Abstract—This paper presents modeling and analysis of 12-phase distribution static compensator (DSTATCOM), which is capable of balancing the source currents in spite of unbalanced loading and phase outages. In addition to balance the supply current, the power factor can be set to a desired value. The theory of instantaneous symmetrical components is used to generate the twelve-phase reference currents. These reference currents are then tracked using current controlled voltage source inverter, operated in a hysteresis band control scheme. An ideal compensator in place of physical realization of the compensator is used. The performance of the proposed DSTATCOM is validated through MATLAB simulation and detailed simulation results are given.

Keywords—DSTATCOM, Modeling, Load balancing, Multi-phase, Power factor correction.

I. INTRODUCTION

MULTI-PHASE (phase order higher than three) system: such as electrical generation, transmission and drives are the focus of research recently due to their inherent advantages compared to their three-phase counterparts [1]-[6].

Multi-phase inverter fed induction motor drives has been found to be quite promising for high power ratings and other specialized applications. The use of such drives and devices presents multi-phase load circuits that may get subjected to phase outages and unbalanced as well as non-linear loading similar to their three-phase counter parts. Such conditions may lead to variety of undesirable effects on the supply system such as additional losses in connecting lines and interfacing devices, oscillatory torques in ac machines, saturation in transformers, increased ripple in rectifiers, malfunctioning in sensitive equipments, harmonic and excessive neutral currents etc. It is therefore desired to have the balanced power system operation with minimum lower order harmonics even in the presence of such operations. A number of methods based on the instantaneous reactive power theory [7]-[9], theory of symmetrical components [10]-[15] and reference frame theory [16]-[18] have been evolved for the compensation of harmonics and unbalances for three phase system. However, instantaneous symmetrical components theory is preferred. Based on this theory the compensator reference currents are generated, which are then realized by hysteresis band control of the inverter circuit and are injected at point of common coupling. The inverter circuit has 12 bridge arms connected to a split capacitor, controlled by chopper. It has been shown that the instantaneous power in the electrical system containing an unbalanced load has an oscillating component that ride a dc value [10]. The objective of the compensator is to supply zero mean oscillating power such that the dc component can be supplied by the source. This paper introduces modeling and analysis of twelve-phase DSTATCOM based on instantaneous symmetrical components for balancing the unbalanced multi-phase load and power factor correction on supply side. The detailed simulation results are given and discussed for verification.

II. THE PROPOSED TWELVE-PHASE COMPENSATOR

A. PRINCIPLE OF DSTATCOM

The basic compensation scheme for 12-phase (multi-phase) load supplied from a balanced stiff multi-phase source is shown in Fig. 1

![Fig. 1 Schematic diagram of the compensator for star connected load (12-phase 13-wire system)](image)

In this scheme, the compensator represented by current sources is connected in shunt with the loads. The goal of the
scheme is to extract the compensator currents,

\[ i_{cp} (p = a, b, c, d, e, f, g, h, i, j, k & l) \]

denoted by

\[ i_{cp}^* (p = a, b, c, d, e, f, g, h, i, j, k & l) \]

from the measure circuit variables, which make the unbalanced loads on the supply side balanced. The proposed scheme can be applied to a 12-phase, 12-wire system or a 12-phase, 13-wire system by isolating or connecting the common points \( N \) or \( n \) respectively. The reference currents are generated using the theory of the instantaneous symmetrical components [11] and these currents by inclusion of loss term \( P_{loss} \) in the power are given as follows:

