlation GARCH [9]. For the sake of simplicity in technical estimation and convey useful statistical inferences, a bivariate asymmetric diagonal BEKK model is selected due to its positive definite covariance matrix and relatively less amount of estimated parameters among the aforementioned models. The outcomes from the multivariate time-varying volatility estimations are used to quantify the market risk, namely the value-at-risk (VaR). The VaR normally defines as the worst loss for a given confidence level (for instance 95%) means one is 95% certain that at the end of a chosen risk horizon (eg. monthly) there will be no greater loss that just the VaR under normal market conditions. In portfolio analysis [10], the VaR often acts as a tool to alert investors for their possible expose risks under a particular portfolio. Most importantly, the VaR can be used as a regulation tool to avoid the self-regulated financial institutes to go bust where some institutes take on extremely high levels of risk (hope for high rewards) may have fantastic current profit records, however facing financial crisis or even going bankrupt in the next day. Besides the market risks, the empirical studies also show on how the time-varying volatility transmission can be used to determine the dynamic hedge ratios and risk minimizing portfolio allocation in a given portfolio investment.

The remainder of this study proceeds as follows: first, we determine the individual one-step ahead VaR for long financial position in each market. Second, the risk minimizing portfolio weights are determined in order to obtain the optimal capital allocation among the markets. Third, the overall diversified VaR for portfolio are computed based on the time-varying cross correlation. Fourth, the undiversified overall VaR is also evaluated to find out the market risk under a possible catastrophic financial crisis. Finally, the time-

¹ According to World Economic Outlook (IMF 1999), the depreciation refers to crisis-output losses relative to hypothetical non-crisis output.
varying cross-correlation is extended to appraise the risk-minimizing hedge ratio among the pair-wise markets.

II. DATA SOURCE

The Stock Exchange of Thailand (SET) is the national stock exchange of Thailand which located at the capital of Thailand, Bangkok. The SET consists of SET Index, SET50 Index and SET100 Index. Further details such as annual market capitalization, trading volume can be obtained from their official website http://www.set.or.th. On the other hand, the public trading shares of the Malaysian stock markets are firstly established in 1960 under the Malayan Stock Exchange (MSE). In 1990, the KLSE and SES finally split into two independent stock markets. The Kuala Lumpur Stock Exchange (KLSE) demutualizes becomes an exchange holding company with the name of Bursa Malaysia Berhad in 2004 which consists of Main Board, Second Board and MESDAQ (market capitalization of US$189 billion). Later, in June 2006 a new index series with the FTSE Group is introduced in the Malaysian stock market. The 100 listed companies in the KLCI are constructed using the weighted average method (with the based year 1977. The details and performances of KLSE are available in http://www.klse.com.my.

III. METHODOLOGY

The continuous compounded return with the percentage natural logarithm is defined as

\[ r_t = 100 \times \ln \left( \frac{P_t}{P_{t-1}} \right), \]  

(1)

where the price \( P_t \) denotes the end of day closing price for a particular trading day. For conditional mean equations, the specification is

\[ \begin{bmatrix} r_{1,t} \\ r_{2,t} \end{bmatrix} = \begin{bmatrix} \theta_{01} \\ \theta_{02} \end{bmatrix} + \begin{bmatrix} \theta_{11} & \theta_{12} \\ \theta_{21} & \theta_{22} \end{bmatrix} \begin{bmatrix} r_{1,t-1} \\ r_{2,t-1} \end{bmatrix} + \begin{bmatrix} a_{1,t} \\ a_{2,t} \end{bmatrix}, \]  

(2)

with the vector representation, \( r_t = \theta_0 + \theta r_{t-1} + a_t \) where \( \theta_0 \) represent the long-term drifts and \( \theta \) capture the impact of own past returns (diagonal elements \( \theta_{ii} \)) and off-diagonal elements \( \theta_{ij} \) (for \( i \neq j \)) quantify the return spillover. For conditional variance specification, the innovations can be written as

\[ a_t | \Omega_{t-1} \sim \text{N}(0, \Sigma), \]  

