World Academy of Science, Engineering and Technology
International Journal of Mechanical and Mechatronics Engineering
Vol:2, No:10, 2008

Dynamic Modeling of Tow Flexible Link Manipulators

E. Abedi, A. Ahmadi Nadooshan, and S. Salehi

***Abstract*—**Modeling and vibration of a flexible link manipulator with tow flexible links and rigid joints are investigated which can include an arbitrary number of flexible links. Hamilton principle and finite element approach is proposed to model the dynamics of flexible manipulators. The links are assumed to be deflection due to bending. The association between elastic displacements of links is investigated, took into account the coupling effects of elastic motion and rigid motion. Flexible links are treated as Euler-Bernoulli beams and the shear deformation is thus abandoned. The dynamic behavior due to flexibility of links is well demonstrated through numerical simulation. The rigid-body motion and elastic deformations are separated by linearizing the equations of motion around the rigid body reference path. Simulation results are shown on for both position and force trajectory tracking tasks in the presence of varying parameters and unknown dynamics remarkably well. The proposed method can be used in both dynamic simulation and controller design.

***Keywords*—**Flexible manipulator, flexible link, dynamic modeling, end point.

I. Introduction

PROGRESS of robot technologies and the development of robotic applications, industrial robots realize many kinds of manipulation tasks. Recent industrial robots are made very big to obtain high stiffness which increases the accuracy and decrease vibration of their motion. Demands for lightweight structure in spatial application, energy efficiency, and high speed motion have increased recently for robot manipulators to carry out complex industrial tasks. Higher speed and high accurate system performance makes it necessary to consider a new generation of lightweight manipulators. The flexible manipulator has been an advanced topic in robotics research. Flexible manipulators are widely used in many applications. Flexible manipulators are good candidates for microminiaturization into micro devices. Demand for lightweight manipulator and use new materials to made manipulator, raising need for more accurate dynamic equation. For such occasions, the flexibility of the mechanical structures of robot manipulators is very important for the design of their control systems [1], [2]. Compared between rigid-arm and flexible manipulators exhibit many advantages among them, have less overall cost, less weight, require smaller actuators, less material and power consumption, high payload to robot weight ratio [3], [4], and more maneuverability. Limits in the robot speed and growth the required energy to motivate the new system, Light weight manipulators need less energy to move and they have larger payload abilities and more maneuver capacity. Therefore, there is a need to develop accurate dynamic models for design and control of such systems. Light members are more likely to vibrate and mistake due to the inertia and external forces. The action of the manipulator may become unacceptable due to the positioning inaccuracy of the end effectors. Traditionally, structural vibration has been avoided by mechanically stiffening each component. However, such a method is not available in the case of lightweight flexible manipulators. Therefore, in order to fully exploit the potential offered by flexible manipulators, the effects of link flexibility must be accounted for in the dynamic model. In this context, it is highly desirable to have an explicit, complete and accurate dynamic model at disposal. Modeling of flexible manipulators to achieve and maintain accurate positioning is challenging. New tasks are Dynamic Modeling and Inverse Dynamic Analysis of Flexible Parallel Robots [5], Dynamic simulation of task constrained of a rigid flexible manipulator [6], Constrained Motion Control of Flexible Robot Manipulators Based on Recurrent Neural Networks [7]. On account of the flexible nature of the system, the dynamics are significantly more complex. The dynamic formulation of flexible multi body systems leads to a set of confuses partial differential equations. Since these equations are space and time dependent, they can not be solved analytically. Mainly Two approximate techniques, assumed mode method and finite element method, have been used in the literature to change these partial differential equations to a set of ordinary differential equations. The assumed mode method was basically introduced [8]. The main disadvantage of this method is the difficulty in finding modes for links with non regular cross sections and for multi-link manipulators. Finite element method was used by many authors to develop approximate models for flexible mechanisms [9] and robots. In these works elastic deformations were analyzed by assuming known rigid body motion and later superposing the elastic deformations on the rigid body motion. Therefore, the coupling effects between the rigid body motion and the elastic deformations are usually neglected. There are several works in literature which consider rigid body motion and elastic motion coupling terms, but they only represent the effect of the rigid body motion on the elastic motion. The dynamics of flexible

Manuscript received September 5; 2008.
Esmaeil Abedi is with the Mechanical Engineering Department, Islamic Azad University, Majlessi Branch (e-mail: esmaeil.abedi@gmail.com).
Afshin Ahmadi Nadooshan is with the Mechanical Engineering Department, Islamic Azad University, Majlessi Branch.
Saeed Salehi is with the Mechanical Engineering Department, Islamic Azad University, Majlessi Branch.

robots working at high speed has been studied by many researchers [10], [11], and a number of approaches have been developed to predict the elastic dynamic behavior of flexible serial robots. A flexible manipulator being a continuous dynamics system has infinite degrees of freedom and is governed by coupled nonlinear partial differential equations [12]. After finding the expression for displacement at any point using the different method the equations of motion is derived using Lagrangian formulation researchers [12], [13]. To meet the requirements for pointing accuracy, flexible body parameters such as elastic displacement are great importance for control tasks and should be continuously identified in the working environment [14], [15]. The objective of the investigation in this paper is to develop a simple and efficient method for dynamic modeling of flexible manipulator. This is achieved using finite element method and Euler Bernoulli Beam theory and considering the elastic displacement of links and dynamic coupling effects. The effects of distributed mass, rotary inertia and bending deformation are all taken into account. The concept of the kinematics Energy and potential Energy constraint conditions of elastic displacement for flexible manipulator is used to decouple dynamic motion equations.

