Multi-objective Optimization of Vehicle Passive Suspension with a Two-Terminal Mass Using Chebyshev Goal Programming

Chuan Li, Ming Liang and Qibing Yu

Abstract—To improve the dynamics response of the vehicle passive suspension, a two-terminal mass is suggested to connect in parallel with the suspension strut. Three performance criteria, tire grip, ride comfort and suspension deflection, are taken into consideration to optimize the suspension parameters. However, the three criteria are conflicting and non-commensurable. For this reason, the Chebyshev goal programming method is applied to find the best tradeoff among the three objectives. A simulation case is presented to describe the multi-objective optimization procedure. For comparison, the Chebyshev method is also employed to optimize the design of a conventional passive suspension. The effectiveness of the proposed design method has been clearly demonstrated by the result. It is also shown that the suspension with a two-terminal mass in parallel has better performance in terms of the three objectives.

Keywords—Vehicle, passive suspension, two-terminal mass, optimization, Chebyshev goal programming

I. INTRODUCTION

Suspension is one of the important vehicle components and influences significantly on comfort, safety and maneuverability of the modern vehicles [1-3]. As a dynamics transmission system between the body and the tire, the conventional vehicle passive suspension can be regarded as a parallel combination of a damper and a spring. Though the mass is also one of the three basic vibration components, it has only one genuine manipulation terminal [4], and thus cannot be directly embedded into a passive suspension where two terminals are required to connect the body and the tire of the vehicle.

Mass components play important roles in dynamical systems including the suspension. Efforts have been made to improve the performance of the passive suspension exploiting inertial (mass) force. For example, a dynamic vibration absorber including a damper, a spring and a mass was mounted on the French subcompact Citroën 2 CV to reduce the wheel resonance without jeopardizing ride comfort [5]. However, the large added mass has limited the use of such absorbers in other types of cars. To reduce the gravitational mass while improving the inertial mass in a single-DOF passive vibration isolation system, Rivin proposed a screw transmission flywheel which is named as motion transformer [6]. A similar system, i.e., an inerter, was introduced by Smith et al. to resolve the synthesis issue of mechanical networks [7-9]. The motion transformer and the inerter are similar in that they both have two manipulation terminals for a flywheel. Later research on two-terminal mass has shown that other rectilinear or rotary mass components such as mass blocks could also realize a mass system with two manipulation terminals [10].

A two-terminal mass differs from traditional mass components such as mass block and flywheel because it has two manipulation terminals [11]. In addition, the two-terminal mass has much greater inertial mass compared to its gravitational mass. Hence it is suitable for practical applications. The two-terminal mass has been introduced for vibration isolation of the passive suspension and numerical simulation result shows that it contributes to better isolation performance [12]. The vibration isolation index, however, is a non-mainstream measure related to the vehicle suspension performance. Tire grip, passenger comfort and suspension deflection are three commonly used criteria to evaluate the suspension performance [13-15]. However, the parametric optimization results obtained separately based on one of the three criteria are often conflicting. Yet, as the three criteria are non-commensurable, it is improper or at least subjective to “optimize” the design based on the weighted sum of the three associated objective functions. The Chebyshev goal programming method is an effective multi-objective optimization approach [16, 17] in dealing with conflicting and non-commensurable objectives. It is therefore adopted in this study to find the optimal tradeoff of the three design objectives.

The rest of this paper is organized as follows. Section II briefly introduces the two-terminal mass and its application to the passive suspension. The dynamics model of the suspension is also proposed in this section. Section III proposes the state-space model of the suspension governing equation for the solution of the three single-objective issues. The Chebyshev method for multi-objective suspension optimization is described in detail in this section. The comparison between the
proposed and the conventional designs is reported in section IV. Conclusions are given in section V.

II. INFLUENCE OF SUSPENSION PARAMETERS ON DYNAMICAL PERFORMANCES

A. Two-Terminal Mass Basics

The two-terminal mass is a mass component or device characterized by the facts that [11]: a) it contains two free, genuine and controllable terminals and none of which is necessary to be fixed to the ground; b) the relative force between two terminals is proportional to the second-order time derivative of the relative displacement between the two terminals of the component; and c) the inertial mass of the component is usually much greater than the gravitational mass of the mass core.

\[ F = m \left( \frac{\sqrt{2} r}{D \tan \alpha} \right)^2 \dot{x} \]  

Comparing (1) and (2) with \( F = m \ddot{x} \) described by Newton’s Second Law indicates that the gravitational mass \( m \) of the flywheel is magnified to inertial masses \( m_r = m (\sqrt{2} r / (D \tan \alpha))^2 \) and \( m_t = m (\sqrt{2} r / (D \tan \alpha))^2 \) respectively, due to the transforms of the hydraulic and screw transmissions. Due to this mass magnification effect, the two-terminal mass is suggested to connect in parallel with the suspension strut, which will be introduced in the following subsection.

