Control of a DC Servomotor Using Fuzzy Logic Sliding mode Model Following Controller

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Abstract—A DC servomotor position control system using a Fuzzy Logic Sliding mode Model Following Control or FLSMFC approach is presented. The FLSMFC structure consists of an integrator and variable structure system. The integral control is introduced into it in order to eliminate steady state error due to step and ramp command inputs and improve control precision, while the fuzzy control would maintain the insensitivity to parameter variation and disturbances. The FLSMFC strategy is implemented and applied to a position control of a DC servomotor drives. Experimental results indicated that FLSMFC system performance with respect to the sensitivity to parameter variations is greatly reduced. Also, excellent control effects and avoids the chattering phenomenon.

Keywords—Sliding mode Model Following Control, Fuzzy Logic, DC Servomotor.

I. INTRODUCTION

RECENTLY advancements in magnetic materials, semiconductor power devices and control theory have made the permanent magnet motor servo drive play an important role in motion control applications [1]. In the servo applications, significant parameter variations arise from often unknown loads; for example, in machine tool drives and robotics. The proposed scheme for DC motor position servo control is shown in Fig. 1. The current control loop is a current controlled pulse width modulation voltage source inverter, which is widely applied in high performance servo drivers. The outer loop controller is designed to achieve a fast and accurate servo tracking response under load disturbance and plant parameter variations. The dynamics characteristic of such systems are very complex and highly nonlinear, a conventional linear controller may not assure satisfactory requirements.

Fig. 1 The DC servomotor control structure.

Variable Structure Control (VSC) or Sliding Mode Control (SMC) reduce the system dynamics to the motion along the sliding surface. The important feature in VSC is what is termed sliding mode. The VSC approach possesses other salient advantages such as high speed of response, good transient performance and no need for precise knowledge of the controlled plant. Although the conventional VSC approach has been applied successfully in many applications[2-5], it cannot perform well in servo applications where the system is designed to track a command input. In order to improve tracking performance, the Integral Variable Structure Model Following Control or IVSMFC approach, presented in [6-7], The IVSMFC approach can eliminate the steady tracking error due to a step command input. However, IVSMFC yields the error when the system has to follow a changing command input, e.g., a ramp input. The Modified Integral Variable Structure Control or MIVSC approach, proposed in [8-9], uses a double integral action to solve this problem. Although, the MIVSC can give a better tracking performance than the IVSMFC does at steady state, its performance during transient period needs to be improved. However, the main problem of VSC, IVSMFC and MIVSC is the chattering phenomenon restricts its application.

Fuzzy control is a practical control method which imitates human being fuzzy reasoning and decision making processes. Fuzzy logic control is derived from the fuzzy logic and fuzzy set theory that were introduced in 1965 by Professor Lotfi A. Zadeh of the University of California at Berkeley. Fuzzy logic can be applied in many disciplines such as economics, data analysis, engineering and other areas that involve a high level of uncertainty, complexity or nonlinearity. In engineering, engineers can use the fundamentals of fuzzy logic and fuzzy set theory to create the pattern and the rules, then design the fuzzy controllers, Finally, the output response of many systems can be improved by using a fuzzy controller [8-9]. The method is applicable to conduct robustness control over target for which a mode is hard to be established. The final program form of the method is simple and easy to achieve. Therefore, combining fuzzy control with the VSC would maintain the insensitivity of sliding mode control to parameter perturbation and external disturbances while in the mean time effectively eliminate the chattering phenomenon.

This paper presents the design and implementation of a DC servomotor position control systems using the FLSMFC approach. This approach, which is the extension of IVSMFC approach, incorporates a feedforward path and fuzzy control to improve the dynamics response for command tracking and strong robustness. An experimental prototype system consisting of a digital signal processor from Texas Instrument is constructed. Experimental results are presented for demonstrating the potential of the proposed scheme and the tracking performance can be remarkably improved.
II. DESIGN OF FLSMFC SYSTEM

The structure of FLSMFC is shown in Fig. 2 can be described by the following equation of state

$$\dot{x}_i = x_{i+1}, i = 1, \ldots, n-1, \dot{x}_n = -\sum_{i=1}^{n} a_i x_i + b U - f(t), \dot{x}_0 = \text{const}$$  \hspace{1cm} (1)

![Fig. 2. The structure of FLSMFC system.](image)

The switching function, $\sigma$, is given by

$$\sigma = c_i (x_i - T x_i - r K_p) + \sum_{i=1}^{n} c_i x_i$$  \hspace{1cm} (2)

where $C_i > 0$ is a constant, $C_n = 1$ and $T$ is the integral time.

