Abstract—This paper presents a multi-objective order allocation planning problem with the consideration of various real-world production features. A novel hybrid intelligent optimization model, integrating a multi-objective memetic optimization process, a Monte Carlo simulation technique and a heuristic pruning technique, is proposed to handle this problem. Experiments based on industrial data are conducted to validate the proposed model. Results show that (1) the proposed model can effectively solve the investigated problem by providing effective production decision-making solutions, which outperforms an NSGA-II-based optimization process and an industrial planning problem with the consideration of various real-world production features. Research on order allocation, release and scheduling in production planning stage, which aims at assigning production processes of each order to appropriate plants and shop floors (assembly lines), has received little attention so far. Ashby and Uzsoy [1] investigated a decision-making problem integrating order release, group scheduling and order sequencing in a single-stage production system. Guo et al. [2] investigated an order scheduling problem which assigns the production processes of each order to different assembly lines in a make-to-order production plant. Chen and Pundoor [3] addressed order allocation and scheduling at supply chain level, which focused on assigning production orders to different plants and exploring appropriate schedules for processing the assigned orders in each plant. Previous studies on order allocation, release and scheduling have usually assumed a simple manufacturing environment with only one production department, which have not consider different types of departments with different production capacities, which are typical features in many labor-intensive manufacturing industries as apparel, and can greatly increase the complexity of production decision-making problems. The MOAP problem investigated in this paper will consider multiple plants, multiple departments, multiple objectives and multiple production uncertainties, which belongs to a type of complicated combinatorial optimization problem with a huge solution space.

Traditional optimization techniques such as brand and bound method, cannot handle this problem effectively. Various intelligent optimization algorithms have been developed and employed to provide complicated production decision-making due to their heuristic nature, such as tabu search method [4], simulated annealing method [5], genetic algorithm [6, 7], ant colony algorithm [8], and immune algorithm [9], in which GA is the most commonly used. In recent years, a type of novel intelligent optimization techniques, called memetic algorithms [10], have been developed and attracted increasing interests. A lot of research has demonstrated that memetic algorithm can provide better optimization performance than GA over a wide variety of applications [10, 11]. To provide effective decision-making solutions for the investigated MOAP problem, this research develops a novel multi-objective hybrid intelligent (MOHI) model, in which a multi-objective memetic algorithm is proposed to seek Pareto optimal MOAP solutions on the basis of the faster non-dominated sorting technique proposed by Deb et al. [12]. In addition, Monte Carlo technique is utilized to handle production uncertainty in the order allocation planning problem since Monte Carlo simulation is intrinsically well equipped to support decision-making when confronting uncertainty [13].

II. PROBLEM FORMULATION

The MOAP problem investigated is formulated in this section. The manufacturing company receives various production orders from different customers. These orders need to be assigned to the company’s $N$ production plants located in different regions, including self-owned or collaborative plants, for production. These plants involve $N$ production departments numbered as 1 to $N$, which perform, respectively, $N$ types of different production processes denoted as process type 1 to process type $N$. That is, production process $i$ ($i = 1,...,N$) can only be produced in production department $i$ ($i = 1,...,N$). These production departments can be classified into two categories: ordinary category and special category. Each category involves multiple production departments. The departments of the ordinary category are fully contained in all plants but it is possible that those of the special category are only partly included (or not included) in some of plants. It is thus possible that different production processes of an order need to be performed in different plants.

The manufacturer receives a group of production order, called an order group, from the customers at a time. Each order group consists of multiple production orders. Each order consists of a maximum of $N$ production processes. Each production process of an order is assigned to only one plant for processing. All finished products are delivered to a distribution
The real-world production environment is subject to the following constraints.

\[
\sum_{i=1}^{N} X_{ij}^k = 1 \quad (2)
\]

\[
X_{ij}^k = 1 \quad (j > 1) \text{ when } X_{ij}^{k-1} = 1 \text{ and } IS_{ij} = 1 \quad (3)
\]

\[
X_{ij}^{1} = 1 \quad \text{ when } X_{ij}^{k} = 1, \quad O_i \in G_k \text{ and } O_j \in G_k \quad (4)
\]

\[
B_{ij} \leq B_{ij(k+1)} \quad (5)
\]

\[
A_{ij} \leq B_{ij} \quad (6)
\]

\[
C_{ij} = B_{ij} + T_{ij} \quad (7)
\]

\[
F_i = C_{iN} + \sum_{k=1}^{n} X_{ij}^k \cdot TT_k \quad (8)
\]