(3)

or with the vector representation \( a_t | \Omega_{t-1} \sim \text{N}(0, \Sigma) \). Under the available of past market information at \( t-1 \), the innovations (shocks) for each market are assumed to be normally distributed with the variance-covariance, \( \Sigma_{it} \). Under the available of past market information at \( t-1 \), the innovations (shocks) for each market are assumed to be normally distributed with the variance-covariance, \( \Sigma_{it} \). The positive definite covariance matrices unrestricted BEKK model specification takes the following form:

\[ \Sigma_{it} = \Lambda'_{it} \Lambda_{it} + \Gamma'_{it} \Lambda_{it} \Lambda_{it} \Gamma_{it} + B' \Sigma_{it} B. \]  

(4)

However, this unrestricted BEKK surfs from parameter interpretation problem where an estimated parameter may affect two conditional equations simultaneously or by the sole number of regressors \([11],[12]\). Thus, a restricted diagonal BEKK model with all the zero off-diagonal elements with the exceptional for the constant matrix is used in order to reduce this severe problem. Besides the clustering volatility, asymmetric news effect is also one of the important empirical stylized facts that often observed in the worldwide financial markets. For asymmetric diagonal BEKK model, the matrix representation for equation (4) under the Threshold ARCH \([13],[14]\) specification is

\[ \Sigma_{it} = \Lambda'_{it} \Lambda_{it} + \Gamma'_{it} \Lambda_{it} \Lambda_{it} \Gamma_{it} + B' \Sigma_{it} B + \gamma' \delta_{it} \delta_{it}, \]  

(5)

where \( \gamma = \begin{bmatrix} \gamma_{11} & d_{1,1} & \cdots & 0 \\ d_{1,2} & \gamma_{22} & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & \gamma_{d,1} \end{bmatrix} \). The additional dummy variables \( d \) denote unity when \( a_{it} < 0 \) and zero otherwise. The diagonal parameters evaluate the effects of market \( i \) (\( i=1,2 \) for
bivariate) to its own past negative shocks. The variance-covariance equation with the asymmetric news effect is given by

\[ \sigma_{1,t}^2 = \alpha_{11,0} + \alpha_{11,1}a_{1,t-1}^2 + \alpha_{12,1}a_{1,t-1} + \gamma_{11}a_{1,t-1}^2a_{2,t-1} + \gamma_{12}a_{2,t-1}^2a_{1,t-1}, \]

\[ \sigma_{2,t}^2 = (\alpha_{22,0} + \alpha_{22,1}a_{2,t-1} + \alpha_{21,1}a_{1,t-1} + \gamma_{22}a_{2,t-1}^2a_{1,t-1} + \gamma_{21}a_{1,t-1}^2a_{2,t-1}). \]

Under the conditional normal assumption, the gradient Marquardt method [15] under a slight modification (correction identity matrix) from the BHHH method [16] is considered to provide a faster convergence result in optimizing the maximum likelihood (ML) parameter estimations using the following log-likelihood function

\[ l_i(\Theta) = -\frac{1}{2} \ln(2\pi) - \frac{1}{2} \ln |\mathbf{H}_i(\Theta)| - \frac{1}{2} a_i'(\Theta) \mathbf{H}_i(\Theta) a_i(\Theta). \]

where \( \Theta \) is the vector parameter to be estimated. However, the non-normality (fat-tail property) of financial time series is often observed in the worldwide financial markets. Although normality assumption ML estimator may fulfill the consistency condition, the departure from normality on the other hand can cause inefficient issue in the estimations. Thus, to circumvent the leptokurtosis ARCH issue, Bollerslev [17] introduces the following log-likelihood function with \( N \) observations:

\[ l_i(\Theta) = -\frac{1}{2} \ln(2\pi) - \frac{1}{2} \ln |\mathbf{H}_i(\Theta)| - \frac{1}{2} a_i'(\Theta) \mathbf{H}_i(\Theta) a_i(\Theta). \]

For model diagnostic, the Ljung-Box Q statistic provides whether the null hypothesis that the noise terms are serially uncorrelated or random.