B. Passive Suspension with a Two-terminal Mass in Parallel

Fig. 2(a) 1/4 car model, (b) the conventional suspension \( S \), and (c) the proposed suspension \( S \) with a two-terminal mass in parallel

Fig. 2(a) displays the conventional 1/4 car model where \( m_1 \) denotes unsprung mass, \( m_2 \) sprung mass, \( d_0 \) road surface excitation, \( d_1 \) tire deformation, \( d_2 \) body displacement, \( k_1 \) tire stiffness and \( S \) the suspension (\( S = S_1 \) or \( S_2 \) respectively for the conventional and proposed suspensions). As shown in Fig. 2(b), a conventional suspension \( S \) is composed of a suspension stiffness \( k \) and a suspension damping \( c \). Having a two-terminal mass \( m_1 \) in parallel, the proposed suspension \( S \) is shown in Fig. 2(c). The governing equations in the Laplace domain for the 1/4 car model shown in Fig. 2(a) are given by

\[
\begin{align*}
\left. m_1 \dot{s}^2 \ddot{d}_1 + G(s)(\ddot{d}_2 - \dot{d}_1) \right) &= 0 \\
\left. m_2 \dot{s}^2 \ddot{d}_2 + G(s)(\ddot{d}_1 - \dot{d}_2) + k_1 (\dot{d}_1 - \dot{d}_0) \right) &= 0 
\end{align*}
\]
where, ‘\(^\ast\)’ represents the Laplace transform of the corresponding variables and \(G(s)\) the transfer function. \(G(s) = G_1(s)\) for \(S_1\) and \(G(s) = G_2(s)\) for \(S_2\) respectively. According to Figs. 2(b) and (c), one has

\[
\begin{align*}
G_1(s) &= k_2 + cs \\
G_2(s) &= k_2 + cs + m_v s^2
\end{align*}
\]  

(4)

A filtered white noise is used to represent the vibration excitation of the road surface, i.e. [18]

\[
\ddot{d}_o(t) = -2\pi f_o \dot{d}_o(t) + 2\pi \sqrt{G_o} v(t)
\]  

(5)

where \(f_o\) is the lower cut-off frequency (Hz), \(G_o\) the road roughness coefficient (m\(^3\)/cycle), \(v\) the vehicle speed (m/s) and \(w(t)\) the zero mean Gaussian noise.

### III. SUSPENSION PARAMETER OPTIMIZATION

#### A. Influence of Suspension Parameters on the Performance

As mentioned before, there are three commonly used criteria to evaluate suspension performance: tire grip \((\ddot{d}_1)\), ride comfort \((\ddot{d}_2)\) and suspension deflection \((\ddot{d}_3)\). To calculate the three performance measures, the Laplace domain governing equations of the proposed suspension \(S_2\) are converted into a state-space model. The state-space model of the suspension \(S_2\) can be expressed as

\[
\begin{align*}
\dot{X} &= AX + BU \\
Y &= CX
\end{align*}
\]  

(6)

where \(X, U,\) and \(Y\) represent respectively the state, input, and output vectors; \(A, B\) and \(C\) are the associated coefficient matrices. According to (3) and (4), we define

\[
\begin{align*}
X &= [x_v\ x_b\ x_c\ x_d]^T \\
Y &= [y_v\ y_b\ y_c]^T
\end{align*}
\]  

(7)

where \(x_v = d_2 - d_1, x_b = \dot{d}_2, x_c = d_3 - d_0, x_d = d_1 - d_0 = x_c, y_v = \dot{d}_2, y_b = \dot{y}_v, y_c = \dot{d}_3 - \dot{d}_1\) and \(y_d = \dot{d}_1 - \dot{d}_0 = x_v\).

Combining (5) and (6) gives

\[
U = \dot{d}_o \leftrightarrow \dot{d}_o = \frac{2\pi \sqrt{G_o} \sqrt{\dot{v}}}{s^2 + 2\pi f_o \dot{v}}
\]  

(8)

For the proposed suspension \(S_2\), combining (3), (4), (6) and (7) leads to

\[
\begin{align*}
A &= \begin{bmatrix} 0 & 1 & 0 & -1 \\ -k_2 m_1 & -cm_1 & -m_k_1 & -cm_1 \\
0 & 0 & 0 & -1 \\
k_2 m_2 & cm_2 & -k_3 m_2 - m_k_3 & -cm_2 \\
0 & 0 & 0 & 0
\end{bmatrix} \\
B &= \begin{bmatrix} 0 \ 0 \ -1 \ 0 \end{bmatrix}^T \\
C &= \begin{bmatrix} -k_2 m_1 & -cm_1 & -m_k_1 & cm_1 \\
0 & 0 & 0 & 0 \\
\Delta & \Delta & \Delta & \Delta \\
1 & 0 & 0 & \Delta \\
0 & 0 & 1 & 0
\end{bmatrix}
\]  

(9)

where \(\Delta = (m_1 + m_v)(m_2 + m_v) - m_v^2\).