II. Dynamic Modeling

Consider the two-link planar manipulator shown in Fig. 1. The first link and second link are assumed to be flexible. The flexible links are assumed to be thin and slender so it can be modeled as an Euler-Bernoulli's beam. Any link is dividing in to such element and any element has 2 nods.

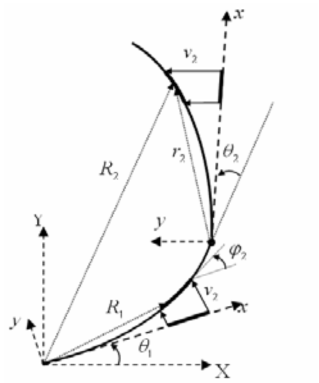

Fig. 1 Mapping Schematic of a planar tow-link flexible manipulator

For any nod assume 2 variables, a longitudinal displacement (v) and angle displacement (φ). The parameters of analyze manipulator are as: *XOY* (Global coordinates), *xoy* (Local coordinates, that is fixed to first point of link), v_1^e (The transversal bending deflection first link at a rigid condition, flexible deflection), v_2^e (The transversal bending deflection second link at a rigid condition), φ_1^e (Rotation of first point of the element), φ_2^e (Rotation of second point of thr element), $\mu(x)$ (Mass per length of link), L (Length of link), l^e (Length of element), EI (Uniform flexural rigidity), τ_1^e (Torque applying to first point of element), τ_2^e (Torque applying to second point of element), θ (Global angular displacement of link), r (Position vector of the particle in local coordinate) and R (Position vector of the particle in global coordinate). Longitudinal deformations are neglected and no damping is assumed and the manipulator moves in the horizontal plane so, the gravity is not considered.

III. Kinetic Energy

Consider the two-link planar manipulator in Fig. 1 that divided into several elements which are composed of equal section beam elements in Fig. 2.

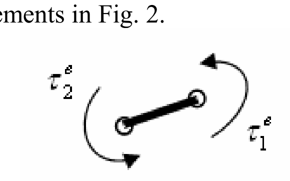

Fig. 2 Schematic of Element and nods

For drive kinetic energy of any element of link 1, consider position vector of an arbitrary point P on link 1 expressed in the inertial coordinate frame is:

$$\mathbf{r}_1 = x\mathbf{i} + v\mathbf{j} \tag{1}$$

Where denotes the displacement vector at the any point. Angles rotation of this point is:

$$\boldsymbol{\omega}_1 = (\dot{\theta}_1 + \dot{v}')\mathbf{k} \tag{2}$$

Absolute position vector and absolute velocity of this point respectively is:

$$\mathbf{R}_1 = x\mathbf{i} + v\mathbf{j} \tag{3}$$

$$\dot{\mathbf{R}}_1 = \dot{v}\mathbf{j} + \dot{\theta}_1\mathbf{k} \times (x\mathbf{i} + v\mathbf{j}) \tag{4}$$

And kinetic energy of any element of first link is:

$$T_1^e = \frac{1}{2}\int_{l^e} \mu\, \dot{\mathbf{R}}_1 \cdot \dot{\mathbf{R}}_1\, dx \tag{5}$$

Substituting Eq. (4) into Eq (5), after multiplying and combining similar terms, the kinetic energy for the first link becomes:

$$T_1^e = \frac{1}{2}\int_{l^e} \mu(v^2\dot{\theta}_1{}^2 + \dot{v}^2 + x^2\dot{\theta}_1{}^2 + 2\dot{v}x\dot{\theta}_1)\, dx \tag{6}$$

For drive kinetic energy of any element of second link, consider position vector of an arbitrary point P on link 2 expressed in the inertial coordinate frame is:

$$\mathbf{r}_2 = x\mathbf{i} + v\mathbf{j} \tag{7}$$

Angles rotation of this point is:

$$\boldsymbol{\omega}_2 = (\dot{\theta}_2 + v')\mathbf{k} \tag{8}$$

Absolute position vector and absolute velocity of this point respectively is:

$$\mathbf{R}_2 = \mathbf{R}_{o2} + \mathbf{r}_2 \quad \text{and} \quad \dot{\mathbf{R}}_2 = \dot{\mathbf{R}}_{o2} + \dot{\mathbf{r}}_2 \tag{9),(10}$$

Component velocity of first point of link 2 is V_{ox} and V_{oy} respectively. By Substituting into Eq (10):

$$\dot{\mathbf{R}}_2 = V_{ox}\,\mathbf{i} + V_{oy}\,\mathbf{j} + \dot{\mathbf{r}}_2 \tag{11}$$

After multiplying and combining similar terms

$$\dot{\mathbf{R}}_2 = (V_{ox} - v\dot{\theta}_2)\mathbf{i} + (V_{oy} + x\dot{\theta}_2 + \dot{v})\mathbf{j} \tag{12}$$