IV. APPLICATION IN RISK MANAGEMENT

The VaR is one of the useful risk management tools in nowadays financial and actuarial industries. The time-varying covariance provides useful information in hedging, portfolio allocation and market risk for multiple financial markets. Under the ARCH estimation, the individual market \( q \)-quantile VaR can be expressed as

\[ \text{long position VaR: } \text{VaR}_i = \mu_{i,t} + D_i \hat{\sigma}_{i,t}, \]

where \( \hat{\mu}_i, \hat{\sigma}_i \), and \( D_i \) are estimated conditional mean, estimated conditional standard deviation and the parametric distributions respectively. For optimal diversified VaR\(^4\) for both the markets under the Markowitz mean-variance equation, the conditional standard deviation is

\[ \sigma_{portfolio} = \sqrt{w_i^2 \sigma_i^2 + (1-w_i)^2 \sigma_j^2 + 2w_i(1-w_i)\rho_{ij}\sigma_i\sigma_j}. \]

where the \( w \) is the portfolio weight for the market \( \rho_{ij} \) is the time-varying cross-correlation coefficient between the two index with the definition \( \rho_{ij} = \frac{\sigma_{ij}}{\sigma_i \sigma_j} \). Under the Kroner and Ng (1998) recommendation, the optimal portfolio holding with the zero expected returns\(^5\), the risk minimizing portfolio weight is derived as

\[ w_{122} = \frac{\sigma_{221}^2 - \sigma_{122}^2 - \sigma_{221}^2}{\sigma_{122}^2 - 2\sigma_{122}^2 + \sigma_{221}^2}. \]

Thus the optimal diversified VaR is

\[ \text{VaR}_{portfolio,i,j} = (w_i r_i + (1-w_i) r_j) + \sigma_{portfolio,i,j} D_q. \]

Besides the market risk determination, the outcomes in the bivariate analysis can be also used to measure the risk-minimizing hedge ratio (Kroner and Sultan,1993) among the two markets with the definition

\[ \beta_i = \frac{\sigma_{221}^2}{\sigma_{221}^2}. \]

V. EMPIRICAL RESULTS

A. Preliminary analysis

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<th>DESCRIBITVE STATISTICS AND HYPOTHESIS TESTS</th>
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<tr>
<td><strong>TABLE I</strong></td>
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<tr>
<td><strong>KLSE</strong></td>
</tr>
<tr>
<td>Mean, ( \bar{r} )</td>
</tr>
<tr>
<td>Std. Dev., s</td>
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<tr>
<td>Variance, ( s^2 )</td>
</tr>
<tr>
<td>Skewness, skew</td>
</tr>
<tr>
<td>Kurtosis, ( k )</td>
</tr>
</tbody>
</table>

\(^*\) This refers to diversified VaR where the correlation (either positive or negative but not equivalent to unity) exists between two markets. The diversified VaR is commonly used to determine the resources limitation therefore the portfolio can minimize the risk and at the same time maximize the profit.

\(^5\) Since all the expected returns are found to be less that 0.5% of their return series (highest for CI-IND with 0.32% for IND return).
Table 1 reports the first four moment statistics which described the central, dispersion, symmetrical property and shape (peak and tail) distribution of the return series for SET and KLCI. Overall, during the recovery period all the financial markets indicate positive expected return. Besides that, both the series are leptokurtic with the kurtosis far away from the mesokurtic normal distribution. A series of normality tests based on the individual statistics are conducted and all the individual tests suggested they are deviated from the standardized normal distribution with mean zero and unity variance. In addition, the Jacque-Bera tests are also conducted and all the series are found to be exceeded the 5% critical value $\chi^2_{(v=2)}$ which implied the rejection of null hypothesis of normal distribution.

For graphical illustration, a series of quantile-quantile plots in Figure 3 indicated that both the series (empirical versus normal distributions) do not lie on the straight line in their respective plots especially at the lower and upper tails. These findings imply that the return series are non-normal and slightly heavy at both the tails as compared to the normal distribution. On the other hand, the plots for return versus the heavy-tailed distribution indicate better fit in a straight line which suggests that the return distributions are somewhat similar to the heavy-tailed distributions. Based on these results, we are motivated to use the heavy-tail assumption in the coming model specifications.

### B. Bivariate GARCH Maximum likelihood estimation

#### TABLE II

MAXIMUM LIKELIHOOD ESTIMATION FOR CONDITIONAL RETURN

<table>
<thead>
<tr>
<th>Conditional mean</th>
<th>SET-KLCI</th>
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<tbody>
<tr>
<td>Coef.</td>
<td>S.E.</td>
</tr>
<tr>
<td>Constant:</td>
<td></td>
</tr>
<tr>
<td>$\theta_{01}$</td>
<td>0.062114** 0.024895</td>
</tr>
<tr>
<td>$\theta_{02}$</td>
<td>0.024063* 0.014405</td>
</tr>
<tr>
<td>Own first lag return SET: $\theta_{11}$</td>
<td>0.044847* 0.023423</td>
</tr>
<tr>
<td>KLCE: $\theta_{22}$</td>
<td>0.143066** 0.022175</td>
</tr>
<tr>
<td>Cross market first lag return SET$\rightarrow$KLCE: $\theta_{12}$</td>
<td>0.157543** 0.03037</td>
</tr>
<tr>
<td>KLCE$\rightarrow$SET: $\theta_{21}$</td>
<td>0.012199 0.011992</td>
</tr>
</tbody>
</table>