Formulae (2)-(4) describe the process allocation constraint, and formula (5) describes the process precedence constraint. Formulae (6)-(8) describe the constraints of beginning time, processing time and completion time respectively. In this research, there exist

\[
A_{ij} = \begin{cases} 
C_{i(j-1)} + TTS_{i(j-1)} & (j > 1) \\
0 & (j = 1)
\end{cases}
\]

where \(TTS_{ij}\) represents the time (days) to transport semi-finished products to the plant performing process \(P_{ij}\)'s downstream process \(P_{i(j-1)}\). If processes \(P_{ij}\) and \(P_{i(j+1)}\) are produced in two different plants, \(TTS_{ij}\) equals the transportation time between the two plants; otherwise \(TTS_{ij}\) is equal to 0. In addition, the processing time \(T_{ij}\) of \(P_{ij}\) is

\[
T_{ij} = \sum_{k=1}^{s} \frac{W_{ij} \cdot X_{ij}^k}{SM_{ij}} \quad (10)
\]

where \(W_{ij}\) indicates the workload of \(P_{ij}\) (unit: standard man days) and \(SM_{ij}\) indicates the available standard manpower in \(S_{ij}\), which equals the summation of each operator’s average efficiency for producing the standard order in department \(S_{ij}\).
\[ \min Z_3(B_y, X^*_y) \quad \text{with} \quad Z_3 = \sum_{i=1}^{n} \sum_{j=1}^{N} TIT_{ij} \]

where \( TD_i = \max(0, F_i - D_i) \), which represents the tardiness of order \( O_i \); \( TPT_i = F_i - B_i \), which represents the throughput time of order \( O_i \); \( TIT_{ij} = \sum_{W_{ij} \geq 0} WT_{ij} \), which represents the total idle time of production department \( S_{ij} \) (where \( WT_{ij} \) is the time to wait for the arrival of process \( P_{ij} \) in a production department). The first two objectives are to minimize the total tardiness and the total throughput time of all orders whereas the third objective is to minimize the total idle time of all production departments.

### III. MULTI-OBJECTIVE HYBRID INTELLIGENT MODEL FOR ORDER ALLOCATION PLANNING

The architecture of the MOHI model is shown in Fig. 1. The model consists of three submodels, including a novel multi-objective memetic optimization (MOMO) submodel, a Monte Carlo simulation (MCS) submodel and a heuristic pruning submodel. The MOMO submodel is firstly utilized to seek the initial Pareto optimal solutions for the deterministic MOAP problem, which assumes that all uncertain orders need to be produced and the processing time of an order in a production department equals the mean of its processing time in this department. The MCS submodel is then utilized to obtain the fitness of each initial Pareto optimal solution for the stochastic MOAP problem with uncertain orders and processing time. Based on the fitness of initial solutions for the stochastic problem, the heuristic pruning submodel (process) is finally employed to generate the final Pareto optimal solutions for order allocation planning practice. The non-numerical objective function ranking preference method, proposed by Taboada and Coit [14], was adopted to implement the heuristic pruning process. The three submodels are described in detail as follows.

#### A. Multi-objective memetic optimization

The MOMO submodel is proposed to generate Pareto optimal solutions for the deterministic MOAP problem, called initial Pareto optimal solutions, in which all production orders and production efficiency (processing time) are deterministic. Fig. 2 illustrates the architecture of the MOMO, which integrates a tabu search [15], a faster non-dominated sorting technique [12] and a self-adaptive population size adjustment method [16] into a canonical MA [10] to generate Pareto optimal solutions for the deterministic MOAP problem. The flowchart of the MOMO is very similar to that of NSGA-II [12] except for inserting a Tabu search process for local improvement and the self-adaptive population size adjustment process. Relevant processes and operators are presented in detail as follows.

