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Hybrid Control of Networked Multi-Vehicle System Considering Limitation of Communication Range

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***Abstract*—** In this research, we study a control method of a multi-vehicle system while considering the limitation of communication range for each vehicles. When we control networked vehicles with limitation of communication range, it is important to control the communication network structure of a multi-vehicle system in order to keep the network's connectivity. From this, we especially aim to control the network structure to the target structure. We formulate the networked multi-vehicle system with some disturbance and the communication constraints as a hybrid dynamical system, and then we study the optimal control problems of the system. It is shown that the system converge to the objective network structure in finite time when the system is controlled by the receding horizon method. Additionally, the optimal control probrems are convertible into the mixed integer problems and these problems are solvable by some branch and bound algorithm.

***Keywords*—**Hybrid system, multi-vehicle system, receding horizon control, topology control.

I. Introduction

RECENTLY, multi-vehicle systems in which each vehicle act cooperatively for a common task are researched [1]. One of the reasons is that technological developments of computation and communication technology enabled the vehicles to communicate by wireless. On the topic of multi-vehicle control, a lot of approaches have been taken for varied tasks [2]. Especially, like Fax et al. [3], formation control has been intensively studied, and distributed receding horizon control was proposed recently [4]. Many of these studies assume that communication network topology is fixed.

However, in general, there is an effective range on wireless communication. In the case of limited communication range, the system needs to change the network topology to keep the network connectivity during the vehicles' locomotion. Although several studies about network topology control exist in the field of sensor network [5] in order to save energy and reduce interference, there are few studies about the network topology control of multi-vehicle system.

Zavlanos et al. proposed the potential field to keep the network connectivity [6] and the connectivity control using the auction algorithm [7] about mobile network. They supposed each vehicle moves by given vector field and the control input is generated by some potential function. These studies didn't explicitly consider to control the network to the desierd structure.

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We aim to propose the control method dealing with both the vehicles' dynamics and the network topology of the multi-vehicle system. We formulate the multi-vehicle system and the network with the effective range of wireless communication as the hybrid dynamical system. Then we consider the optimal control problems of the system with several cases of distrubance, and show that the system controlled by the receding horizon method [8] converge to the objective network structure in finite time. To solve the optimal control problems, we additionally show the probrems are convertible into the mixed integer problems and introduce the branch and bound algorithm. Unlike previous studies, our method makes it possible to consider the control of the vehicles and the network structure at the same time.

The framework of this paper is following. Section II defines the dynamics of each vehicle and the network structure with the effective range of communication. In Section III, we study the optimal control problems of the system with several disturbance. Then, the convergence to the desierd network structure in finite time is analysed here with respect to the system controlled by the receding horizon controller. In addition we show the optimal control problems are convertible into the mixed integer problems, and introduce the branch and bound algorithm to solve the problems. In Section IV, we show the validity of our approach through computer simulation of the system with three vehicles. Section V gives the conclusion of this paper.

II. Formulation of a networked multi-vehicle system

In this section, we formulate the networked multi-vehicle system investigated in our study. The dynamics of each vehicle and the network structure with a limitation of communication range are formulated as a hybrid system.

We consider N mobile vehicles locomoting in a d dimensional space. Each vehicle is numbered from 1 to N, and the set of the vehicles' number is described $\mathcal{V} = \{1, 2, \ldots, N\}$.

The dynamics of the vehicle $i \in \mathcal{V}$ is given by the discrete time state equation as

$$x_i(t+1) = A_i x_i(t) + B_i u_i(t) + F_i w_i(t), \qquad (1)$$

where $x_i = (q_i^T, \dot{q}_i^T)^T \in \mathbb{R}^{n_x = 2d}$ denotes the state vector of position $q_i \in \mathbb{R}^d$ and the velocity $\dot{q}_i \in \mathbb{R}^d$ of vehicle i, $u_i \in \mathbb{R}^{n_u}$ denotes the control input of vehicle i , and $w_i \in \mathbb{R}^{n_w}$ denotes the disturbance of vehicle i we define later. We assume (A_i, B_i) is controllable.

Collecting up the state, the control input and the disturbance of all vehicles, the multi-vehicle system's state equation is given as

$$x(t+1) = Ax(t) + Bu(t) + Fw(t), \tag{2}$$

where $x = (x_1^T, \dots, x_N^T)^T$, $u = (u_1^T, \dots, u_N^T)^T$ and $w = (w_1^T, \dots, w_N^T)^T$.