The aforementioned equations show that the three performance indices are given by the three components of the output vector \(Y\) of the state-space model. With any given road input shown in (5), one may calculate the tire grip, the ride comfort and the suspension deflection from (6) to (9). The optimal suspension parameters \((k_2, c, m)\) should ensure the following are achieved:

\[
\begin{align*}
\min J_1 &= \frac{1}{T} \int_0^T \sqrt{v^2} dt \\
\min J_2 &= \frac{1}{T} \int_0^T \sqrt{\dot{y}_v^2} dt \\
\min J_3 &= \frac{1}{T} \int_0^T \sqrt{\dot{y}_c^2} dt
\end{align*}
\]  

(10)

where \(\min J_i\) denotes the optimal tire grip, \(\min J_2\) the optimal ride comfort, and \(\min J_3\) the optimal suspension deflection.

In the actual design of the vehicle suspension, the stiffness is usually specified first according to the use of the vehicle. Then the other dynamics parameters of the suspension are optimized afterwards. In our research, suppose the 1/4 car model suspension parameters are [19]: \(m_v = 60kg, m_c = 375kg,\) and \(k_v = 200kN/m\). Let \(k_v = 80kN/m,\) the road excitation parameters \(G_o = 8\times 10^{-5}m^3/cycle, \dot{v} = 25m/s, f_o = 0.2Hz\) and \(w(t)\) be 20dB zero-mean white noise.

It should be noted that \(\min J_i\) is zero if \(c \rightarrow \infty\). To achieve a meaningful design results, we specify the \(c\) and \(m\) ranges as \(c \in [1,20]kN/m\) and \(m_v \in [20,500]kg\). Simulating 2500m of the road length yields the running time 100s with the given vehicle speed \(v\). The sampling frequency is set as 1kHz. Calculating from (6)-(9), the optimal suspension parameters in terms of the three objectives are respectively displayed in Table I.

**TABLE I**

<table>
<thead>
<tr>
<th>Objective</th>
<th>(c_v(kN/m))</th>
<th>(m_v(kg))</th>
<th>(\min J_i)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Tire grip ((J_1))</td>
<td>5.108</td>
<td>135</td>
<td>3.2382\times10^4 m</td>
</tr>
<tr>
<td>Ride comfort ((J_2))</td>
<td>5.262</td>
<td>183</td>
<td>0.1071 m/s</td>
</tr>
<tr>
<td>Suspension deflection ((J_3))</td>
<td>20</td>
<td>20</td>
<td>3.2400 \times 10^4 m</td>
</tr>
</tbody>
</table>

Note: a) Subscript ‘opt’ means the optimal parameter; and b) \(i = 1, 2, 3\) respectively for \(\min J_i\).
Table I reveals that it is impossible to simultaneously obtain min$J_1$, min$J_2$ and min$J_3$ with a fixed combination of suspension parameters. The parametric optimization results in terms of the three single criteria are noncommensurable and conflicting. Optimizing one objective separately may jeopardize the other two. To achieve the optimization design tradeoff with respect to all the three objectives, the Chebyshev goal programming method is adopted to find the preferred compromise between these objectives as detailed in the following.

B. Multi-objective optimization using the Chebyshev goal programming method

The multi-objective suspension parameter optimization problem is expressed as

$$\min Q = f(J_1, J_2, J_3)$$  \hspace{1cm} (10)$$

Besides the best single-objective performances min$J_1$, min$J_2$ and min$J_3$ resulting from the optimal parameters, there are also the worst performances expressed as max$J_1$, max$J_2$ and max$J_3$ for the suspension in the given parameter ranges. With the best and worst single objective values, the Chebyshev goal programming model is formulated as follows:

$$\text{Min} Q,$$

subject to

$$Q \geq \frac{J_{1\text{mop}} - \min J_1}{\max J_1 - \min J_1}$$

$$Q \geq \frac{J_{2\text{mop}} - \min J_2}{\max J_2 - \min J_2}$$

$$Q \geq \frac{J_{3\text{mop}} - \min J_3}{\max J_1 - \min J_3}$$

(11)

where $\min Q$ is the multi-objective performance index incorporating the three single criteria, and subscript ‘mop’ represents the multi-objective optimum. Solving the above model yields an overall optimal solution in terms of all these three performance criteria.

Since we have already found out three optimal performance with respect to optimal parameters $c_{opt}$ and $m_{opt}$. Calculating in the same way described in last subsection, we hereafter present the worst performance resulting from the ‘worst’ parameters $c_{wor}(\text{mop})$ and $m_{wor}(\text{mop})$ (subscript ‘wor’ represents the worst parameter) in Table II.

