Calculation of the Forces Acting on the Knee Joint When Rising from Kneeling Positions
(Effects of the Leg Alignment and the Arm Assistance on the Knee Joint Forces)

S. Hirokawa, M. Fukunaga, and M. Mawatari

Abstract—Knee joint forces are available by in vivo measurement using an instrumented knee prosthesis for small to moderate knee flexion but not for high flexion yet. We created a 2D mathematical model of the lower limb incorporating several new features such as a patello-femoral mechanism, a thigh-calf contact at high knee flexion and co-contracting muscles' force ratio, then used it to determine knee joint forces arising from high knee flexions in four kneeling conditions: rising with legs in parallel, with one foot forward, with or without arm use. With arms used, the maximum values of knee joint force decreased to about 60% of those with arms not used. When rising with one foot forward, if arms are not used, the forward leg sustains a force as large as that sustained when rising with legs parallel.

Keywords—Knee joint force, kneeling, mathematical model, biomechanics.

I. INTRODUCTION

The relationship between knee forces and physical activity is becoming increasingly important in understanding joint injuries and diseases, evaluating treatment outcomes, planning rehabilitation programs and designing more durable Total Knee Arthroplasty (TKA).

At present, knee joint forces are determined either by direct measurement using an instrumented knee prosthesis [1], [2] or through mathematical modeling, i.e. inverse dynamics [3]-[6]. The advent of instrumented knee prostheses has made it possible to measure knee joint force in vivo; however, the in vivo direct measurement data mainly concerns forces generated by small to moderate flexion. Data from instrumented prostheses about forces at high flexion are not yet available. A comparison of data from mathematical models reveals significant differences in predicted knee joint forces. The causes of these differences remain unknown; however, once the data are published, they serve as the current gold standard for determining the possible cause of the differences.

II. MATHEMATICAL MODEL AND EXPERIMENT

A. Mathematical Model

To measure the kinetics and kinematics required to kneel, it would be necessary to look at all the different ways to move from a standing to a kneeling position and back. However, formulating all the ways is difficult. Instead, we decided to assess forces on the knee joint when rising from a kneeling position in four different ways (Fig.1). To avoid confusion between the expressions "kneeling" and "deep squatting", we will use, "kneeling" to refer to the act of sitting on a floor with one or both knees touching the floor surface (Fig.1).

Our 2D mathematical model is composed of three segments: upper leg, lower leg and foot. The muscle groups incorporated into our model are shown in Fig.2. The forces acting on the hip, knee and ankle joints are illustrated in Fig.3 with the variables representing the tensile forces generated by the muscles and tendons.

We incorporate three features into the Dahlkvist et al. model [3]: a patello-femoral mechanism, a thigh-calf contact at high knee flexion and co-contracting muscles' force ratio. First, we introduce the ratio between the force in the quadriceps, \( Q \), and the force in the patella tendon, \( Q' \) as a function of knee flexion angle, as described in the literature [4]. We set the directions of pull on the quadriceps, \( Q \) and the patellar tendon, \( Q' \) as a function of knee angle based on the patello-femoral mechanism [8], [9]. When knee angle exceeds 125°, we take the...
thigh-calf contact force, \( P \) and the position, \( d \) of this contact force after Zelle et al. [10]. Per our definition of kneeling above, we incorporate the contact force between the knee and the floor exerted when the knee is touching the floor; \( N_z \) and \( N_x \) represent the normal and tangential components of the floor reacting force on the knee respectively.

The variables in Fig.3 mean as follows.

The variables representing the tensile force generated by the muscles and tendons:

- \( H: \) hamstrings, \( GM: \) glutaeus, \( R: \) rectus femoris, \( V: \) the vasti, \( Q: \) quadriceps (the vector sum of \( R \) and \( V \)), \( Q': \) patella tendon, \( GAS: \) gastrocnemius, \( S: \) soleus.