Next, we fomulate the dynamics of the network structure considering the communication range. We assume each vehicle communicates to other vehicles via radio waves and the communication link is variable by time t.

The communication network structure at time t is given by an undirected graph $\mathcal{G}(t) = (\mathcal{V}, \mathcal{E}(t))$ where $\mathcal{V}$ is the set of the vehicles as vertices and $\mathcal{E}(t)$ is the set of edges at time t. Furthermore, we represent the graph $\mathcal{G}(t)$ as the adjacency matrix $Adj(t) = [a_{ij}(t)] \in \{0,1\}^{N\times N}$, in which the element $a_{ij}(t) = 1$ if there is communication between the vehicle i and the vehicle j, and $a_{ij}(t) = 0$ otherwise.

We consider there is the upper bound of communication range between each vehicle. Here we define the distance between vehicle i and vehicle j as $d_{ij}(x(t)) = \|q_i(t) - q_j(t)\| = \|Q_{ij}x(t)\|$ and the upper bound of communication range as r. Then we give the dynamics of the network structure $Adj(t)$ as

$$a_{ij}(t+1) = \begin{cases} v_{ij}(t) & \text{if } d_{ij}(x(t+1)) \le r, \\ 0 & \text{otherwise,} \end{cases} \tag{3}$$

where $v_{ij} \in \{0,1\}$ is the command input of the link a_{ij}, and we define the matrix V that the ijth element is v_{ij}. This equation means we require the distance between vehicle i and j is less than r ($d_{ij}(x(t+1)) \le r$) whenever we give the communication link between vehicle i and j ($a_{ij}(t+1) = v_{ij}(t) = 1$).

III. Optimal control problem and algorithm

In this paper, we define the control objective is to achive the objective vehicles states x_f and/or the objective network structure $Adj_f = [a_{ij,f}]$ from the given initial state $x(0) = x_0$ and initial network structure $Adj(0) = Adj_0$. Especially we focus on attaining the objective network structure because we may control the vehicles by any constraint control method (for example [8],[9]) without controlling the network once the network structure Adj become Adj_f. Hence we introduce the following assumption in this study.

Assumption 1: If w is bounded, there exist $u(t) \in \mathcal{U}$ that

$$x(t+1) = Ax(t) + Bu(t) + Fw(t) \in \mathcal{X}_f$$

whenever $x(t) \in \mathcal{X}_f$, where $\mathcal{X}_f = \{x | d_{ij}(x) \le r, \ ^\forall ij, a_{ij,f} = 1\}$.
This assumption suggests that we can keep $Adj(t) = Adj_f$ by some control input $u(t) \in \mathcal{U}$ for all $t \ge t_0$ once we get $Adj(t_0) = Adj_f$.

We consider the cost function

$$\begin{aligned} &J(u, V; x(t), Adj(t)) \\ &\quad = \sum_{k=t}^{t+T-1} \Big(\|x(k) - x_f\|_Q^2 + \|u(k)\|_R^2 \\ &\qquad + c_{Adj}\|Adj(k) - Adj_f\|_{Frob}^2 \Big) \end{aligned} \tag{4}$$

where $\|x\|_P^2$ denotes $x^T P x$, $Q = Q^T > 0, R = R^T > 0$, $\|A\|_{Frob}^2$ is the square of the Frobenius norm, $c_{Adj} \ge 0$ is the weighting coefficient and $T \ge 1$ is the integer denoting finite interval. Hereafter, we consider the optimal control problem based on the cost function, and discuss the finite time optimal control and the receding horizon control [8] simultaneously. The receding horizon control is a feedback control because the controller refer the current state $x(t), Adj(t)$, solve a optimal control problem whose initial value are $x(t), Adj(t)$, and input the optimal solution into the system every time step t.

In this research we assume a single vehicle solves the optimal control problem or all vehicles solve it by a parallel distributed algorithm. When the entire network is not connected, some vehicles are unable to translate the information and then the optiomal control problem can't be solved. Therefore, we require each vehicle locomotes to keep the graph $Adj(t)$ connected for all time.

A. The case without disturbunce

In this subsection, we consider the system (2) and (3) without disturbance ($w = 0$). We consider the following constrained finite time optimal control problem about the system (2) and (3),

$$\begin{aligned} \min_{u,V} \quad & J(u, V; x(t), Adj(t)) \\ \text{subject to} \quad & (2),\ (3), \\ & x(k) \in \mathcal{X}, u(k) \in \mathcal{U}, \\ & Adj(k) \text{ is connected}, \\ & Adj(t+T) = Adj_f, \\ & k \in \{t, \dots, t+T-1\}, \end{aligned}$$

where $\mathcal{X}$ and $\mathcal{U}$ are the closed convex set simultaneously. We suppose this optimal control problem is feasible at $t = 0$.