\[ i_{el}^* = i_{el} - \frac{\{v_{sa} + \xi (v_{sa} - v_{ad})\}}{4(v_{sa}^2 + v_{se}^2 + v_{si}^2)} (P_{lavg} + P_{loss}) \]  
\[ i_{eh}^* = i_{eh} + \frac{\{v_{sh} + \xi (v_{sh} - v_{sd})\}}{4(v_{sa}^2 + v_{se}^2 + v_{si}^2)} (P_{lavg} + P_{loss}) \]  
\[ i_{ec}^* = i_{ec} + \frac{\{v_{sl} + \xi (v_{sl} - v_{se})\}}{4(v_{sa}^2 + v_{se}^2 + v_{si}^2)} (P_{lavg} + P_{loss}) \]  
\[ i_{ed}^* = i_{ed} - \frac{\{v_{sd} + \xi (v_{sd} - v_{se})\}}{4(v_{sa}^2 + v_{se}^2 + v_{si}^2)} (P_{lavg} + P_{loss}) \]  
\[ i_{ef}^* = i_{ef} + \frac{\{v_{sn} + \xi (v_{sn} - v_{sd})\}}{4(v_{sa}^2 + v_{se}^2 + v_{si}^2)} (P_{lavg} + P_{loss}) \]  
\[ i_{es}^* = i_{es} + \frac{\{v_{sl} + \xi (v_{sl} - v_{se})\}}{4(v_{sa}^2 + v_{se}^2 + v_{si}^2)} (P_{lavg} + P_{loss}) \]  
\[ i_{eh}^* = i_{eh} - \frac{\{v_{sh} + \xi (v_{sh} - v_{sd})\}}{4(v_{sa}^2 + v_{se}^2 + v_{si}^2)} (P_{lavg} + P_{loss}) \]  
\[ i_{ec}^* = i_{ec} - \frac{\{v_{sl} + \xi (v_{sl} - v_{se})\}}{4(v_{sa}^2 + v_{se}^2 + v_{si}^2)} (P_{lavg} + P_{loss}) \]  
\[ i_{ed}^* = i_{ed} + \frac{\{v_{sd} + \xi (v_{sd} - v_{se})\}}{4(v_{sa}^2 + v_{se}^2 + v_{si}^2)} (P_{lavg} + P_{loss}) \]  
\[ i_{ef}^* = i_{ef} - \frac{\{v_{sn} + \xi (v_{sn} - v_{sd})\}}{4(v_{sa}^2 + v_{se}^2 + v_{si}^2)} (P_{lavg} + P_{loss}) \]  
\[ i_{es}^* = i_{es} - \frac{\{v_{sl} + \xi (v_{sl} - v_{se})\}}{4(v_{sa}^2 + v_{se}^2 + v_{si}^2)} (P_{lavg} + P_{loss}) \]

where \( \xi = \tan \frac{\phi}{\sqrt{3}} \), \( \phi \) is the desired phase angle between supply voltages and compensated sources currents in the respective phases. \( P_{lavg} \) is the dc value of load power and is determined by employing moving average filter having mean time of half cycle. The term \( P_{loss} \) is account for losses in the inverter. To generate \( P_{loss} \) a suitable closed-loop dc-link voltage controller is used, which regulate the dc voltage to the reference value. Conventionally a proportional-plus-integral (PI) controller is used to maintain the dc-link voltage.

**B. TOPOLOGY OF TWELVE-PHASE DSTATCOM**

The topology of the proposed twelve-phase DSTATCOM has been derived from its three-phase counterpart where an inverter circuit is employed with a self-charged capacitor as DC source and unbalanced load circuit connected at point of common coupling (PCC) as load to the inverter. The proposed twelve-phase compensator consists of twelve arms of a single-phase H-inverter circuits as shown in Fig. 2 with split capacitor for providing the neutral point \( n' \) of the compensator.

A single capacitor instead of two capacitors can be used if neutral point of the converter is not required as 12-phase 12-wire system. In case of two-capacitor topology, it has been reported in literature that capacitor voltages becomes unbalanced and therefore may lead to unstable operation in a three-phase circuit. To avoid the unbalanced operation of capacitors, a chopper can be used to balance the capacitor voltage. The chopper is controlled in such a way that capacitor voltages are dynamically regulated to have equal average voltage. It is to be noted that like any inverter circuit, this
converter circuit has the same switching constraint, i.e. any switch pair $S$ and $\overline{S}$ of the bridge arm are never be closed together to avoid shoot through fault across the charged capacitors. The converter can be operated as inverter by switching a pair of switching elements of this circuit. One of the simplest ways is pulse width modulated switching where a reference voltage is compared with a fixed amplitude and frequency carrier voltage and the switching instants of the switching elements are decided. This technique is however employed if the converter is desired to be operating as voltage source inverter. The proposed circuit is required to be operating as current controlled voltage source inverter and this may be achieved by hysteresis band control of the inverter or converter current.