The control signal, $U$, can be determined as follows, from (1) and (2), we have

$$\dot{\sigma} = -c_i T (r - x_i) + \sum_{i=1}^{n} a_i x_i + b U - f(t)$$  \hspace{1cm} (3)

Let $a_i = a_i^0 + \Delta a_i$, $i = 1, \ldots, n$ and $b = b^0 + \Delta b$, $b^0 > 0$, $\Delta b > -b^0$.

The control signal can be separated into

$$U = U_{eq} + U_{fb}$$  \hspace{1cm} (4)

This condition results in

$$U_{eq} = c_i T (r - x_i) - \sum_{i=1}^{n} a_i x_i + b^0 \sqrt{\text{b}^2 + \text{b}^2}$$  \hspace{1cm} (5)

The transfer function when the system is on the sliding surface can be shown as

$$H(s) = \frac{X_i(s)}{R(s)} = \alpha_s$$  \hspace{1cm} (6)

The transient response of the system can be determined by selecting the poles of the transfer function.$$

\begin{align*}
\frac{s^n + \alpha_1 s^{n-1} + \ldots + \alpha_{n-1} s + \alpha_n}{s^n + \alpha_1 s^{n-1} + \ldots + \alpha_{n-1} s + \alpha_n}
\end{align*}$$  \hspace{1cm} (7)

be the desired characteristic equation (closed-loop poles), the coefficient $C_i$ and $T$ can be obtained by

$$C_{n+1} = \alpha_1, C_1 = \alpha_{n+1} \text{ and } T = \alpha_0/\alpha_{n+1}.$$  \hspace{1cm} (8)

III. DESIGN OF FUZZY LOGIC CONTROLLER

By the definition

$$U_{fb} = k_i (x_i - T x_i - r K_p) + \sum_{i=1}^{n} k_{i+1} x_i + K_i (x_i - T x_i - r K_p) + \sum_{i=1}^{n} k_{i+1}$$  \hspace{1cm} (9)

$U_{fb}$ is required to guarantee the existence of the sliding mode under the plant parameter variations in $\Delta A_i$ and $\Delta b$ and the disturbances $f(t)$. Among them,

$$k_i = \begin{cases} \alpha_i & \text{if } x_i > T x_i - r K_p \\ \beta_i & \text{if } x_i < T x_i - r K_p \\ 0 & \text{if } x_i = T x_i - r K_p \end{cases}$$  \hspace{1cm} (9a)

and $k_{n+1} = \begin{cases} \alpha_{n+1} & \text{if } \sigma > 0 \\ \beta_{n+1} & \text{if } \sigma < 0 \\ 0 & \text{if } \sigma = 0 \end{cases}$

According to (3), we know

$$\dot{\sigma} = -c_i T (r - x_i) + \sum_{i=1}^{n} a_i x_i + b U - f(t)$$

and

$$U = U_{eq} + k_i T (r - x_i) - \sum_{i=2}^{n} k_{i+1} x_i.$$  \hspace{1cm} (9b)

The condition for the existence of a sliding mode is known to be.

$$\sigma \dot{\sigma} < 0.$$  \hspace{1cm} (10)

In order for (10) to be satisfied, the following conditions must be met,

$$k_i = \begin{cases} \alpha_i & \text{if } |\Delta a_i - a_i^0| > c_i (1 + |\Delta b/b^0|) \\ \beta_i & \text{if } |\Delta a_i - a_i^0| < c_i (1 + |\Delta b/b^0|) \end{cases}$$  \hspace{1cm} (10a)

where $i = 1, \ldots, n-1, c_0 = 0$

and

$$k_{n+1} = \begin{cases} \alpha_{n+1} & \text{if } \sigma > -N \\ \beta_{n+1} & \text{if } \sigma < -N \end{cases}$$  \hspace{1cm} (10b)

Now we consider the effect of $\Delta k_i (i = 1, \ldots, n), \Delta k_i$ is the function of to eliminate the chattering phenomenon of the control system and find out $\Delta k_i$ by making use of fuzzy set theory. Firstly take positive constants $\alpha$ and $\beta$, normalize switching function $\sigma$ and its rate of change against time.

Suppose

$$\sigma_i = \alpha \sigma, \text{ and } \sigma_i = \beta \sigma$$  \hspace{1cm} (11)

The input variable of the fuzzy controller is

$$\sigma_i = \text{sign}(x_i - T x_i - r K_p), \sigma_i = \text{sign}(x_i - T x_i - r K_p), \sigma_i = \text{sign}(x_i)$$

And $\sigma_i = \text{sign}(x_i) (i = 2, \ldots, n)$, the output of the controller is $\Delta k_i$.
Define the following fuzzy control regularity Table 1.

\[
\Delta k_i = \left( \int \Delta k_i \, \tilde{\mu}_{\Delta k_i} \, d\Delta k_i \right) \left( \int \tilde{\mu}_{\Delta k_i} \, d\Delta k_i \right)^{-1}
\]