Clearly, it is found that the main control objective $Adj(t) = Adj_f$ is satisfied in the case of finite time optimal control. Here we show the convergence to Adj_f when the system (2) and (3) are controlled by the receding horizon method.

Theorem 1: Suppose x_f is in interior of $\mathcal{X}_f$, $(x_f, 0)$ is a equibrium pair of (2), and

$$\max_{x\in\mathcal{X}_f, u\in\mathcal{U}} (\|x - x_f\|_Q^2 + \|u\|_R^2) \le c_{Adj}. \tag{5}$$

Then, $Adj(t)$ of the system (2) and (3) controlled by the receding horizon method with the above optimal control problem converge to Adj_f in finite time.

Proof: We first remark

$$\|Adj(k) - Adj_f\|_{Frob}^2 \ge 1 \text{ if and only if } Adj(k) \ne Adj_f \tag{6}$$

because of $Adj \in \{0,1\}^{N\times N}$. We define the optimal solution

$$\begin{aligned} u_t^* &= \{u_t^*(t), \dots, u_t^*(t+T-1)\},\\ V_t^* &= \{V_t^*(t), \dots, V_t^*(t+T-1) = Adj_f\}, \end{aligned}$$

at time t, then we can give the candidate of solution at $t+1$ is

$$\begin{aligned} u_{t+1} &= \{u_t^*(t+1), \dots, u_t^*(t+T-1), u_{t+1}(t+T)\},\\ V_{t+1} &= \{V_t^*(t+1), \dots, V_t^*(t+T-1) = Adj_f, Adj_f\}. \end{aligned}$$

Then it is found that the feasible solution exists at $t+1$ from Assumption 1, and the probrem is feasible $t \geq 0$. Hence the cost function J satisfy

$$\begin{aligned} &J(u_{t+1}^*, V_{t+1}^*; x(t+1), Adj(t+1)),\\ &-J(u_t^*, V_t^*; x(t), Adj(t))\\ &\leq \|x(t+T) - x_f\|_Q^2 + \|u(t+T)\|_R^2\\ &+c_{Adj}\|Adj(t+T) - Adj_f\|_{Frob}^2\\ &-\|x(t) - x_f\|_Q^2 - \|u_t^*(t)\|_R^2\\ &-c_{Adj}\|Adj(t) - Adj_f\|_{Frob}^2. \end{aligned}$$

This inequality shows J is non-increasing and $x(t)$ is getting close to x_f whenever $Adj(t) \neq Adj_f$ from (5) and (6). We can estimate the lower bound of J that

$$J(u, V; x(t), Adj(t)) \geq c_{Adj}$$

when $Adj(t) \neq Adj_f$ by (6). However when $x(t)$ is sufficiently close to x_f, some stabilizing feedback $u(k) = Kx(k)$ and $V(k) = Adj_f$ are feasible and

$$J(Kx(k), Adj_f; x(t), Adj(t)) \leq c_{Adj}.$$

This means the optimal solution is $V_t^*(k) = Adj_f$ and therefore $Adj(t+1) = Adj_f$. These show $Adj(t)$ converge to Adj_f in finite time. ■

Theorem 1 shows the receding horizon control method we proposed satisfies our control objective. We discuss how to solve this optimal control problem next.

To solve the constrainted optimal control problem we translate it into an equivalent mixed integer programming problem. We define $\hat{u} = (u^T(t), \dots, u^T(t+T-1))^T$, $\hat{w} = (w^T(t), \dots, w^T(t+T-1))^T$, and similarly $\hat{v} = (v^T(t), \dots, v^T(t+T-1))^T$ where $v(t) = (v_{12}(t), \dots, v_{(N-1)N}(t))^T$. In order to decrease computation, we ignore v_{ii} that is no concern of our discussion and $v_{ij}(i > j)$ that is equal to v_{ji}. $x(k)$ can be rewritten as

$$\begin{aligned} x(k) &= A^{k-t}x(t) + \sum_{l=t}^{k-1} A^{l-t}\left(Bu(l) + Fw(l)\right)\\ &= A^{k-t}x(t) + X_k^{AB}\hat{u} + X_k^{AF}\hat{w}, \end{aligned} \tag{7}$$