Kinetic energy of any element of second link is

$$T_2^e = \frac{1}{2}\int_{l_e} \mu\, \dot{\mathbf{R}}_2 \cdot \dot{\mathbf{R}}_2 dx \tag{13}$$

Substituting Eq. (12) into Eq. (13), after multiplying and combining similar terms, the kinetic energy for the 2ed link becomes

$$\begin{aligned} T_2^e = \frac{1}{2}\int_{l^e} \mu(V_{ox}^2 + v^2\dot{\theta}_2{}^2 - 2V_{ox}v\dot{\theta}_2 + V_{oy}^2 + \\ x^2\dot{\theta}_2{}^2 + \dot{v}^2 + 2V_{oy}x\dot{\theta}_2 + 2V_{oy}\dot{v} + 2x\dot{\theta}_2\,\dot{v})\,dx \end{aligned} \tag{14}$$

The kinetic energy of the overall system is obtained by adding the kinetic energy of all links.

IV. Potential Energy

The potential energy of the manipulator comprises tow components, U_1 and U_2, due to elasticity of any link. Noting that the potential energy only includes the strain energy and the gravitational effect in this derivation, the longitudinal and tensional deformations are neglected. Potential energy of any element of 1st link due to elasticity from Euler- Bernoulli theory is:

$$U_1^e = \frac{1}{2}\int_{l^e} EI\,(\frac{\partial^2 v}{\partial x^2})dx \tag{15}$$

Where EI is the total flexural rigidity of the 1st link. Potential energy of any element of 2ed link due to elasticity from Euler-Bernoulli theory is:

$$U_2^e = \frac{1}{2}\int_l^e EI(\frac{\partial^2 v}{\partial x^2})dx \tag{16}$$

The potential energy of the overall system is obtained by adding the potential energy of all elements and links.

V. The FE Method

The FE method involves decomposing a structure into several simple pieces or elements. The elements are assumed to be interconnected at certain points, known as nodes. For each element, an equation describing the behavior of the element is obtained through an approximation technique. The elemental equations are then assembled to form the system equation. It is found that by reducing the element size of the structure, that is, increasing the number of elements, the overall solution of the system equation can be made to converge to the exact solution. Similarly, the case of a flexible manipulator clamped to a hub as shown in Fig. 1 is considered. The Bernoulli-Euler beam theory is used to model the elastic behavior of the manipulator. Moreover, the beam is considered to have a constant cross-section and uniform material properties throughout.

Considering linear displacements, the total displacement *y(x,t)* at a distance *x* from the frame origin in the *OX* direction can be described as a function of both the rigid body motion *θ(t)* and elastic deflection $v(x,t)$ as:

$$y(x,t) = x\theta(t) + v(x,t)$$

Using FE method to solve dynamic problems, leads to the well known equation:

$$v(x,t) = [\mathbf{N}(x)]\{\mathbf{q}(t)\} \tag{17}$$

Where **[N(X)]** and **[q(t)]** represent the shape function and nodal displacement respectively. The manipulator is approximated by partitioning it into n elements. As a consequence of using the Bernoulli-Euler beam theory, the FE method requires each node to possess two degrees of freedom, a transverse deflection and a rotation. These necessitate the use of Hermit cubic basis functions as the element shape function. Hence, for any element, the shape function can be obtained as:

$$[\mathbf{N}(x)] = [N_1(x)\ \ N_2(x)\ \ N_3(x)\ \ N_4(x)\ ]$$

For element i the nodal displacement vector is given as:

$$\{\mathbf{q}(t)\} = \{v_{1i}(t)\ \ \varphi_{1i}(t)\ \ v_{2i}(t)\ \ \varphi_{2i}(t)\}^{-1}$$

Where $v_{1i}(t)$ and $v_{2i}(t)$ are the elastic deflections of the node 1 and node 2 of element i and $\varphi_{1i}(t)$ and $\varphi_{2i}(t)$ are the corresponding rotations. Substituting for $v(x,t)$ from (17) into (6), (14), (15), (16) and simplifying yields:

$$T_1^e = \frac{1}{2}\mathbf{q}^T\mathbf{M}_1\mathbf{q}\dot{\theta}_1{}^2 + \frac{1}{2}\dot{\mathbf{q}}^T\mathbf{M}_1\dot{\mathbf{q}} + \frac{1}{2}I\dot{\theta}_1{}^2 + \dot{\mathbf{q}}^T\mathbf{a}^T\dot{\theta}_1 \tag{18}$$

$$\begin{aligned} T_2^e = \frac{1}{2}(M_4V_{ox}^2 + \mathbf{q}^T\mathbf{M}_1\mathbf{q}\dot{\theta}_2{}^2 - 2V_{ox}\mathbf{q}^T\mathbf{M}_2^T\dot{\theta}_2 \\ + M_4V_{oy}^2 + \dot{\mathbf{q}}^T\mathbf{M}_1\dot{\mathbf{q}} + I\dot{\theta}_2{}^2 + 2V_{oy}\dot{\mathbf{q}}^T\mathbf{M}_2^T + \\ 2V_{oy}M_3\dot{\theta}_2 + 2\dot{\mathbf{q}}^T\mathbf{a}^T\dot{\theta}_2) \end{aligned} \tag{19}$$

$$U_1^e = \frac{1}{2}\mathbf{q}^T\mathbf{K}_1\mathbf{q} \quad \text{and} \quad U_2^e = \frac{1}{2}\mathbf{q}^T\mathbf{K}_2\mathbf{q} \tag{20}$$