Table 1: Fuzzy Control Regularity

<table>
<thead>
<tr>
<th>Fuzzy Control Regularity</th>
<th>( N )</th>
<th>( Z )</th>
<th>( P )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( N )</td>
<td>PB</td>
<td>PM</td>
<td>PS</td>
</tr>
<tr>
<td>( Z )</td>
<td>NS</td>
<td>ZE</td>
<td>NS</td>
</tr>
<tr>
<td>( P )</td>
<td>NB</td>
<td>NB</td>
<td>NB</td>
</tr>
</tbody>
</table>

According to the above form, use the fuzzy calculation method introduced in [12] and gravity method to turn fuzzy output into precise control quantity

\[
\Delta k_i = \left( \int \Delta k_i \, \tilde{\mu}_{\Delta k_i} \, d\Delta k_i \right) \left( \int \tilde{\mu}_{\Delta k_i} \, d\Delta k_i \right)^{-1}
\]

when

1. \( \sigma_n \leq -\frac{1}{3} \), \( \bar{\sigma}_n \leq -\frac{1}{3} \); it is easy to get \( \Delta k_i = 1 \),

and when

2. \( \sigma_n \leq -\frac{1}{3} \), \( \bar{\sigma}_n \leq 0 \): \( \sigma_n \), \( \bar{\sigma}_n \), \( \sigma_n \) (N), \( \bar{\sigma}_n \) (N,Z).

The subordinate function of \( \Delta k_i \) (PB, PM) corresponding to is shown in [13].

Thus, points \( P_1 \) and \( P_3 \)'s abscessa are \( \bar{\sigma}_n + \frac{2}{3} \), \( \bar{\sigma}_n + 0 \);

\( \bar{\sigma}_n \) and \( \bar{\sigma}_n \)'s abscessa are \( -\bar{\sigma}_n + \frac{2}{3} \), \( -\bar{\sigma}_n + \frac{4}{3} \); then

\[
\Delta k_i = \frac{5}{2} \sigma_n^2 - \frac{7}{6} \sigma_n + \frac{2}{9}
\]

Using the same method we get the precise output \( \Delta k_i \) under other circumstances to be

for \( i = 1 \), \( \sigma_n \) is \( \sigma_n \) sign(\( x_i \) - \( T_{K_i} \) - \( R_{K_i} \)) and \( \bar{\sigma}_n \) is \( \bar{\sigma}_n \) sign(\( x_i \) - \( T_{K_i} \) - \( R_{K_i} \)); for \( i = 2 \), \( \sigma_n \) is \( \sigma_n \) sign(\( x_i \)) and \( \bar{\sigma}_n \) is \( \bar{\sigma}_n \) sign(\( x_i \)). Finally, the control function of FLMSFC approach for simulate is obtained as
\[ U = U_{eq} + k_i(x_i - T_K - r K_i) + \sum \Delta K_i x_i + K_a(x_a - T_a - r K_a) + \sum \Delta K_a x_a \] \hspace{1cm} (16) \]

Among them, \( U_{eq} \) is given by (5), \( k_i \) is given by inequality (11), \( \Delta K_i \) is given by (15), therefore \( U \) is a continuous function.

IV. DC SERVOMOTOR MODELING

The DC servomotor considered in the paper is a permanent magnet DC motor as shown in Fig. 6. The voltage and electromagnetic torque equation can be expressed as:

\[ V_a = R_a i_a + L_a \frac{di_a}{dt} + V_s \] \hspace{1cm} (17)

\[ V_b = K_i o_a \] \hspace{1cm} (18)

\[ T_n = K_i i_a(t) \] \hspace{1cm} (19)

The torque, velocity and position can be related by:

\[ T_n = J_a \frac{d\omega_n}{dt} + B_m \omega_n + T_L \] \hspace{1cm} (20)

\[ \omega_n = \int \omega_a dt \] \hspace{1cm} (21)

where \( i_a \) is the armature current, \( R_a \) is the armature resistance, \( L_a \) is the armature inductance, \( K_i \) is the back-EMF constant, \( K_b \) is the back-EMF constant, \( J_a \) is the inertia of the motor rotor, \( B_m \) is the viscous coefficient, \( V_s \) is the armature voltage, \( V_a \) is the back-EMF, \( T_n \) is the motor torque, \( T_L \) is the load torque, \( \omega_n \) is the motor velocity, \( \omega_a \) is the motor displacement.

\[ \hat{x}_1 = x_2, \quad \hat{x}_2 = x_3 \text{ and } \hat{x}_3 = -a_1 \hat{x}_2 - a_2 \hat{x}_3 - a_3 x_3 + b_1 U_{eq}. \] \hspace{1cm} (23)