In a hysteresis band control, the converter current is forced to remain in a hysteresis band around the reference current. The reference currents for all the twelve phase for a twelve-phase compensator are given by (1)-(12). The converter current for each phase is forced to follow the reference current by closing or opening of switches of the converter. By closing the upper switch of the arm, the current rises, whereas closure of the lower switch reduces the current. If the bandwidth is kept constant, the switching frequency of the converter is variable with maximum at zero transitions and minimum near the peak. It has been assumed that the switching devices are able to meet the highest frequency requirement of the converter.

C. ANALYSIS OF SWITCHING BEHAVIOR

To illustrate the working of the converter, phase -a of the twelve-phase compensator shown in Fig. 3 has been analyzed. The equivalent circuit of phase “a” with switch $S_{a1}$ closed is shown in Fig. 3. The current through the switch $S_{a1}$ is the series current $i_{ca}$ which can be determined by applying KVL for the circuit.

\[
\begin{align*}
\frac{di_{ca}}{dt} &= -\frac{R}{L}i_{cb} + S_aV_{cl} - \overline{S_a} V_{c2} - \frac{v_{sa}}{L} \\
&= -\frac{R}{L}i_{cb} + S_aV_{cl} - \overline{S_a} V_{c2} - \frac{v_{sa}}{L}
\end{align*}
\]  

(13)

where $S_a$ is switching function defined by (14) and is complement of the switching function $\overline{S_a}$.

\[
S_a = \begin{cases} 
1 & \text{if switch is on} \\
0 & \text{otherwise} 
\end{cases}
\]  

(14)

similar equations can be written for other phases by replacing the script a by b, c, d, f, g, h, i, j, k or l.

D. DSTATCOM Mathematical Model

The state space equations have been written in terms of the switching functions ($S_a, S_b, \ldots, S_i$) of the converter and ($S_u$ & $S_j$) of chopper of the DSTATCOM and solved by Runga-kutta method in MATLAB by ode45 function. The state space equations of the twelve-phase compensator system are given in (15-26).

\[
\begin{align*}
\frac{di_{ca}}{dt} &= -\frac{R}{L}i_{cb} + S_aV_{cl} - \overline{S_a} V_{c2} - \frac{v_{sa}}{L} \\
\frac{di_{ch}}{dt} &= -\frac{R}{L}i_{cb} + S_hV_{cl} - \overline{S_h} V_{c2} - \frac{v_{sh}}{L} \\
\frac{di_{ce}}{dt} &= -\frac{R}{L}i_{cb} + S_eV_{cl} - \overline{S_e} V_{c2} - \frac{v_{se}}{L} \\
\frac{di_{ed}}{dt} &= -\frac{R}{L}i_{cb} + S_dV_{cl} - \overline{S_d} V_{c2} - \frac{v_{sd}}{L} \\
\frac{di_{ef}}{dt} &= -\frac{R}{L}i_{cb} + S_fV_{cl} - \overline{S_f} V_{c2} - \frac{v_{sf}}{L} \\
\frac{di_{eg}}{dt} &= -\frac{R}{L}i_{cb} + S_gV_{cl} - \overline{S_g} V_{c2} - \frac{v_{sg}}{L} \\
\frac{di_{eh}}{dt} &= -\frac{R}{L}i_{cb} + S_hV_{cl} - \overline{S_h} V_{c2} - \frac{v_{sh}}{L} \\
\frac{di_{ei}}{dt} &= -\frac{R}{L}i_{cb} + S_iV_{cl} - \overline{S_i} V_{c2} - \frac{v_{si}}{L}
\end{align*}
\]  

(15-26)

The chopper dynamics has been realized by writing the state space equations for the circuit shown in Fig. 3. Let $i_1$ and $i_2$ be the currents in circuit of the chopper (Fig. 3), then (27) and (28) represents the relation of these in terms of switching function and the converter currents.
The isolation transformer parameters are as follows:

\[ R = 2 \Omega \quad L = 200 \text{ mH} \]