Defining

$$M_1 = \int_{-1}^1 \mu(\eta)(\mathbf{N}^T(\eta).\mathbf{N}(\eta))\,J\,d\eta \tag{21}$$

$$M_2 = \int_{-1}^1 \mu(\eta)\mathbf{N}^T(\eta)J\,d\eta$$

$$M_3 = \int_{-1}^1 \mu(\eta)(\frac{1}{2}(2i-1+\eta)l^e)J\,d\eta$$

$$M_4 = \int_{-1}^1 \mu(\eta)\,J\,d\eta$$

$$I = \int_{-1}^1 \mu(\eta)(\frac{1}{2}(2i-1+\eta)l^e)^2\,Jd\eta \tag{22}$$

$$\mathbf{a} = \int_{-1}^1 \mu(\eta)(\frac{1}{2}(2i-1+\eta)l^e)\mathbf{N}(\eta)J\,d\eta$$

$$\mathbf{K} = \int_{-1}^{1} EI(\eta)\mathbf{N}''(\eta)\mathbf{N}''(\eta)\, J^{-3} d\eta$$

VI. Manipulator Dynamic Model

The equations of motion of the total manipulator system can be drive by applying the Hamilton principle.

$$\delta\int_{t_1}^{t_2} (T-U)dt + \int_{t_1}^{t_2} \delta w\, dt = 0 \tag{23}$$

Where $L=T\text{-}U$ is the Lagrangian of the total system, and δw is virtual work of conservative forces of element corresponding to the generalized coordinates and is given as follows.

$$\delta w = \tau_1^e(\theta + v'(x_1^e)) + \tau_2^e(\theta + v'(x_2^e)) \tag{24}$$

By using the Hamilton approach, the dynamic equations of motion the flexible manipulator can be derived using the Hamilton principle, by substituting Eqs. (18), (19), (20) and (21) into Eq. (23) and simplifying yields:

$$\begin{aligned} &\mathbf{M}_{11}\ddot{\mathbf{q}}_1 + \mathbf{a}_1{}^T\ddot{\theta}_1 + [\mathbf{K}_1 - \mathbf{M}_{11}\dot{\theta}_1{}^2]\mathbf{q}_1 = \mathbf{F}_{11}^e \\ &\mathbf{a}_1\ddot{\mathbf{q}}_1 + I_1\ddot{\theta}_1 + 2\dot{\mathbf{q}}_1{}^T\mathbf{M}_{11}\mathbf{q}_1\dot{\theta}_1 + \mathbf{q}_1{}^T\mathbf{M}_{11}\mathbf{q}_1\ddot{\theta}_1 = F_{21}^e \end{aligned} \tag{25}$$

$$\begin{aligned} &I_2\ddot{\theta}_2 + \mathbf{q}_2{}^T\mathbf{M}_{12}\mathbf{q}_2\ddot{\theta}_2 + \mathbf{a}_2\ddot{\mathbf{q}}_2 - \mathbf{q}_2{}^T\dot{V}_{ox}\mathbf{M}_{22} + \dot{V}_{oy}M_{32} + \\ &2\dot{\mathbf{q}}_2{}^T\mathbf{M}_{12}\mathbf{q}_2\dot{\theta}_2 - \dot{\mathbf{q}}_2{}^T V_{ox}\mathbf{M}_{22} = F_{22}^e \end{aligned}$$

and

$$\begin{aligned} &\mathbf{a}_2{}^T\ddot{\theta}_2 + \mathbf{M}_{12}\ddot{\mathbf{q}}_2 + \mathbf{K}_2\mathbf{q}_2 - \mathbf{M}_{12}\dot{\theta}_2{}^2\mathbf{q}_2 \\ &+ \mathbf{M}_{22}\dot{V}_{oy} + \mathbf{M}_{22}V_{ox}\dot{\theta}_2 = \mathbf{F}_{12}^e \end{aligned} \tag{26}$$

By assembling all equation of elements, the desired dynamic equations of motion of the system can accordingly be obtained as:

$$\mathbf{M}\ddot{\mathbf{Q}} + \mathbf{K}\dot{\mathbf{Q}} + \mathbf{C} = \mathbf{F} \tag{27}$$

That the matrixes can be represented as:

$$\mathbf{M} = \begin{bmatrix} I_1 + \mathbf{q}_1{}^T\mathbf{M}_{11}\mathbf{q}_1 & \mathbf{a}_1 & 0 & 0 \\ \mathbf{a}_1{}^T & \mathbf{M}_{11} & 0 & 0 \\ 0 & 0 & I_2 + \mathbf{q}_2{}^T\mathbf{M}_{12}\mathbf{q}_2\ \mathbf{a}_2 & \mathbf{a}_2 \\ 0 & 0 & \mathbf{a}_2{}^T & \mathbf{M}_{12} \end{bmatrix} \tag{28}$$

$$\mathbf{K} = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & \mathbf{K}_1 - \mathbf{M}_{11}\dot{\theta}_1{}^2 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \mathbf{K}_2 - \mathbf{M}_{12}\dot{\theta}_2{}^2 \end{bmatrix}$$