from the system equation (2), and $a_{ij}(k)$ is rewriten as

$$a_{ij}(k+1) = v_{ij}(k) \in \begin{cases} \{0,1\} & \text{if } d_{ij}(x(t+1)) \leq r,\\ \{0\} & \text{otherwise} \end{cases} \tag{8}$$

from (3). Because $w = 0$, we can describe the cost function (4) as

$$J = \hat{u}^T S_1 \hat{u} + \hat{v}^T S_2 \hat{v} + S_3 \hat{u} + S_4 \hat{v} + \text{const.}, \tag{9}$$

so the cost function is quadratic.

Next, we reformulate the case constraint of (8) as a convex constraint. In the work of Bemporad et al. they translate the hybrid system with some logic like a piecewise linear system into a linear system with linear inequality [10]. However they treat the linear inequality constraint for generality, we extend it to our system and treat the convex constraint. To keep (3) or (8), we add the following logical constraint

$$\{v_{ij}(k) = 1\} \Rightarrow \{d_{ij}(x(k+1)) \leq r\}. \tag{10}$$

Using the upper bound of the distance between each vehicle $M = \max_{x\in\mathcal{X}} d_{ij}(x)$, the expression (10) is rewritten as the inequality

$$d_{ij}(x(k+1)) \leq M(1 - v_{ij}(k)) + r. \tag{11}$$

The expression (10) and the expression (11) are equivalent, as in Table I.

From above, we can describe the constrained optimal control problem as the following mixed integer programming problem

Problem 1:

$$\begin{aligned} \min_{\hat{u},\hat{v}} \quad & J,\\ \text{s.t.} \quad & d_{ij}(x(k+1)) \leq M(1 - v_{ij}(k)) + r,\\ & Adj(k) \text{ is connected},\\ & Adj(t+T) = Adj_f,\\ & i, j \in \mathcal{V},\ k \in \{t, \dots, t+T-1\},\\ & \hat{u} \in \hat{\mathcal{U}},\ \hat{v} \in \{0,1\}^{\frac{1}{2}N(N-1)T}, \end{aligned}$$

where $\hat{\mathcal{U}}$ is the modified set to satisfy $x \in \mathcal{X}$ and $u \in \mathcal{U}$. We can see this probrem is a mixed integer convex problem except for the constraint "$Adj(k)$ is connected". In the lattar subsection we describe how to solve this problem.

B. Robust optimal control problem

In this subsection, we consider the system (2) and (3) with the unknown disturbance $w \in \mathcal{W}$ where $\mathcal{W}$ is a closed convex

TABLE I
TRUTH TABLE OF EXPRESSION (10) AND (11)

$\{v(k)_{ij} = 1\}$	$\{d_{ij}(x(k+1)) \leq r\}$	$\{v_{ij}(k) = 1\} \Rightarrow \{d_{ij}(x(k+1)) \leq r\}$, (10)	$\{d_{ij}(x(k+1)) \leq M(1 - v_{ij}(k)) + r\}$, (11)
T	T	T	T
T	F	F	F
F	T	T	T
F	F	T	T

set that $\|w\| \le 1$. We consider the following min-max optimal control problem,

$$\begin{aligned}\min_{u,V}\max_{w\in\mathcal{W}} \quad & J(u,V;x(t),Adj(t)),\\ \text{subject to} \quad & x(k+1)=Ax(k)+Bu(k)+Fw(k),\\ & a_{ij}(k+1)\\ & \quad = \begin{cases} v_{ij}(k) & \text{if } \max_{w\in\mathcal{W}} d_{ij}(x(k+1)) \le r,\\ 0 & \text{otherwise,}\end{cases}\\ & \max_{w\in\mathcal{W}} x(k)\in\mathcal{X}, u(k)\in\mathcal{U},\\ & Adj(k) \text{ is connected,}\\ & Adj(t+T)=Adj_f,\\ & k\in\{t,\dots,t+T-1\}.\end{aligned}$$

The constraint $\max_{w\in\mathcal{W}} d_{ij}(x(k+1)) \le r$ is set with the purpose to prepare for the worst case. We suppose this optimal control problem is feasible at $t=0$.

Theorem 2: Suppose the same assumption of Theorem 1. Then, $Adj(t)$ of the system (2) and (3) controlled by the receding horizon method with the above optimal control problem converge to Adj_f in finite time.