Defining \( e = x_n - x_{eq} \): (1, 2, 3), the FLSMFC system can be represented as \( \dot{e}_1 = e_2, \quad \dot{e}_2 = e_3 \) and

\[ \begin{align*}
\dot{e}_1 &= -a_1 e_1 - a_2 e_2 - a_3 e_3 + (a_1 - a_3) x_3 \\
&\quad + (a_2 - a_3) x_2 - (a_3 - a_3) x_3 - b_1 U_{eq} + b_2 U_p - f(t). 
\end{align*} \] \hspace{1cm} (24)

Following the design procedure we have the control law to implement as

\[ U_p = e_1 (K_i x_a + a_1' e_1 + a_2' e_2 - a_3' x_3) \] \hspace{1cm} (25)

\[ + (a_2 - a_3) x_2 + (a_3 - a_3) x_3 + b_1 U_{eq} + b_2 U_p - f(t). \]

The switching function, \( \sigma \) from (2), is

\[ \sigma = (e_1 - K_i x_a) \] \hspace{1cm} (26)

V. THE FLSMFC FOR DC SERVOMOTOR DRIVES

The implementation of FLSMFC system is shown in Fig. 8. The nominal values of the FLSMFC controller and machine parameters are listed in Table 2 and Table 3, respectively. The simplified dynamic model of the DC servomotor position control can be described as

\begin{center}
\textbf{TABLE II}

\begin{tabular}{|c|c|}
\hline
\textbf{Parameter} & \textbf{Value} \\
\hline
\hline
\( A_1, A_2 \) & -15.834, 18.749i \\
\( A_3, A_4 \) & -32.518, -14.723 \\
\( C_1, C_2 \) & 1.525, 58.72 \\
\( K_1, K_2 \) & 28.37, 18.83 \\
\( \phi_1, \phi_2 \) & -1, -0.01 \\
\( \phi_3, \phi_4 \) & -0.0001, -0.001 \\
\( a_{11}, a_{12} \) & 12.000, 1.220 \\
\( a_{21}, b_{m} \) & 42, 13,000 \\
\( a_{22}, a_{33} \) & 9, 143.74, 2.248 \\
\( b_{p} \) & 50, 838.39 \\
\( \delta_{1}, \delta_{2} \) & 2, 20 \\
\hline
\end{tabular}
\end{center}
### TABLE III

**PARAMETERS OF DC SERVOMOTOR**

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Dimension</th>
</tr>
</thead>
<tbody>
<tr>
<td>$R_a$</td>
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<td>$\Omega$</td>
</tr>
<tr>
<td>$L_a$</td>
<td>0.012</td>
<td>H</td>
</tr>
<tr>
<td>$K_a$</td>
<td>6.5</td>
<td>dimensionless</td>
</tr>
<tr>
<td>$g_t$</td>
<td>5</td>
<td>dimensionless</td>
</tr>
<tr>
<td>$B_m$</td>
<td>0.00</td>
<td>N-m/s</td>
</tr>
<tr>
<td>$J_m$</td>
<td>0.0014</td>
<td>Kg-m$^2$</td>
</tr>
<tr>
<td>$K_h$</td>
<td>0.43</td>
<td>V-s/rad</td>
</tr>
<tr>
<td>$K_t$</td>
<td>0.43</td>
<td>N-m/A</td>
</tr>
</tbody>
</table>

### VI. EXPERIMENT RESULTS AND DISCUSSION

The experimental results of the dynamic response are shown in Fig. 9, where a ramp command is introduced and the motor is applied with a variable load torque and parameters variation. The results are compared with obtained from the IVSMFC and MIVSC approaches, respectively. Fig. 10, compares the position tracking errors. It is clear from the curves that FLSMFC can track the command input extremely well during steady state as well as transient periods. Among others, the FLSMFC approach gives the minimum tracking error and fast accurate tracking.

![FLSMFC Tracking Error and Control Signal](image)

![MIVSC Tracking Error and Control Signal](image)

![IVSMFC Tracking Error and Control Signal](image)

Fig. 9 Comparison of ramp position tracking.

Fig. 10 Comparison of position tracking error and control signal.

### VII. CONCLUSIONS

This paper has presented the FLSMFC configuration for a DC servomotor driver. A design procedure has also been proposed for determining the control function and switching plane by using model following control. The application of FLSMFC to the DC servomotor position control system has illustrated that the FLSMFC approach can improve the tracking performance by 75% and 85% when compared to the MIVSC and IVSMFC approaches, respectively. Experimental results demonstrate that the proposed approach can achieve accurate position tracking in face of wide plant parameter variations and external load disturbance. Furthermore, The FLSMFC approach is robust and applicable to electrical machine control systems. It is therefore expected that the proposed control scheme can be applied to the high performance applications of an induction
motor such as the high precision control of machine tools and industrials.

REFERENCES


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