#### 1. Representation and population initialization

Each memetic individual represents a feasible MOAP solution. To handle the MOAP problem addressed, a feasible solution needs to be able to determine the assignment of each production process of each order to an appropriate plant. According to formulation (4), the individual can be determined by the assignment of each order group to an appropriate plant. Each memetic individual represents a feasible MOAP solution needs to be able to determine the assignment of each production process of each order to an appropriate plant. The non-numerical objective function ranking preference method, proposed by Taboada and Coit [14], was adopted to implement the heuristic pruning process. The three submodels are described in detail as follows.

#### 2. Population initialization

The initial population is generated randomly, with each individual representing a feasible MOAP solution. The initial population size is determined based on the number of orders and the number of plants. Each individual encodes the assignment of each order group to an appropriate plant. The encoding scheme is illustrated in Fig. 3.
groups to 4 plants. According to this solution, only one order group (order group 3) is assigned to plant 1 for the production of its production process 1 while 3 order groups (order groups 2, 5 and 8) are assigned to plant 2.

<table>
<thead>
<tr>
<th>Solution individual</th>
<th>4</th>
<th>2</th>
<th>1</th>
<th>3</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>2</th>
<th>4</th>
<th>3</th>
</tr>
</thead>
<tbody>
<tr>
<td>Order group No.</td>
<td>1</td>
<td>2</td>
<td>3</td>
<td>4</td>
<td>5</td>
<td>6</td>
<td>7</td>
<td>8</td>
<td>9</td>
<td>10</td>
</tr>
</tbody>
</table>

Fig. 3 An example of the solution representation

The initial population is generated randomly with a specified initial population size according to the solution representation described above.

2. Genetic operators

In this research, the tournament selection [17] is adopted. A multi-parent recombination operator was utilized, which modified the fitness-based scanning crossover [18], to adapt the proposed presentation. A modified mutation operator is proposed based on the uniform mutation [17] usually used for binary and real-coded representations. The mutation is implemented by randomly changing the values of several randomly selected genes.

3. Performance evaluation

During the memetic evolution process, the performance of each individual is evaluated by calculating the values of objective functions to be optimized. On the basis of the values of objective functions, performances of solutions are then sorted by the faster non-dominated sorting technique [12]. To obtain the values of objective functions, one needs to firstly determine the values of variables $B_{ij}$ and $X_{ij}$. Since the individual only represents the assignment of production process 1 of each order group to an appropriate plant, the assignment and processing sequence of the subsequent production processes of each order group need to be deduced further by heuristic rules. Four rules are presented to handle the assignment of subsequent processes. The process allocation constraints formulated in formulae (6) and (7) are used as rules (1) and (2). For the cases the two rules cannot handle, the following rules (3) and (4) are employed.

Rule 3) For an order, if the plant, which is assigned to process the last process of the current production process, has the production department processing the current production process, the process must be assigned to the same plant for processing. Otherwise go to rule 4.

Rule 4) Randomly assign the current production process to another plant capable of processing it.

The processing sequence of processes $P_{ij}$ ($j \geq 2$) in a production department is determined by the beginning time $B_{ij}$ of this production process, which depends on this process’s arrival time, the completion time of its preceding process $P_{i,j-1}$ and the processing priority of its corresponding order. In the situation that the production department is idle and waiting for the arrival of production orders, the order arrives firstly should be processed first. In the situation that multiple production orders have arrived a production department waiting for being processing, the order with highest processing priority should be processed first. The processing priority of each order and each order group is determined in terms of following rules:

- Rule 1) The order group with an earlier due date needs to be processed in a higher priority.
- Rule 2) If multiple order groups have the same due date, the order group with the less workload needs to be processed in a higher priority.
- Rule 3) In an order group, the order with the larger number of production processes needs to be processed in a higher priority.
- Rule 4) In an order group, if multiple orders have the same number of processes, the order with less workload needs to be processed in a higher priority.

4. Tabu search-based local improvement and replacement

On the basis of the individual $x$ newly generated in genetic optimization process, this research uses a simplified tabu search process, described below, to seek the local optimal solution $X^*$ in its neighborhood $N(x)$.