Diagnostic (std $\sigma_i$) Q-statistic P-value

- Q(6) – SET 0.0028 1.000
- Q(6) – KLCI 1.7268 0.943
- Q(12) – SET 0.0077 1.000
- Q(12) – KLCI 2.3088 0.999

* denotes the significance level at 5%; $\Rightarrow$ represents uni-directional effect; Q statistics denote the Ljung-Box serial correlation test for standardized residual.

Conditional mean:

$$r_{t,1} = 0.0621140803319 + 0.0448469472624* r_{t-1,1} + 0.157542515223 r_{t-1,2}$$

$$r_{t,2} = 0.0240629718778 + 0.0121992195437* r_{t-1,2} + 0.143065910695* r_{t-1,1}$$

* indicates 5% significance level.

Table 2 reports the estimated conditional mean equation which allows us to understand how the linkages in terms of returns across the two markets. First, the long-term drifts are all statistically significant at 10% level with positive values. This finding implies that there is a tendency of upward drift in long-run during the recovery period. Second, according to the diagonal elements ($\theta_{ii}$), both the returns of SET and KLCI are depended on their first lag. The first-order correlation may cause by the economic growth [19] or infrequent trading effect [20] that often occurred in emerging market. Third, in order to investigate the cross-market return relationships, only the off-diagonal parameters $\theta_{12}$ is statistically significant at 5% level. In short, there is an uni-directional return spillovers from KLSE to TSE. This result suggests that the KLCI plays an important role in news transmission into the pricing process in the SET.
driving the conditional variance of the three markets. For the own market asymmetric response to negative shocks\(^7\) (bad news) is only found to be statistically significant at 5% level in the SET. This leverage effects imply that downward movements (shock) in the respective financial market are followed by greater volatilities than upward movements of the same magnitude. From economic view point, this is an expected phenomenon since most of the worldwide financial market participants are tended to be more sensitive to bad news in the stock market. On the other hand, contrary result is indicated in the KLSE where during the recovery period, the good news contributes higher impact to the market volatility.

As a conclusion, the empirical results evidenced significant volatility transmission between the markets. Under the Ljung-Box serial correlation tests, all the series are failed to reject the null hypothesis with no serial correlation (lag 6 and lag12) at 1% significant level for the standardized squared.

C. Risk management implications

<table>
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<tr>
<th>TABLE IV</th>
<th>THE RISK MANAGEMENT APPLICATION</th>
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<tbody>
<tr>
<td></td>
<td>Individual Value-at-risk, VaR</td>
</tr>
<tr>
<td>SET (%)</td>
<td>3.354013</td>
</tr>
<tr>
<td>KLSE (%)</td>
<td>0.864526</td>
</tr>
<tr>
<td></td>
<td>Pairwise Value-at-risk (SET-KLSE)</td>
</tr>
<tr>
<td></td>
<td>time-varying correlation coefficient, ( \rho )</td>
</tr>
<tr>
<td></td>
<td>risk minimizing portfolio weight, ( \omega )</td>
</tr>
<tr>
<td></td>
<td>optimal diversified VaR, VaR(_{\text{optimal}})</td>
</tr>
<tr>
<td></td>
<td>non-optimal diversified VaR, VaR(_{\text{non-optimal}})</td>
</tr>
<tr>
<td></td>
<td>optimal undiversified VaR, VaR(_{\text{undiversified}})</td>
</tr>
</tbody>
</table>

The individual long position, \( D_t = \omega \alpha_{12} + \beta_1 \beta_{12} \sigma_{12} \), is 2.015048.