Proof: Because the optimal solution u_t^* and V_t^* at t is robustly feasible, the problem at $t+1$ is feasible from the same argument of the proof of Theorem 1, and then the problem is feasible at $t \ge 0$. Here we define the optimal cost J_w^* with fixed disturbance sequence $\{w\}$, then J_w^* satisfy

$$J_w^*(t+1)-J_w^*(t) \le -\|x(t)-x_f\|_Q^2 - \|u_t^*(t)\|_R^2,$$

from (5) and (6), the optimal cost J^* can be described as $J^* = \max_{w\in\mathcal{W}} J_w^*$, we define w_{t+1} that $J^*(t+1)=J_{w_{t+1}}^*(t+1)$ and then

$$J^*(t+1)-J_{w_{t+1}}^*(t) \le -\|x(t)-x_f\|_Q^2 - \|u_t^*(t)\|_R^2.$$

Of cource $J_{w_{t+1}}^*(t) \le \max_{w\in\mathcal{W}} J_w^*(t) = J^*(t)$, this inequality shows

$$J^*(t+1)-J^*(t) \le -\|x(t)-x_f\|_Q^2 - \|u_t^*(t)\|_R^2,$$

J^* is non increasing and $x(t)$ is getting close to x_f whenever $Adj(t) \neq Adj_f$. The remainder of proof is same as the proof of Theorem 1 and therefore $Adj(t)$ converge to Adj_f in finite time. ∎

Next, we translate the min-max optimial control problem into mixed integer problem like the above subsection. We can rewrite J as quadratic form

$$\begin{aligned}J =& \hat{u}^T S_1 \hat{u} + \hat{v}^T S_2 \hat{v} + S_3\hat{u} + S_4\hat{v}\\ &+ \hat{w}^T S_5 \hat{w} + S_6\hat{w} + \hat{w}^T S_7 \hat{u} + \text{const.}\\ =& \hat{u}^T T_1 \hat{u} + \hat{v}^T T_2 \hat{v} + T_3\hat{u} + T_4\hat{v}\\ &+ \|T_5\hat{w} + T_6\hat{u} + T_7\|^2 + \text{const.}.\end{aligned}$$

Here we trancelate the min-max problem into the following minimization problem

$$\begin{aligned}&\min_{u,v}\max_{w} J,\\ \Leftrightarrow\ & \min_{u,v,\tau} J = \hat{u}^T T_1\hat{u} + \hat{v}^T T_2 \hat{v} + T_3\hat{u} + T_4\hat{v} + \tau^2,\\ & \text{subject to } \max_{w} \|T_5\hat{w} + T_6\hat{u} + T_7\| \le \tau. \qquad (12)\end{aligned}$$

The constraint about $v_{ij}(k)$ can be rewriten like the above subsection,

$$\max_{w} d_{ij}(x(k+1)) \le M(1-v_{ij}(k)) + r. \qquad (13)$$

Recalling $d_{ij}(x(k)) = \|Q_{ij}x(k)\|$ and $x(k) = A^{k-t}x(t) + X_k^{AB}\hat{u} + X_k^{AF}\hat{w}$, the constraint (13) can be expressd as

$$\begin{aligned}&\|Q_{ij}A^{k+1-t}x(t) + Q_{ij}X_{k+1}^{AB}\hat{u} + Q_{ij}X_{k+1}^{AF}\hat{w}\|\\ &\qquad \le M(1-v_{ij}(k)) + r.\end{aligned} \qquad (14)$$

The constraints (12) and (14) can be represent like the following lemma.

Lemma 1: Denote

$$\begin{aligned}X\hat{w} &= (X(t),\dots,X(t+T-1))\begin{pmatrix} w(t)\\ \vdots \\ w(t+T-1)\end{pmatrix}\\ &= \sum_{l=t}^{t+T-1} X(l)w(l).\end{aligned}$$

Then

$$\|X_1\hat{w} + X_2\hat{u} + X_3\| \le \tau,\ {}^\forall w(k)\in\mathcal{W} = \{w : \|w\| \le 1\},$$

is equivalent to

$$\|X_2\hat{u} + X_3\| + \left(\sum_{l=t}^{t+T-1} \|X_1(l)\|\right) \le \tau. \qquad (15)$$

