Multi-criteria Optimization of Square Beam using Linear Weighted Average Model

Ali Farhaninejad, Rizal Zahari, Ehsan Rasooliyazdi

Abstract—Increasing energy absorption is a significant parameter in vehicle design. Absorbing more energy results in decreasing occupant damage. Limitation of the deflection in a side impact results in decreased energy absorption (SEA) and increased peak load (PL). Hence a high crash force jeopardizes passenger safety and vehicle integrity. The aims of this paper are to determine suitable dimensions and material of a square beam subjected to side impact, in order to maximize SEA and minimize PL. To achieve this novel goal, the geometric parameters of a square beam are optimized using the response surface method (RSM), multi-objective optimization is performed, and the optimum design for different response features is obtained.

Keywords—Crashworthiness, side impact, energy absorption, multi-objective optimization, Square beam, SEA

I. INTRODUCTION

GLOBAL accident statistics demonstrate that nearly 30% of accidents and 35% of fatalities are caused by side impact [1, 2]. Side impact is more significant than frontal impact due to the reduced crash zone.

For this reason thin-walled structures is increasingly used and a lot of research work has been carried out in past decades on the energy absorption of thin-walled structures under loading [3-10]. Kecman conducted experimental and theoretical analysis of the bending performance of rectangular beams. Niknejad [11] studied the fold creation in square columns under axial loading. The effect of web corrugation under bending was investigated by C. L. Chan et al [12]. However, they have not considered the side impact on a square beam. Most of the research has analyzed the axial crash of a square beam but neglected the lateral crash of a square beam, which is analyzed in this research. Langseth et al. [13, 14] studied local buckling and the crush behavior of square beams.

Finding the optimum point, considering maximum SEA and minimum PL with respect to their simultaneous limitation of deflection, is a major challenge. This optimum design point is critically important for vehicle components subjected to side impact. Meanwhile, a conflict between the criteria for these objectives is inevitable.

This paper aims to present optimization method to find the optimum point. The modeling, meshing and crash analysis were done using the LS-DYNA suite of programs, and at a crash speed of 5 m/s.

The thickness of the square beam is 1 mm. Figure 1 shows the dimensions of the structure and the condition of the impactor.

Fig. 1 Mapping nonlinear data to a higher dimensional feature space

Two steps in this research are considered. In the first step, the effects of steel and aluminum alloys are investigated to find the maximum SEA with reasonable deflection. In the second step, to choose the optimum structure design, the optimization method is investigated. This optimum design should result in the maximum SEA and minimum impact force simultaneously, considering the limitations of deflection.

II. SPECIFIC ENERGY ABSORPTION
The energy $E$ which is absorbed by the objects during the collision can be obtained from the following Equations:

$$E = \int A(\varepsilon)d\varepsilon$$

where $A(\varepsilon)$ implies the total strain energy density of the corresponding structure. The specific energy absorption (SEA), which is the energy absorbed per unit mass of the structure part, can be defined by:

$$SEA = \frac{E_{total}}{M}$$

where $E_{total}$ is the total energy and $M$ is the mass of the corresponding structure under impact

III. FINITE ELEMENT MODELING
The CAD data of the square beam is modelled, meshed and simulated using LS-DYNA 3.1 Beta software from LSTC Co. In the analysis, the square beam is constrained with a rigid wall on one side, while the other side is impacted by a rigid wall of 10 kg mass moving with a constant velocity of 5 m/s.
The four-node quadrilateral element (Belytschko-Tsay) is chosen because of its appropriate application in shell elements with the formulation of 3 integration points to mesh the model [15].

IV. MATERIALS PROPERTIES

The properties of aluminium, steel and magnesium are assigned to the square beam. The mechanical properties of the materials are given in Table I.

### TABLE I

<table>
<thead>
<tr>
<th>Material types</th>
<th>E (GPa)</th>
<th>Poisson's Ratio</th>
<th>Yield stress(MPa)</th>
<th>Ultimate stress(MPa)</th>
<th>Strain at failure</th>
<th>Density (kg/m³)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Aluminium 3105-H18</td>
<td>68.9</td>
<td>0.33</td>
<td>193</td>
<td>214</td>
<td>0.03</td>
<td>2720</td>
</tr>
<tr>
<td>Magnesium AZ31B</td>
<td>45</td>
<td>0.35</td>
<td>190</td>
<td>275</td>
<td>0.1</td>
<td>1740</td>
</tr>
<tr>
<td>Steel AISI1006</td>
<td>200</td>
<td>0.3</td>
<td>190</td>
<td>320</td>
<td>0.3</td>
<td>7860</td>
</tr>
</tbody>
</table>

Fig. 2 shows the lateral deflection for the square beam made of different materials. The maximum deflection occurs for aluminium 3105 with deflection of 16 mm and 5 mm for aluminium 2011. The minimum deflection occurs to the steel due to its high rigidity compared with the aluminium alloys. Fig. 3 shows a comparison of the SEA for each material. It can be seen that the maximum SEA occurs with aluminium 3105, which is about 5.47276 (N.mm/ton E+8). However, the amount of deflection for aluminium 3105 is high. Thus, aluminium 2011 is a good choice considering the less deflection compared with aluminium 3105. In addition, the amount of SEA for aluminium 2011 is about 5.15484 (N.mm/ton E+8) which is reasonable. Fig. 4 shows impact forces for these three materials. It is observed that the maximum force belongs to the steel which is about 320434 N.