$$\mathbf{C} = \begin{Bmatrix} 0 \\ 0 \\ -\mathbf{q}_2{}^T\dot{V}_{ox}\mathbf{M}_{22} + \dot{V}_{oy}M_{32} \\ \mathbf{M}_{22}\dot{V}_{oy} \end{Bmatrix}$$

$$\mathbf{F} = \begin{Bmatrix} F_{21} - 2\dot{\mathbf{q}}_1{}^T\mathbf{M}_{11}\mathbf{q}_1\dot{\theta}_1 \\ \mathbf{F}_{11} \\ F_{22} - 2\dot{\mathbf{q}}_2{}^T\mathbf{M}_{12}\mathbf{q}_2\dot{\theta}_2 + \dot{\mathbf{q}}_2{}^T V_{ox}\mathbf{M}_{22} \\ \mathbf{F}_{12} - \mathbf{M}_{22}V_{ox}\dot{\theta}_2 \end{Bmatrix}$$

$$\mathbf{Q} = \{\theta_1, v_2^1, \varphi_2^1, v_2^2, \varphi_2^2, \ldots, \theta_2, v_2^1, \varphi_2^1, v_2^2, \varphi_2^2 \ldots\}$$

In this study, the robot manipulators are highly nonlinear and heavily coupled complex systems. Therefore, its accurate dynamic model is difficult to obtain.

VII. Numerical Simulations

In order to verify the validity of the model, one simulation for flexible planar manipulators is performed. Simulation is about a two-link flexible planar manipulator. The links of the manipulator are initially at rest and parameters of the flexible manipulator are shown in Table I. Initial position is corresponding to Fig. 3 and driving torques at the two joints is shown in Fig. 4 for move in the special path. The manipulator moves under the effect of torques. This case corresponds to an elastic motion in planar. The flexible displacements at the tips of the links are calculated by using the model developed in this paper. The results are illustrated in Figs. 5 to 16. It can be observed from Figs. 5 to 16 that the flexible displacements of the two links are in very good agreement with those in rigid manipulator (Rigid motion are shown with dash in this Figures).

TABLE I
PARAMETER OF THE FLEXIBLE MANIPULATOR SYSTEM

parameter	Quantity
EI_1	105 $(N.M^2)$
L_1	1.4142 (m)
μ_1	0.61732 (Kg/m)
Number element of link 1	2
EI_2	105 $(N.M^2)$
L_2	1 (m)
μ_2	0.61732 (Kg/m)
Number element of link 2	1