Proof:

$$\begin{aligned}& \|X_1\hat{w} + X_2\hat{u} + X_3\| \le \tau,\ {}^\forall w\in\mathcal{W}\\ \Leftrightarrow\ & \|X_2\hat{u} + X_3\| + \|X_1\hat{w}\| \le \tau,\ {}^\forall w\in\mathcal{W}\\ \Leftrightarrow\ & \|X_2\hat{u} + X_3\| + \left\|\sum_{l=t}^{t+T-1} X_1(l)w(l)\right\| \le \tau,\ {}^\forall w\in\mathcal{W}\\ \Leftrightarrow\ & \|X_2\hat{u} + X_3\| + \sum_{l=t}^{t+T-1} \|X_1(l)w(l)\| \le \tau,\ {}^\forall w\in\mathcal{W},\end{aligned}$$

By $\mathcal{W} = \{w : \|w\| \le 1\}$, it is equivalent to (15). ∎

From above, we can describe the min-max optimal control problem as the following mixed integer programming problem

Problem 2:

$$\begin{aligned}\min_{\hat{u},\hat{v},\tau} \quad & J = \hat{u}^T T_1\hat{u} + \hat{v}^T T_2\hat{v} + T_3\hat{u} + T_4\hat{v} + \tau^2,\\ \text{s.t.} \quad & \|T_6\hat{u} + T_7\| + \left(\sum_{l=t}^{t+T-1} \|T_5(l)\|\right) \le \tau,\\ & \|Q_{ij}A^{k+1-t}x(t) + Q_{ij}X_{k+1}^{AB}\hat{u}\| + c_{ijk}\\ & \le M(1-v_{ij}(k)) + r,\\ & c_{ijk} = \left(\sum_{l=t}^{t+T-1} \|Q_{ij}X_{k+1}^{AF}(l)\|\right),\\ & Adj(k) \text{ is connected,}\\ & Adj(t+T) = Adj_f,\\ & i,j\in\mathcal{V},\ k\in\{t,\dots,t+T-1\},\\ & \hat{u}\in\hat{\mathcal{U}},\ \hat{v}\in\{0,1\}^{\frac{1}{2}N(N-1)T},\end{aligned}$$

where $\hat{\mathcal{U}}$ is the modified set to satisfy $\max_w x\in\mathcal{X}$ and $u\in\mathcal{U}$. This probrem is also a mixed integer convex problem except the constraint "$Adj(k)$ is connected".

C. Probabilistic case

In this subsection, we consider the system (2) and (3) with white Gaussian noise w whose mean is 0 and variance matrix is I (we denote $w \sim \mathcal{N}(0, I)$). Notice this disturbance is unbounded and then Assumption 1 is not applicable in this subsection. Meanwhile, even though w is probablistic, we can apply the above subsection approach if w is bounded. Since $x(t)$ is a random variable here, we additionally define the mean of $x(t)$ is $\mu(t)$ and the variance matrix of $x(t)$ is $\Sigma(t)$.

We consider the following probabilistic optimal control problem,

$$\begin{aligned}
\min_{u,V} \quad & \mathbf{E}\left\{J(u, V; x(t), Adj(t))\right\}, \\
\text{subject to} \quad & x(k+1) = Ax(k) + Bu(k) + Fw(k), \\
& a_{ij}(k+1) \\
& = \begin{cases} v_{ij}(k) & \text{if Prob}(d_{ij}(x(k+1)) \le r) \ge p, \\ 0 & \text{otherwise}, \end{cases} \\
& \mu(k) \in \mathcal{X}, u(k) \in \mathcal{U}, \\
& Adj(k) \text{ is connected}, \\
& Adj(t+T) = Adj_f, \\
& k \in \{t, \ldots, t+T-1\},
\end{aligned}$$

where $\mathbf{E}$ denotes expectation operator and the expression Prob$(E) \ge p$ means the event E occurs with the probability greater than or equal to p. In this case, we can not guarantee the feasibility at $t > 0$ even if the problem at $t = 0$ is feasible, due to the unboundedness of w.