Step 1. Initialize the tabu list, $x^* \leftarrow x$, count $\leftarrow 0$.
Step 2. Select the solution among its neighborhood $N(x)$ that are not tabu;
Step 3. Update the tabu list according to the move of the selected solution;
Step 4. If the performance of the selected solution is superior to $x$, $x^* \leftarrow x$;
Step 5. count $\leftarrow$ count $+1$;
Step 6. Check if the termination condition is met. If so, go to Step 7, otherwise go to Step 2;
Step 7. Return $x^*$ as the best neighbor of $x$.

The neighborhood $N(x)$ consists of solutions which are generated by swapping the positions of any two elements (genes) in an individual (chromosome). For a individual with $n$ elements, its neighborhood contains $n(n-1)/2$ solutions. The termination condition is defined as an instance: (1) a specified number max SimTimes of moves are performed without improving the best solution obtained; (2) a solution which is close to the given lower bound of the goal function value is found.

B. Monte Carlo simulation

Let max SimTimes denotes the maximal simulation times. The MCS process to get the fitness of each initial Pareto optimal solution for the stochastic MOAP problem with uncertain production orders and uncertain processing time can be outlined as follows:

Step 1. Initialize max SimTimes, set count $=1$;
Step 2. Generate deterministic problem inputs randomly based on the probability distributions of uncertain production orders and uncertain processing time in production departments, including the production orders processed and the processing time of each production process of each order;
Step 3. Obtain and save the values of objective functions of
solution x on the problem inputs generated in Step 2 according to the performance evaluation method described in sub-section III.A.3.

Step 4. \( \text{count} = \text{count} + 1 \);
Step 5. Check if \( \text{count} \) is greater than \( \max \text{SimTimes} \). If so, go to Step 6, otherwise go to Step 2;
Step 6. Return the average value of each objectives function, for \( \max \text{SimTimes} \) repetitive simulations, as the values of objective functions of solution x.

IV. SIMULATION EXPERIMENTS AND COMPARISON

A. Experimental results

A series of simulation experiments were conducted to validate the effectiveness of the proposed MOHI model. This section highlights one of these experiments in detail.

In this experiment, 12 order groups with 76 production orders were performed. The experimental data were collected from a make-to-order labor-intensive manufacturing company producing knitwear products in China. The company comprises 4 plants located in different regions, in which 5 different production departments are involved.

For simplicity, it is assumed that the production departments discussed are empty initially in the 3 experiments. The proposed model was established based on the settings: the initial population sizes of memetic optimization processes were all equal to 500 while the maximum numbers of generations was 100. In each generation, the crossover probability changed randomly between 0.5 and 0.8 while the mutation probability changed randomly between 0.01 and 0.05. The length of tabu list equals 15; \( \max \text{MVTimes} \) equals 30, and \( \max \text{SimTimes} \) equals 10000. For determinative MOAP problems, the random value \( \text{randValue} \) in formula (1) is set as 1.

The ranking preference of objective functions applied to experiments 1-3 is the case in which objective 1 is more important than objective 2, and objective 2 is more important than objective 3. To highlight the importance of objective 1, we set \( \nu_1 \geq 2 \nu_2 \). This ranking preference is consistent with the policies and priorities of the investigated company.

The Pareto optimal solutions generated by the proposed model are shown in Fig. 4 in a three-dimensional space. There are 71 solutions in total, which is a very large set of solutions and it is difficult for the production planner to choose an appropriate solution for real production schedule. Based on these Pareto optimal solutions, the pruning process then generated 9 pruned solutions as shown in Table I. The pruned solutions are marked by ‘c’ points in Fig. 4 while the Pareto optimal solutions are marked by ‘a’ points. The columns OG1-OG12 of Table I show the numbers of plants that production process 1 of the corresponding order groups’ are assigned to. Take solution 1 as an example, the production process 1 of order groups 1-3 are assigned to plants 2, 1 and 3 respectively.