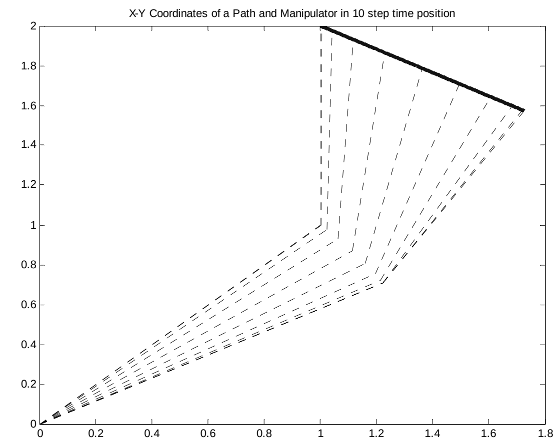

Fig. 3 X-Y coordinates of a path and Manipulator in 10 step time position

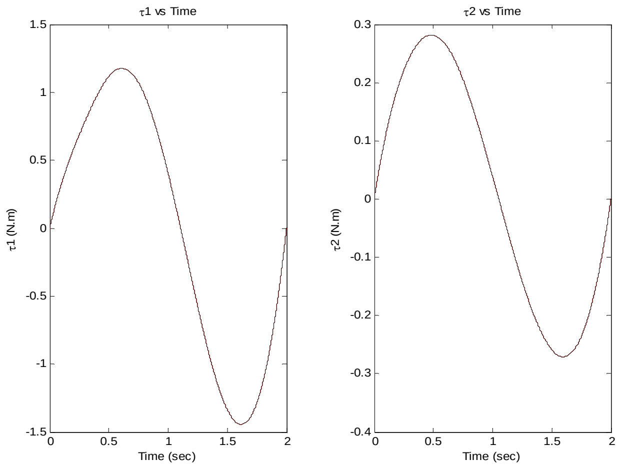

Fig. 4 Torque input for the two joints of manipulator

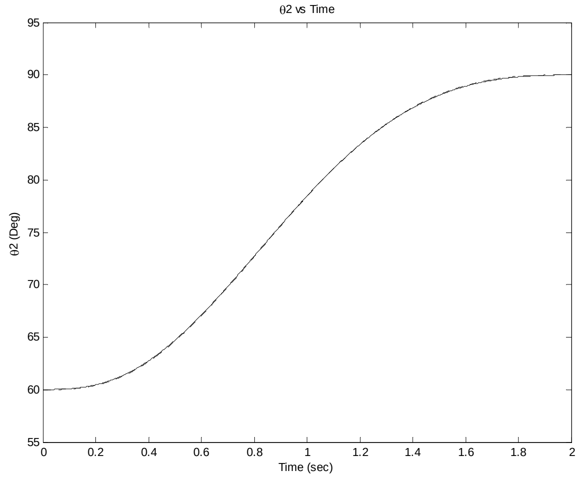

Fig. 7 Link 2 angle of the manipulator

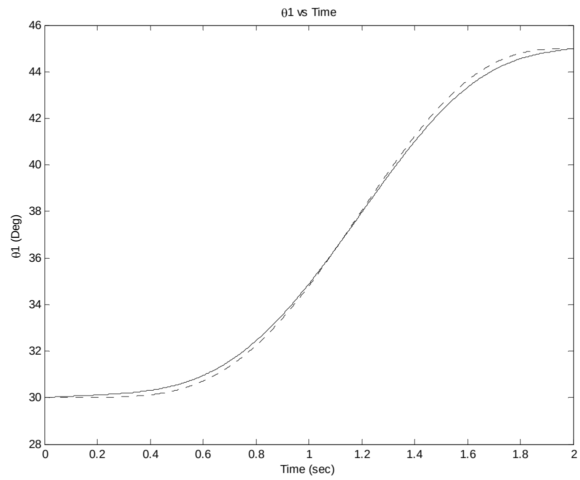

Fig. 5 Link 1 angle of the manipulator

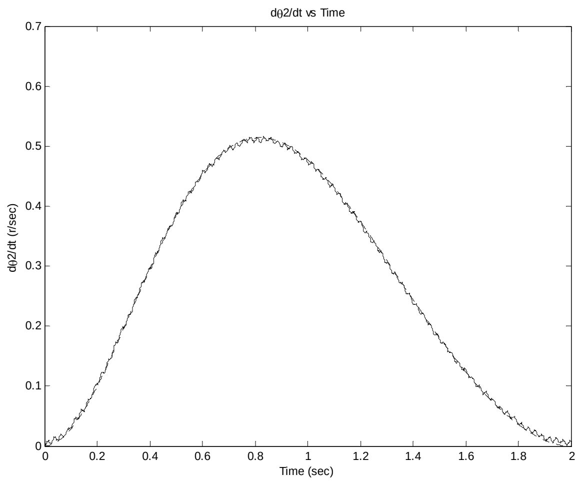

Fig. 8 link 2 velocity angle of the manipulator

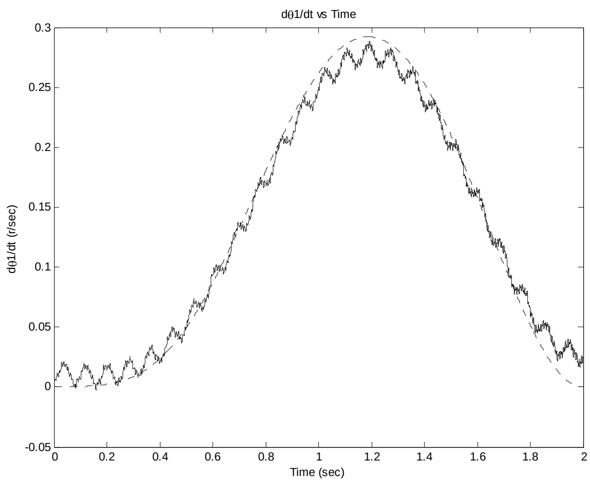

Fig. 6 link 1 velocity angle of the manipulator

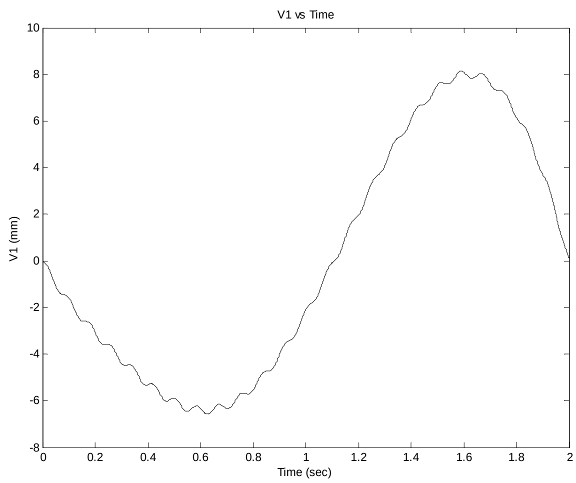

Fig. 9 Flexible displacement of link 1

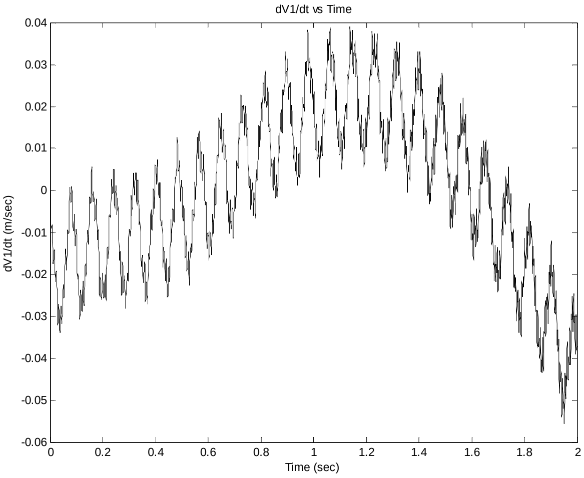

Fig. 10 End-point velocity of the link 1

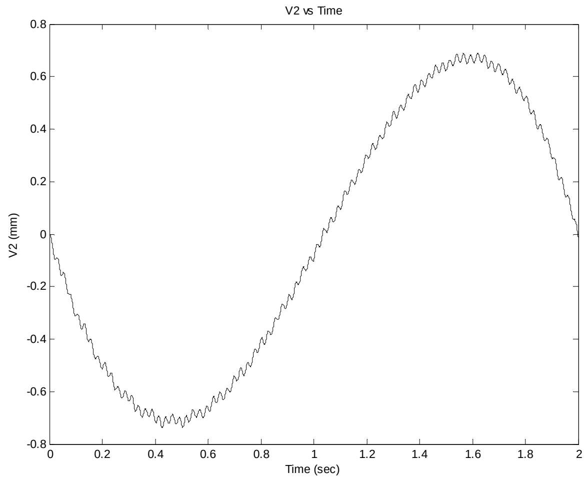

Fig. 11 Flexible displacement of link 2

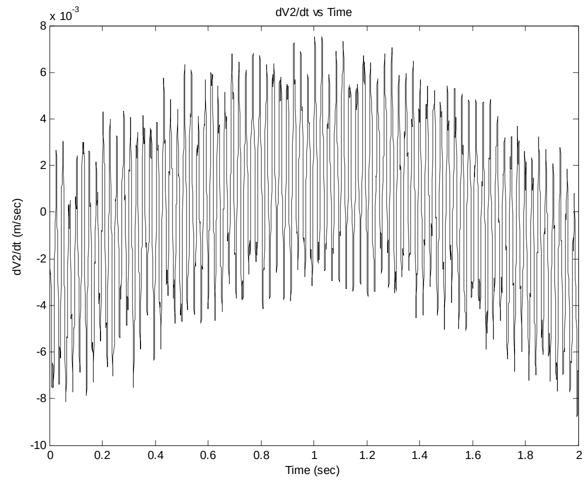

Fig. 12 End-point velocity of the link 2

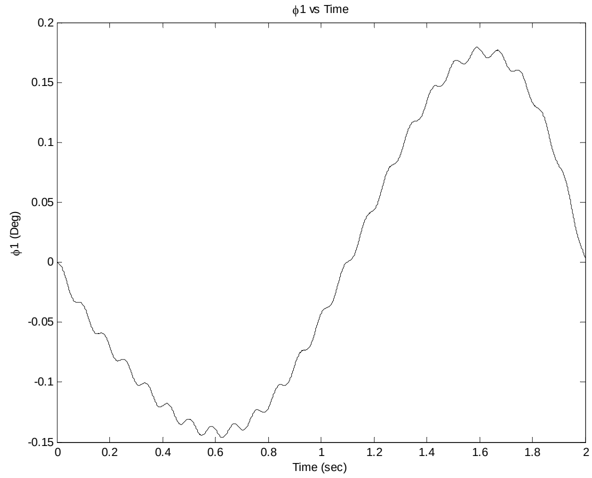

Fig. 13 End-point angle of the link 1

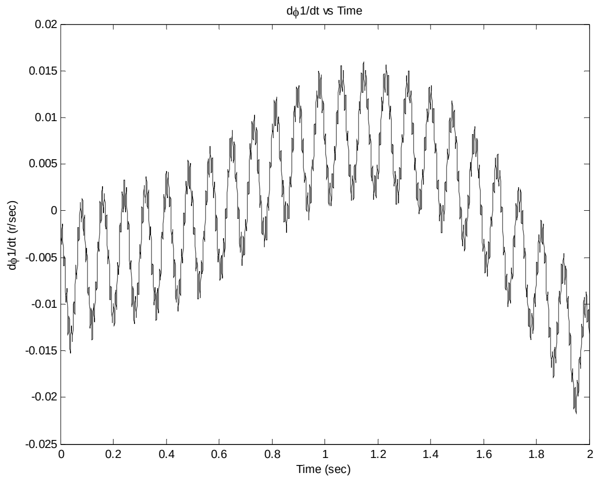

Fig. 14 End-point velocity angle of the link 1

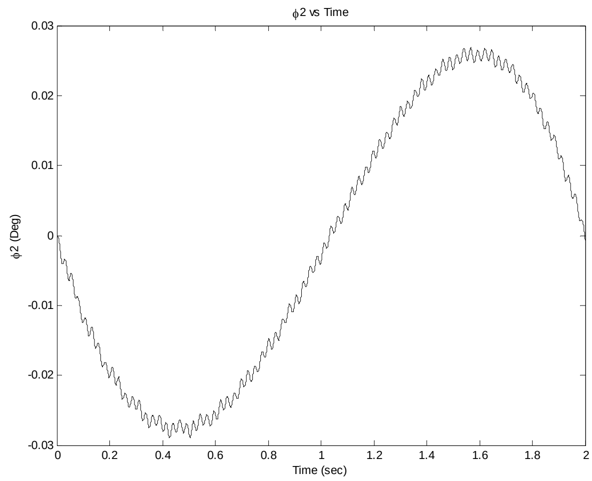

Fig. 15 End-point angle of the link 2

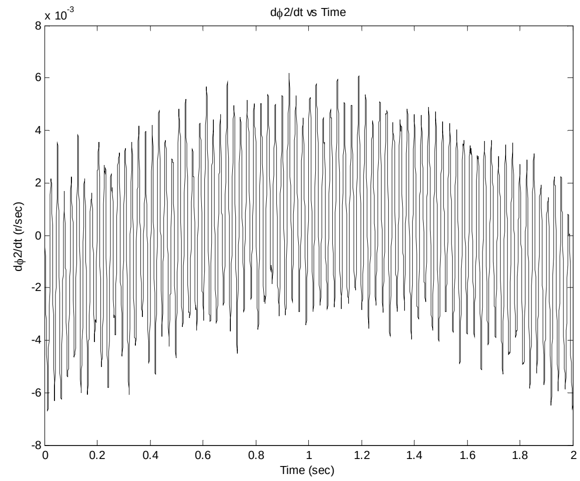

Fig. 16 End-point velocity angle of the link 2

VIII. Conclusion

A dynamic model for a general planar multi-link flexible manipulator is established in this investigation. The FE method is used to model the flexible deformation of the links and Hamilton principle is adopted to establish the equations of motion for the total manipulator system. The motion of each link is separate as the nonlinear gross rigid-body motion upon which small elastic deviations and vibrations are superimposed. The rigid-body motion and elastic