From the evolution equations of $x(k) \sim \mathcal{N}(\mu(k), \Sigma(k))$ is

$$\begin{aligned}
\mu(k+1) &= A\mu(k) + Bu(k), \\
\Sigma(k+1) &= A\Sigma(k)A^T + FF^T,
\end{aligned}$$

$\Sigma(k)$ does not depend on $u(k)$ and then is the constant matrix in the above optimal control problem. Therefore we can get

$$\begin{aligned}
& \text{Prob}(d_{ij}(x(k+1)) \le r) \ge p \\
\Leftarrow \quad & d_{ij}(\mu(k+1) + h\Sigma^{\frac{1}{2}}(k+1)\eta) \le r, \ {}^{\forall}\eta : \|\eta\| \le 1,
\end{aligned} \tag{16}$$

where $h \ge 0$ is the constant depending on p and $\Sigma^{\frac{1}{2}}(k)$ is the Cholesky decomposition $\Sigma(k) = (\Sigma^{\frac{1}{2}}(k))(\Sigma^{\frac{1}{2}}(k))^T$. In the similar way to the above subsection and Lemma 1, we get (16) is equivalent to

$$\begin{aligned}
\|Q_{ij}A^{k+1-t}\mu(t) + Q_{ij}X^{AB}_{k+1}\hat{u}\| + \|Q_{ij}\Sigma^{\frac{1}{2}}(k+1)\| \\
\le M(1 - v_{ij}(k)) + r,
\end{aligned}$$

and then the probabristic optimal control problem can be modified as Problem 2.

D. How to solve the problems

We slate branch and bound method to solve above problems. Branch and bound is a method by which we can efficiently find a optimal solution by splitting the solution space into some subspaces and figuring out if there exists a optimal solution in the divided subspaces [11], and is often applied to a mixed integer problem. In a case of solving a mixed integer problem, $\{0, 1\}$ integer values are often used as a binary tree and the lower bound of the subspace is estimated by solving the continuous relaxation problem [10].

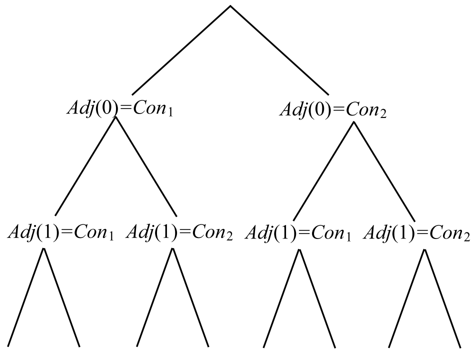


Fig. 1. A tree of adjacency matrices that are connected graph.

In our case, we use the value of the adjacency matrix $Adj(k)$ for each time k as the tree Fig. 1 shows. For this purpose, we collect the set of $N \times N$ connected adjacency matrices $\mathcal{C} = \{Con_1, \ldots, Con_C\}$ in advancce. To compute the set $\mathcal{C}$ here we introduce the Laplacian matrix Lap, that is constructed by the elements of the adjacency matrix Adj (as $Lap = Diag(\sum_j a_{ij}) - Adj$). It is known the graph $\mathcal{G}$ is connected if and only if the second smallest eigenvalue $\lambda_2(Lap)$ of the Laplacian matrix Lap satisfies the inequality[12]

$$\lambda_2(Lap) > 0.$$

Therefore we can compute the set $\mathcal{C}$ in advance by using this inequality and then the difficult constraint "$Adj(k)$ is connected" is gone.

The remainder of problem is convex. A convex programming problem can be efficiently solved by algorithms such as an interior point method [13].

IV. Computer simulation

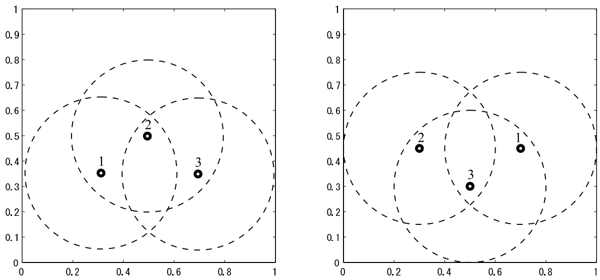
In this section, we consider the networked multi-vehicle system consisted of $N = 3$ vehicles. All the vehicles locomote in $d = 2$ dimensional plane and the vehicle i moves following the equation of motion

$$\ddot{q}_i + \dot{q}_i = u_i + w_i,$$

and we treat the discrete time multi-vehicle system (2) using zero-order hold on the control input with the sampling time 0.1. We also define the effective range of wireless communication $r = 0.3$, each vehicle locomotes in $0 \le q_i \le 1$, $-0.7 \le \dot{q}_i \le 0.7$, and the initial velocity $\dot{q}_i(0) = 0$. The bound of control input is defined $-1 \ge u_i \ge 1$ and the bound of disturbance is $\|w_i\| \le 1$.

About the networked multi-vehicle system defined above, we consider the control from the initial position (a) to the objective formation (b) as Fig. 2 shows, where points denote each vehicle's position and circles of dashed line denote range limitation of communication of each vehicle.