The variables representing external forces:

- \( W_{1,2,3}: \) the gravity force acting on the thigh, shank and foot, respectively
- \( F_Z, F_S: \) the normal and tangential components of the floor reacting force respectively

The variables representing moment:

- \( M_h: \) about the hip joint
- \( M_k: \) about the knee joint
- \( M_a: \) about the ankle joint

In the following equations, the symbols \( a, b \) and \( c \) stand for the lengths of the moment arm about the hip, the knee and the ankle joints respectively. Thus \( a_{r_z} \) means the moment arm of \( F_z \) about the hip joint, and \( b_{r} \) means the moment arm of thigh-calf contact force \( P \) about the knee joint; the equivalent of variable \( d \) above.

Moment \( M_h \) created by external forces is expressed as,

\[
M_h = F_z a_{r_z} + F_s a_{r_s} - W_z a_{w_z} - W_s a_{w_s} - W_a a_{w_a}, \quad (1)
\]

Moment \( M_k \) created by muscle forces is expressed as,

\[
M_k = GM a_{GM} + H a_h - R a_s + N_z a_{s_z} + N_x a_{s_x}, \quad (2)
\]

Since the values from equations (1) and (2) must be equivalent to each other, we can eliminate \( M_k \), which gives the following equation,

\[
F_z a_{r_z} + F_s a_{r_s} - W_z a_{w_z} - W_s a_{w_s} - W_a a_{w_a} - GM a_{GM} - H a_h + R a_s - N_z a_{s_z} - N_x a_{s_x} = 0 \quad (3)
\]

The equation for the knee joint is,

\[
-F_z b_{r_z} + F_s b_{r_s} + W_z b_{w_z} + W_s b_{w_s} - GAS b_{GM} + Q' b_q - H b_h - N_z b_{s_z} - N_x b_{s_x} + P b_r = 0 \quad (4)
\]

Fig. 1 Ways of rising from various kneeling conditions
(a) rising with legs parallel (arms not used), (b) rising with one foot forward (arms not used), (c) rising with legs parallel (arms used), (d) rising with one foot forward (arms used)

Fig. 2 Illustration of the muscles included in the model. H: hamstrings, GM: glutaeus, R: rectus femoris, V: the vasti, GAS: gastrocnemius, S: soleus. The same symbols in Italic shown in Fig.3 will be the variables representing the forces exerted by the respective muscles, thus \( GM \) stands for the forces exerted by GM and so on

Fig. 3 Two dimensional mathematical models for the moments around the hip (left), knee (center) and ankle (right) joints.
and for the ankle joint as,

\[ F_{z}c_{n} + F_{z}c_{n} - Wc_{n} - (GAS + S)c_{n} = 0 \]  

(5)

The three equations (3), (4) and (5) contain six variables, i.e. six muscle forces: \( GM, H, R, GAS, Q' \) and \( S \). To solve this indeterminate equation, it is necessary to decrease the number of variables from six to three. To do this we will assume, as Dahlkvist et al. [3] did that a moment that tends to extend the hip is shared by the gluteal muscles \( GM \) and the hamstrings \( H \), a moment that tends to flex the ankle dorsally is shared by the soleus \( S \) and the gastrocnemius \( GAS \), and the four muscles in the quadriceps are active simultaneously. Dahlkvist et al. [3] set the moment at any joint is shared equally by the muscles involved. On the other, we set the moment is shared by the muscles’ forces according to each muscle’s PCSA since the muscle forces are known to be in proportion to their PCSAs [11, 12]. Also, the PCSAs of four muscles in the quadriceps are almost equal [11, 12], the forces exerted by each one of these muscles would be one-quarter of the total quadriceps force. Thus, \( R : V = 3 : 1 \). By considering the force triangle composed of \( R, V \) and \( Q \), we can calculate the force \( Q \) (see Fig.3).

From equations (3) through (5) and the muscles’ force ratios above, we can introduce the muscle forces acting on the hip, knee and ankle joints respectively. We can then use the values for muscle forces around the knee joint to introduce the forces acting on the knee joint.