V. THE EFFECT OF MATERIALS ON CRASHWORTHINESS

VI. PERFORMANCE OF SQUARE BEAM FOR DIFFERENT GEOMETRY

In this step the effect of changing geometry such as thickness and dimension of square beam on the two parameters of SEA and PL are investigated. The results are shown in Table II.

### TABLE II

<table>
<thead>
<tr>
<th>L1 (mm)</th>
<th>L2 (mm)</th>
<th>T (mm)</th>
<th>Mass (kg)</th>
<th>E-4</th>
<th>E+8 (N.mm/ton)</th>
<th>PL (N)</th>
</tr>
</thead>
<tbody>
<tr>
<td>50</td>
<td>50</td>
<td>1</td>
<td>2.26</td>
<td>5.15484</td>
<td>224945</td>
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<tr>
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<td>50</td>
<td>0.8</td>
<td>1.81</td>
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<td>50</td>
<td>0.6</td>
<td>1.35</td>
<td>8.68824</td>
<td>154678</td>
<td></td>
</tr>
<tr>
<td>50</td>
<td>50</td>
<td>0.4</td>
<td>0.95</td>
<td>12.7332</td>
<td>108661</td>
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<tr>
<td>46</td>
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<tr>
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<tr>
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<tr>
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<tr>
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<tr>
<td>38</td>
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<td>0.4</td>
<td>0.688</td>
<td>16.4822</td>
<td>101925</td>
<td></td>
</tr>
</tbody>
</table>

VII. OPTIMIZATION PROBLEM DESCRIPTION

Structural optimization techniques have been used recently for optimizing the energy absorption and peak load of structures under impact.
There are a number of methods for optimization. The response surface method (RSM) is one of the methods most commonly used for crashworthiness optimization [16-20]. Yamazaki, Lee [21] and Allahbakhsh [22] have applied an RSM method for crashworthiness optimization. In this paper, for optimizing specific energy absorption and peak load, multi-objective optimization is applied. In the present paper, RSM as described by [23] is used and is described in this section.

Multi-objective optimization can be formulated in two different ways, one of which is the linear weighted average as given in Equation (3):

\[
\min F_w = (1-w)f_1^* + w f_2^*
\]

(3)

where \( f_1^*, f_2^* \) are the normalizing values of \( f_1 = \text{SEA}(x) \) and \( f_2 = \text{PL}(x) \) respectively [24-26]. \( w \) is the weight factor for emphasizing the different importance of each of the objectives.

VIII. RESPONSE SURFACE MODEL

In this paper the second order polynomial function is used for SEA(x) and PL(x) and these can be expressed as Equations (4) and (5) respectively.

\[
\text{SEA} = 1.079 \times 10^9 + 1.8 \times 10^8 x(1) - 7.953 \times 10^7 x(2) - 3.77 \times 10^6 x(1)^2 + 1.55 \times 10^5 x(1) x(2) + 6.96 \times 10^4 x(2)^2
\]

(4)

\[
\text{PL} = 5.39 \times 10^5 - 2.06 \times 10^5 x(1) - 4.614 \times 10^4 x(2) + 2125 x(2)^2
\]

(5)

Where \( x(1) \) and \( x(2) \) are the dimension and thickness of structure respectively.

IX. DESIGN OPTIMIZATION RESULT

By varying weight \( w \) in Equation (3), the Pareto sets for the square beam are obtained as plotted in Fig. 4. The Pareto front provides a range of optimal solutions. The Pareto plot shows the relation between SEA and PL and any further improvement in SEA must sacrifice the PL and vice versa. In fact, any point in the Pareto frontier can be an optimal point, meaning that it is up to the designer to determine which factor is more important. For generating the Pareto frontier, the Genetic Algorithm (GA) multi-objective optimization solver of MATLAB is used.

From the results obtained and the discussion presented, the following conclusions are made:

- Analyzing the effect of material on crashworthiness leads to choose aluminum 2011 due to its reasonable SEA and deflection compared to steel and aluminum 3105.

- The multi-objective Pareto graph enables the designer to make a better decision on the design point. Having various optimum points based on two contrary objectives (SEA, PL) enables the designer to have a group of solutions to find the optimum point, which is considered to be the maximum SEA and minimum PL with respect to deflection.

REFERENCES