deformation are decoupled by linearizing the equations of motion around the rigid-body reference trajectory. Numerical simulations are included to verify the validity of the established model. Steeper driving torques tends to excite larger flexible deformations of the link. But the difference between the magnitudes of the flexible deformations is very remarkable. Comparison between the rigid motion and the flexible manipulator dynamic model is investigated.

References

[1] Yazdizadeh, A. & Khorasani, K. & Patel, R V. (2000). Identification of a two-link flexible manipulator using adaptive time delay neural networks: IEEE Trans. Syst., Man, Cybern. B, vol. 30, pp. 165–172.

[2] Su, Z. & Khorasani, K. (2001). A neural-network-based controller for a single-link flexible manipulator using the inverse dynamics approach: IEEE Trans. Ind. Electron., vol. 48, pp. 1074–1086, Apr.

[3] Martins, J M. & Mohamed, Z. & Tokhi, M O. & Costa, J. & Botto, M A. (2000). Approaches for dynamic modeling of flexible manipulator systems: Inst. Elec. Eng. Proc. Contr. Theory and Applications, vol. 150, no. 4, pp. 401–411.

[4] Yang, H. & Krishman, H. & Ang Jr, M H. (2000). Tip-trajectory tracking control of single-link flexible robots by output redefinition: Inst. Elec. Eng. Proc. Contr. Theory Applicat., vol. 147, no. 6, pp. 580–587.

[5] Du, Zhaocai. & Yu, Yueqing. (2008). Dynamic Modeling and Inverse Dynamic Analysis of Flexible Parallel Robots: International Journal of Advanced Robotic Systems, Vol. 5, no. 1, ISSN 1729-8806, pp. 115-122.

[6] Ata, A. A. & Johar, H. (2004). Dynamic simulation of task constrained of a rigid-flexible manipulator: International Journal of Advanced Robotic Systems, Vol. 1, no. 2, ISSN 1729-8806, pp. 61 – 66.