**B. Experiment**

Ten healthy males: (age 26±4 years, height 175.1±5.5 cm, and weight 76.6±21.1 kg) and five healthy females (25±3 years, 160.1±7.1 cm, 47.7±6.2 kg) participated in the measurement experiment. Before the experiment, we obtained the approval of the Saga University ethics committee and informed consent from all subjects. To obtain the physical parameters, the length of each subject’s upper leg, lower leg and foot was measured directly. The mass of each segment and its center of gravity were determined by referring to the literature [14].

First, three subjects who have similar builds performed activities requiring small/middle knee flexion: standing on one leg, level walking, ascending and descending stairs, and knee bending. Ground reaction force data and the angles of each joint during the motions were collected by a force plate walkway (Model BP400600, AMTI Co., USA) and a video recording system (Vicon Motion Systems, Vicon Co., UK) respectively. Next, the same subjects performed an activity requiring high knee flexion: rising from a squatting position with legs in parallel. As four more plates were installed on the midway of the walkway, a subject was able to place his/her right and left hands, knees and feet on six individual plates respectively. Finally, all fifteen subjects performed the four different rising motions depicted in Fig.1.

In all the measurements above, the subjects repeated each activity three times, and the three sets of data were averaged. The muscle and joint forces were then calculated through our 2D mathematical model.

**III. RESULTS**

First, using the data from the above three subjects, the maximum values of the net resultant force (the vector sum of \( F_{z} \) and \( F_{x} \) in Fig.3) on the knee joint of one leg during small/middle knee flexion activities were compared with the in vivo data [1, 2] (Table I).

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Secondly, the maximum knee joint forces when rising from a squatting position were compared with predictions reported in the literature [3, 5, 6] (Table I).

Thirdly, the maximum knee joint force from all fifteen subjects when rising from kneeling with legs parallel with arms not used (Fig. 1(b)) was compared with the prediction by Nagura et al. [4] (Table I). After that, knee joint forces when rising from a kneeling position were graphed as follows: with legs in parallel (Fig. 4) and with one foot forward (Fig. 5). In

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TABLE I

<table>
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<tr>
<th>Approach</th>
<th>Authors</th>
<th>level walking</th>
<th>stair descent</th>
<th>stair ascent</th>
<th>knee bend</th>
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<td>—</td>
<td>3</td>
<td>2.1</td>
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<td></td>
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<td>3.06</td>
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<td>3.46</td>
<td>3.16</td>
<td>2.53</td>
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<td>—</td>
<td>—</td>
<td>4.6 ~ 5.2</td>
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<tr>
<td></td>
<td>Zheng (1998)</td>
<td>—</td>
<td>—</td>
<td>—</td>
<td>4.3</td>
<td>—</td>
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<td></td>
<td>Smith (2008)</td>
<td>—</td>
<td>—</td>
<td>—</td>
<td>3.73(0.56)</td>
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<tr>
<td></td>
<td>Nagura (2006)</td>
<td>—</td>
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<td>—</td>
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<td>7.3(1.9)</td>
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<td>2.5 ~ 2.8</td>
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<td>3.1 ~ 3.4</td>
<td>2.2 ~ 2.3</td>
<td>3.5 ~ 4.1</td>
<td>4.1 ~ 4.5</td>
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</table>

They described "kneel bend" as "squatting". ††mean (SD)
Figs 4 and 5, the curves of the mean and standard deviations for the variations in knee joint forces are drawn as a function of knee angle, because the time when rising began and the duration from start to finish varied from subject to subject. In Fig.6, the graphs of variation in forces on the forward leg and that on the trailing leg are shown separately in order avoid the impression that both legs share the forces over the same knee angle. In Fig.6, for the sake of clarity, the standard deviation curves are drawn in terms of knee angles or joint forces, depending on whether an inclination in the graph is steep or not.