![Fig. 4 Pareto optimal set in a three-dimensional space](image)

<table>
<thead>
<tr>
<th>Solution No.</th>
<th>Assignment of production process 1 of each order group (OG)</th>
<th>Values of objective functions = ( \langle x, y, z \rangle )</th>
<th>Average values of objective functions = ( \langle \bar{x}, \bar{y}, \bar{z} \rangle )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>(2,1,4,2,3,3,3,3,3,3,3,2)</td>
<td>179 ( \text{value} ) \text{OG1}</td>
<td>27.7 ( \text{value} ) \text{OG1}</td>
</tr>
<tr>
<td>2</td>
<td>(2,1,4,2,3,3,3,3,3,3,3,2)</td>
<td>616 ( \text{value} ) \text{OG1}</td>
<td>26.7 ( \text{value} ) \text{OG1}</td>
</tr>
<tr>
<td>3</td>
<td>(2,1,4,2,3,3,3,3,3,3,3,2)</td>
<td>648 ( \text{value} ) \text{OG1}</td>
<td>27.0 ( \text{value} ) \text{OG1}</td>
</tr>
<tr>
<td>4</td>
<td>(3,1,4,2,3,3,3,3,3,3,3,2)</td>
<td>1303 ( \text{value} ) \text{OG1}</td>
<td>25.5 ( \text{value} ) \text{OG1}</td>
</tr>
</tbody>
</table>

It can be easily found from the experimental results that the number of pruned solutions is much less than that of original Pareto solutions so that the production planner can choose an appropriate solution more conveniently for MOAP practice. In addition, the performance of production planning is probably significantly different if different MOAP solutions are adopted. It is thus important to obtain appropriate MOAP solutions according to a specified production objective preference.

It can be found that from Table I, when production uncertainties are considered, optimization performances (represented by 3 objective functions) generated by the MCS process are quite different from those of determinative MOAP problems. The values of each objective of stochastic MOAP problems are usually less than the values of its corresponding objective of determinative MOAP problems. It shows that the optimization performances for the determinative problem cannot reflect the real remaining production capacity, which will thus inevitably affect frontline production decision-making such as order acceptance and production scheduling.

B. Performance comparison

To validate the effectiveness of the proposed method, this research compares its optimization results with those generated by an NSGA-II model and an industrial method in terms of the determinative MOAP problems with the consideration of all production orders and determinative processing time. To reduce the effects of randomness of evolutionary processes in the proposed MOMO submodel and NSGA-II model, this research repeatedly runs the two models 50 times to achieve the minima of each objective in each experiment. The solutions generated by the industrial method are called industrial solutions.
The NSGA-II model used for performance comparison is similar to the proposed MOMO submodel. The only difference is that the former does not include the tabu search-based local improvement and replacement process. The parameter settings of the NSGA-II model are the same with those of the MOMO submodel except with different maximum numbers of generations. In the NSGA-II, the maximum numbers of generations were 1000, which are much greater than those of the MOMO submodel.

Table II shows the comparison results, generated by the 3 different methods in terms of the determinative MOAP problems in the above experiment. Columns of ‘Min’ and ‘Mean’ show the minimum, mean of the corresponding objectives generated by different methods whereas ‘Times’ columns show the times of getting the corresponding minimum objective value in 50 repetitive runs. Taking objective 1 as an example, the minimal value of objective 1 could converge to the global minimum in experiments 1-3 because the value of objective 1 cannot be less than 0. It indicates that the proposed MOMO model has the capacity of finding the globally optimal solutions. In addition, there are 34 times that the proposed MOMO can get the minimum 0 while there are 5 times that the NSGA-II can get the minimum. It is clear that the proposed MOMO can generate much superior results to the NSGA-II because it can reach the responding minima more frequently and generate less means. In addition, the MOMO and NSGA-II can generate much better results than the industrial methods because they generate less objective values. These results show that the proposed MOMO has the best optimum-seeking ability.

TABLE II

<table>
<thead>
<tr>
<th>Method</th>
<th>Objective 1</th>
<th>Objective 2</th>
<th>Objective 3</th>
<th>Objective 4</th>
</tr>
</thead>
<tbody>
<tr>
<td>MOMO</td>
<td>Min: 0</td>
<td>Times: 34</td>
<td>Mean: 1.9</td>
<td>1342.8</td>
</tr>
<tr>
<td>NSGA-II</td>
<td>Min: 0</td>
<td>Times: 5</td>
<td>Mean: 18.9</td>
<td>1342.8</td>
</tr>
<tr>
<td>Industrial method</td>
<td>Min: 20.3</td>
<td>Times: /</td>
<td>Mean: /</td>
<td>1364.1</td>
</tr>
</tbody>
</table>