[7] Tian, L. & Wang, J. & Mao, Z. (2004). Constrained Motion Control of Flexible Robot Manipulators Based on Recurrent Neural Networks: IEEE Transactions on systems, man, and cybernetics- part B: cybernetics, Vol. 34, no. 3.

[8] Book, W J. (1984). Recursive lagrangian dynamics of flexible manipulator arms: International Journal of Robotics Research, 3:87±101.

[9] Erdman, A G. & Sandor, G N. & Oakberg, R G. (1972). A general method for Kineto-Elastodynamic analysis and synthesis of mechanisms: ASME Journal of Engineering for Industry, 94:1193±205.

[10] Book, W J. (1990). Modeling, design, and control of flexible manipulator arms: A tutorial review, Proceedings of the 29th IEEE Conference on Decision and Control, IEEE Control Systems Society, pp.500-506, ISBN: 90CH2917-3.

[11] Santosha, K D. & Peter, E. (2006). Dynamic analysis of flexible manipulators, a literature review. Mechanism and Machine Theory, Vol.41, pp.749-777, ISSN: 0094-114X

[12] Kim, Hyoung-Kyu. Choi, Seung-Bok. & Thompson Brian S. (2000). Compliant control of two-link flexible manipulator featuring piezoelectric actuators: Mechanism and Machines Theory 36, 411-424.

[13] Ho, Cheol Shin. & Seung, Bok Choi. (2001). Position control of a two link flexible manipulator featuring piezoelectric actuators and sensors: Mechatronics 11, 707-729.

[14] Trigolo, Kuga, De Souza. (2003). Parameter Identification of a Rigid Flexible Satellite Using Kalman Filter: Proceedings, COBEM, November 10-14, Sao Paulo

[15] Turcic, D A. & Midha, A. (1984). Generalized equations of motion for the dynamic analysis of elastic mechanism systems: ASME Journal of Dynamic Systems, Measurements, and Control, 106:243±8.