Finally, the numerical values of the maximum knee joint forces acting on a single knee were tabulated, as were the knee angles at which those forces are exerted when rising from various kneeling conditions (Table II). Note that the maximum values in Table II differ from the maximum values of the mean curves in Fig. 4 and 5, because the mean curves were created from individual curves, per Acker et al. [15].

IV. DISCUSSION

Although studies on knee joint kinetics and kinematics are extensive, there is still little data on high knee flexion. We created a mathematical model of the lower limb and used it to analyze knee joint forces for all types of flexion from small to high.

Despite the current notion that large variations exist among the reported knee joint forces, our results for small/middle knee flexion did not differ significantly from the in vivo data (Table I). Our results for rising from a squatting position did not differ significantly from the literature data [5], [6] either but Dahlkvist et al.’s [3] (Table I). Zheng et al. [6] produced their results from their detailed 3D model which incorporated micro and macro structures of the knee. Although Smith et al.’s 2D model [5] was rather simple, they applied a unique scaling method to their analysis. As already mentioned, we added three features to Dahlkvist et al.’s model [3]. By removing these three features one at a time and recalculating the forces each time, we found that the main cause of the difference was the method used to set the muscles’ force ratio. When we set the ratios back to the Dahlkvist et al.’s [3], then the force values became equivalent to theirs.
When rising from a kneeling with legs parallel with arms not used (Fig.1(b)), we assessed the maximum knee joint force as 4.5 times Body Weight [BW]. On the other hand, Nagura et al. [4] using a simple 2D model of the knee joint, reported a significantly high value as 7.3 ± 4 [BW]. Using the principal component analysis (PCA) and the forward leg, we assessed the maximum knee joint force as 146°, the tibio-femoral surfaces do not maintain a complete articulation (subluxation) [16] and therefore the knee joint would not sustain such a large force as they reported. Another question is that they determined the force ratio between the extensor and flexor muscles groups from each group's EMG data by simplifying each of the extensor and flexor muscles to only one string respectively. It is doubtful whether the EMG data could be used to predict the force of a group of many muscles with different lengths and insertions. In view of these weaknesses in their study, we consider their predictions of the maximum knee joint force are impractically large. By factoring in each mathematical reason and recalculating the forces each time, we found several possible reasons for the large variations among the reported predictions did not have decisive influence on the results. We found a patellar-femoral mechanism had a little effect on the prediction of knee joint force. The thigh-calf contact had little effect. Actually the value of the thigh-calf contact force itself is reported to be less than 0.5 [BW] [10]. In predicting knee joint force, we found one of the most influential factors was the method how to determine the co-contracting muscles' force ratio to address the indeterminate problem. To this respect many optimization techniques have been reported. Yet, the choice of optimization criteria depends on researchers and its validation is indirect. Brand et al. [11] had already mentioned that the optimization criteria have only a small influence on the calculated joint contact forces. Besides optimization techniques, a substantial criterion is needed for determining the muscles' force ratio. Determining it according to each muscle's PCSA could be one of the solutions as shown in this study. Kumamoto et al. [17] claimed the bi-articular muscle function could fill the role to solve the indeterminate problem. If their claim is correct, future models should incorporate Kumamoto et al.'s idea [17] in order to introduce further accurate knee joint forces. Although various problems still remain in model analyses, we believe our mathematical predictions for knee joint force are reasonable. We provided the reasons to some literature data why they are extreme, thereby contributing to decrease significant differences in predicted knee joint forces. From Figs 4 and 5 and Table II, we know how the knee joint forces differ, depending on the alignment of the leg and/or on whether the arms are used or not. Overall, with arms used, the maximum values of knee joint force decreased to about 60% of those with arms not used (Fig. 4, Fig. 5(a), (c), Table II). When rising with one foot forward, if arms are not used, the forward leg sustains a force as large as that sustained when rising with legs parallel (Fig.4, Fig.5(a), Table II). The maximum force on the trailing leg does not change with arms used or not (Fig.5(b), (d)). The results concern the influence of the legs' alignment and the arms' assistance on the joint force should be of use in rehabilitation and the design of TKA.

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