The control parameters for regulating capacitor voltage of the chopper are adjusted heuristically and are given as follows:

\[ K_p = 50 \quad K_j = 1000 \]

In the simulation results, the system works with unbalanced load for one cycle (0.02sec) and run for another three cycle 0.08sec) with the proposed compensator. It can be seen from Fig 4 (b) and Fig. 4 (c) that the source currents and load currents are equal and unbalanced when compensator is off. After 0.02sec., the compensator is turned on and the instantaneous compensator currents become balanced accordingly as shown in Fig. 4 (d).
Fig. 4 Variation of source voltages, load currents and compensator currents for 12-phase 13- wire supply system

The instantaneous powers (source, load and compensator powers) and neutral currents (source and load neutral currents) are depicted in Fig. 5. It can be seen from Fig. 5 (a) that before compensation, that is, when compensator is switched off, source power ($P_s$) and load power ($P_l$) are equal in magnitude and oscillating in nature due to unbalance in load currents. But after compensation when compensator is turned on the oscillating component of power in source attains a steady state value as can be seen from Fig. 5 (a) while load power is oscillating in nature as shown in Fig. 5 (a). Moreover, the source neutral current ($i_{sn}$) attains zero value when the compensator is turned on and can be seen from Fig. 5 (b) as it balances the source currents. It can thus be inferred that the sum of the instantaneous compensator currents is equal to load neutral ($i_{ln}$).

Fig. 5 Variation of powers and neutral current for 12-phase 13-wire supply system

The co-phasors variation of voltage and current shows unity power factor operation with compensator on as evident from Figs. 6 (a)-(l). It can be seen from (10b) that impedances are unbalanced and have reactive elements, but the currents are not only balanced but also operate at unity power factor.

Fig. 6 (a)-(l) (1:18) scaled source voltages (solid line), source currents (dashed line), load currents (hard solid line) and variation of compensator currents, portraying u.p.f. operation for 12-phase 13-wire supply system

The D.C. link capacitor maintains the constant voltage through the PI controller. The voltage variation of individual capacitor is shown in Fig. 7. Please note that the capacitor voltage is assumed to be initially charged or recharged.
B. Load Compensation for Phase Outages

As stated earlier that advantage of multiphase loads like motor is in its capability to operate with phase outages. The motor operates with degraded performance and source sees an unbalanced operation and therefore other loads connected to such a source get affected. The proposed compensator has also been tested with various combination of phase outages and it has been found that it works satisfactorily even with six phases open (with phase outages (a, b, c, d, e, and f)) in a 12-phase source. The results are shown in Fig. 8 to Fig. 11 where entire source currents are balanced with compensator on.

Fig. 7 Capacitor voltage variation

Fig. 8 Variation of load currents, source currents and compensator currents when six phases (a, b, c, d, e, f) are out (from load side) from 12-phase 13-wire supply system

Fig. 9 Variation of power and current when six phases (a, b, c, d, e & f) are out in 12-phase 13-wire supply system

Fig. 10 (a)-(l) (1:18) Scaled source voltages (solid line), source currents (dashed line), load currents (hard solid line) and variation of compensator currents for the phase outage (a, b, c, d, e, f) of load for 12-phase 13-wire supply system.
Fig. 11 Capacitor voltage variation for phase outage (a, b, c, d, e, f) of the load for a 12-phase 13-wire supply system supply system

IV. CONCLUSION

In this paper a current controlled VSI topology for 12-phase DSTATCOM is presented for load compensation of 12-phase circuit. The mathematical modeling of proposed DSTATCOM is discussed for carrying out simulation studies. A detailed study has been made on source voltages, load currents, source currents, and compensator currents during non-compensating and compensating period. The variations of the load, source, compensator power, and capacitor voltage are also analyzed. The analysis has confirmed unity power factor operation of the source in all cases as evidenced by the voltage and current waveforms. An interesting aspect of the simulation study has been to observe that multi-phase (12-phase) system equipped with multi-phase compensator can work with phase outages as high as 6 in 12-phase system without any degradation in supply system quality-- a feature most valuable for multiphase sources and multiphase induction motor drives. The experimental result will be offered in the near future for further verifications.

REFERENCES

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