The linkages among the financial market can be further investigated in the area of risk management. We begin with the individual VaR for each market and the overall results are presented in Table 4. Suppose that an investor holds a long position in TSE and KLSE with arbitrary capital says \( C \) in each of the market. The VaR is forecasted using the one-step ahead forecast with 95% confidence interval from the forecast origin at \( t = 1826 \). Thus the one-step ahead forecast for long position VaR are 3.354013% and 0.864526% in SET, and KLSE. It is found that the SET provided the higher risk than the KLSE. This finding implies that with the probability of 0.95, the potential loss encounters by the long holder of the TSE and KLSE financial position over the one day time horizon are less than or equal to 3.354013% and 0.864526% of the total invested capital, \( C \) in each market respectively. If a portfolio consists exclusively of $1Million for each of the assets, the VaR in term of value will $33540.13 and $8645.26.

Besides, the individual VaR, one may investigate the pairwise market risks for between two markets. Now, for the...
overall pair-wise VaR, the investor (says with the overall capital SC) should determine the portfolio weight before making optimal portfolio allocation decisions. With multiple positions, the risk minimizing portfolio weight can be computed with the value

\[ w_{1,2} = \frac{\sigma_{21,2}^2 - \sigma_{12,2}}{\sigma_{11,2} + 2\sigma_{12,2} + \sigma_{22,2}} = 0.201835. \]

In other words, the investor should hold $0.201835C in the SET over $0.798165C in the KLSE for the optimal portfolio holding. Based on the above results, the overall diversified VaR of the positions can be determined by

\[ VaR_{\text{portfolio},q} = (w_q'r_q + (1-w_q)r_q_1) + \sigma_{\text{portfolio},q}D_q = -1.17689. \]

For non-optimal overall diversified, the value is calculated as

\[ VaR = \sqrt{VaR_{\text{TSE}}^2 + VaR_{\text{KLSE}}^2 + 2\rho_{12}VaR_{\text{TSE}}VaR_{\text{KLSE}}} = 2.923746. \]

Besides the diversified VaR, the investor is also interested to know the undiversified VaR if any catastrophic events occur in the financial markets. Under the perfectly correlated condition, the overall non-optimal undiversified VaR is

\[ VaR = VaR_{\text{TSE}} + VaR_{\text{KLSE}} = 3.893519. \]

The undiversified optimal VaR is higher than the diversified VaR due to the fact that the perfectly correlated condition contributes additional cross-markets impact to the overall undiversified VaR. This phenomenon often observed during the economic crisis where severe pressure of selling spree causes all assets and derivatives to depreciate and consequently it is possible that the situation of perfectly correlation between asset prices occurs during the crisis. In other words, the panic-stricken investors who radically pull out the short-term capital are paying less attention to the diversification. Thus, the quantified market risk during the crisis is greater than normal market conditions.

Finally for risk minimizing hedge ratio, the beta is found to be 0.048355 which implies that for every capital SC in long for SET, the investor should short $0.048355C of the KLSE market. Since the KLSE is less riskier than the TSE, almost all the capitals should invest in the long trading position in the TSE.

VI. CONCLUSION

This study investigates the return and volatility linkages between the Thailand and Malaysian equity markets. The hidden dynamics of interactions among the markets are evaluated by using a bivariate asymmetric BEKK model. In short, the major empirical findings that may attract the interest of investors and policy makers are three-fold: First, the bivariate return series analysis evidenced the presence of linkages in term of return and volatility among the markets in term of uni-directional impact. This finding suggests that the financial markets share common information and give impact to each other according to their interrelations effects. Second, the optimal portfolio holding within the pair-wise markets is based on the risk minimizing weight which calculated from the time varying conditional variance-covariance estimations. This information provide useful guide to provide the optimal cross-market pair-wise value-at-risk. Third, the cross-market risk minimizing hedge ratio provides useful guide on how to hold the long and short financial positions in the pair-wise markets. It is worth noting that the stock market linkage (return and volatility) can be interpreted as the information transmission among the markets. Thus, investors and researchers should monitor all the markets closely because a shock (good or bad news) eventually will transmit across the markets through their interdependence.

As a summary, this study provides useful information to understand the return and volatility transmission mechanism over time and across markets in the selected Southeast Asian stock exchange during the economic discovery period. The empirical findings for VaR in the pair-wise markets have demonstrated the importance of hedge ratios and portfolio weights under the diversified and undiversified conditions. For future research, one may extend the trivariate to n-variate framework in order to simultaneously examine all the ASEAN stock markets. In addition, it will be interesting to compare the empirical work by using the more powerful multivariate modelling such as dynamic conditional correlation [9], flexible dynamic correlations [11] and fractionally integrated multivariate ARCH [21].

REFERENCES