<table>
<thead>
<tr>
<th>Objectives</th>
<th>$c_{wor}(\text{kN.s/m})$</th>
<th>$m_{wor}(\text{kg})$</th>
<th>max$J_i$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Tire grip ($J_1$)</td>
<td>1</td>
<td>300</td>
<td>5.771×10^4</td>
</tr>
<tr>
<td>Ride comfort ($J_2$)</td>
<td>1</td>
<td>500</td>
<td>0.2424m^2/s</td>
</tr>
<tr>
<td>Suspension deflection ($J_3$)</td>
<td>1</td>
<td>350</td>
<td>9.639×10^4</td>
</tr>
</tbody>
</table>

Substituting the parameters shown in Table I and II into (11) yields the multi-objective optimized parameters $c_{wor} = 11.220\text{kN.s/m} \text{ and } m_{mop} = 177\text{kg}$. The corresponding suspension performances are summarized in Table III. The relationship between the multi-objective performance and the suspension parameters in the given ranges is shown in Fig. 3.

IV. COMPARISON WITH THE CONVENTIONAL SUSPENSION

As a comparison, the Chebyshev goal programming method is also applied for parametric optimization design of the conventional suspension $S_1$ shown in Fig. 2(b). With the same given parameters, the design optimization results are shown in Table IV. The influence of the damping on the multi-objective performance of the suspension $S_1$ in the simulation case is shown in Fig. 4.

<table>
<thead>
<tr>
<th>Objective</th>
<th>Performance</th>
<th>$c(\text{kN.s/m})$</th>
</tr>
</thead>
<tbody>
<tr>
<td>min$J_1$</td>
<td>2.4432×10^4</td>
<td>5.736</td>
</tr>
<tr>
<td>min$J_2$</td>
<td>0.1163</td>
<td>6.551</td>
</tr>
<tr>
<td>min$J_3$</td>
<td>3.239×10^4</td>
<td>20.0</td>
</tr>
</tbody>
</table>

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<tr>
<th>Objective</th>
<th>Performance</th>
<th>$c(\text{kN.s/m})$</th>
</tr>
</thead>
<tbody>
<tr>
<td>max$J_1$</td>
<td>3.666×10^4</td>
<td>1.0</td>
</tr>
<tr>
<td>max$J_2$</td>
<td>0.1830</td>
<td>1.0</td>
</tr>
<tr>
<td>max$J_3$</td>
<td>8.485×10^4</td>
<td>1.0</td>
</tr>
</tbody>
</table>

TABLE III

<table>
<thead>
<tr>
<th>Objective</th>
<th>Performance</th>
<th>$c(\text{kN.s/m})$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$J_{\text{max}}$ (m)</td>
<td>2.5651×10^4</td>
<td>10.579</td>
</tr>
<tr>
<td>$J_{\text{wor}}$ (m)</td>
<td>0.1201</td>
<td>1.0</td>
</tr>
<tr>
<td>$J_{\text{mop}}$ (m)</td>
<td>3.7619×10^4</td>
<td>1.0</td>
</tr>
<tr>
<td>min$Q$</td>
<td>0.0997</td>
<td></td>
</tr>
</tbody>
</table>

TABLE IV

TABLE II

<table>
<thead>
<tr>
<th>Objectives</th>
<th>$c_{wor}(\text{kN.s/m})$</th>
<th>$m_{wor}(\text{kg})$</th>
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</tbody>
</table>
Fig. 4 Multi-objective performance variety resulting from the damping change of the conventional suspension $S_1$.

Fig. 5 displays the comparison between the two suspensions, $S_1$ and $S_2$, in terms of these three performance criteria. As compared to the conventional design, the proposed suspension $S_2$ has led to improvements of 0.53% $=\frac{(2.5651 \times 10^{-4}-2.5516 \times 10^{-4})}{2.5651 \times 10^{-4}}$, 2.58% $=\frac{(0.1201-0.1170)}{0.1201}$, and 1.39% $=\frac{(3.7619 \times 10^{-4}-3.7096 \times 10^{-4})}{3.7619 \times 10^{-4}}$ in terms of tire grip, ride comfort, and suspension deflection, respectively. For this example case, the proposed design has also improved the overall multi-objective performance by 26.38% $=\frac{(0.0997-0.0734)}{0.0997}$ over the conventional design $S_1$.

V. CONCLUSION

In this paper, we examined the performance of a passive vehicle suspension with a two-terminal mass in parallel in terms of tire grip, ride comfort, and suspension deflection. As the three performance measures are non-commensurable and often conflicting, the Chebyshev goal programming method is applied to solving this multi-objective problem. For our example case, the proposed new design has outperformed the conventional one in all the three criteria, leading to 26.38% overall improvement.

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