If the system is controlled by static network structure method, there is possibility that the control fails because the



(a) Initial positions (b) Objective position

Fig. 2. Initial positions and objective position of the control simulation.

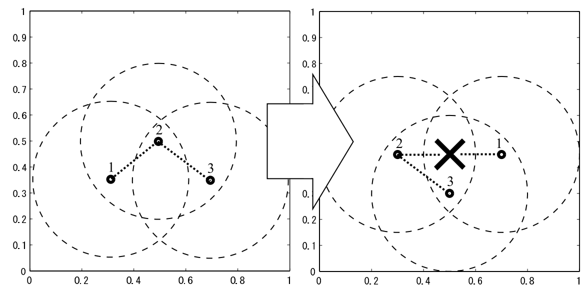


Fig. 3. Connectivity violation in the case of the control on the static network structure.

network connectivity breaks as Fig. 3 shows. In this case, therefore, it is needed to control the vehicles and the network structure at the same time.

Here we show the result of the computer simulation of the receding horizon control. The finite time of the optimal control problem is given as $T = 10$, and the cost function is given by

$$J(t) = \sum_{k=t}^{t+9} \Big(\|q(t) - q_f\|_{Q_q}^2 + \|\dot{q}(t)\|_{Q_{\dot{q}}}^2 + \|u(k)\|_R^2 + c_{Adj}\|Adj(k) - Adj_f\|_{Frob}^2 \Big)$$

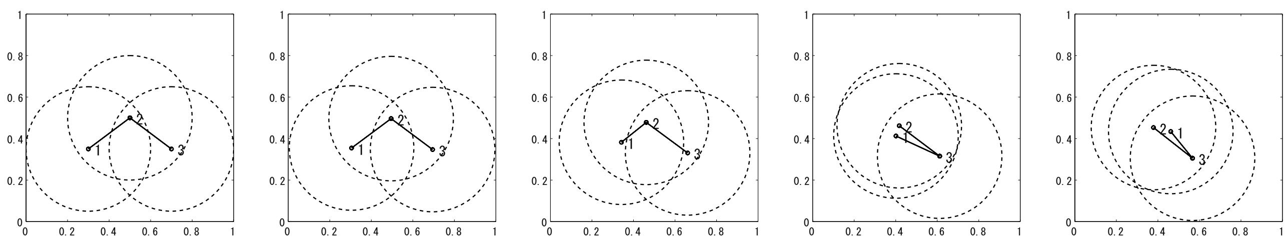
where $q = (q_1^T, q_2^T, q_3^T)^T, \dot{q} = (\dot{q}_1^T, \dot{q}_2^T, \dot{q}_3^T)^T$ and $Q_q = 10I, Q_{\dot{q}} = I, R = I, c_{Adj} = 30$. Fig. 4 shows the result of the receding horizon control simulation. Points denote each vehicle's position, solid lines between robots denote communication links and circles of dashed line denote range limitation of communication of each vehicle. It is found that the network converge to the objective structure in finite time ($t = 6$) and each vehicle converges to the objective position satisfying the connectivity constraint.

V. Conclusion

In this paper, we propose the control method dealing with both the vehicles' dynamics and the network topology of the multi-vehicle system. First we formulate the multi-vehicle system and the network with effective range of wireless communication as the hybrid dynamical system. Then we consider the optimal control problems of the system with no distrubance, bounded disturbance and probablistic disturbance. It was shown the systems with no disturbance or bounded disturbance and controlled by the receding horizon method converge to the desierd network structure in finite time. And to solve the optimal control problems, we additionally showed the probrems are convertible into the mixed integer problems and introduced the branch and bound algorithm.

Though we consider the communication range as a constant, it can be formulated as variables supposing the range varies with electric power. Several robust optimal control problem may also be trancelated into a convex problem in the way of the robust optimization [14].

A multi-vehicle system is often regarded as consisting of many vehicles. As the number of vehicles of the system increase, it seems to require longer computation time because the number of the optimal problem variables (especislly integer variables) also increase. Tedesco et al. propose the sequential method that only one vehicle per decision time solve the mixed integer problem about collision avoidance control [15].



(a) t=0 (b) t=2 (c) t=4 (d) t=6 (e) t=8

(f) t=10 (g) t=12 (h) t=14 (i) t=16 (j) t=18

Fig. 4. Result of the receding horizon control simulation.

It remains as future works to present a distributed control method or a distributed solving method of the optimal control problem in order to reduce calculations and communications.

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