Intuitionistic Fuzzy Points in Semigroups

Sujit Kumar Sardar  Manasi Mandal  Samit Kumar Majumder

Abstract—The notion of intuitionistic fuzzy sets was introduced by Atanassov as a generalization of the notion of fuzzy sets. Y.B. Jun and S.Z. Song introduced the notion of intuitionistic fuzzy points. In this paper we find some relations between the intuitionistic fuzzy ideals of a semigroup S and the set of all intuitionistic fuzzy points of S.

Keywords—Semigroup, Regular(intra-regular) semigroup, Intuitionistic fuzzy point, Intuitionistic fuzzy subsemigroup, Intuitionistic fuzzy ideal, Intuitionistic fuzzy interior ideal, Intuitionistic fuzzy semiprime ideal, Intuitionistic fuzzy prime ideal.

I. INTRODUCTION

A semigroup is an algebraic structure consisting of a non-empty set S together with an associative binary operation[4]. The formal study of semigroups began in the early 20th century. Semigroups are important in many areas of mathematics, for example, coding and language theory, automata theory, combinatorics and mathematical analysis. The concept of fuzzy sets was introduced by Lofti Zadeh[19] in his classic paper in 1965. A subsemigroup was also introduced by Jun[18] in [17]. In [9], Kuroki characterized several classes of semigroups in terms of fuzzy left, fuzzy right and fuzzy bi-ideals. Others who worked on fuzzy semigroup theory, such as X.Y. Xie[16], [17], Y.B. Jun[5], [6], are mentioned in the bibliography. X.Y. Xie[16] introduced the idea of extensions of fuzzy ideals in semigroups. The notion of intuitionistic fuzzy sets was introduced by Atanassov[1], [2], [3] as a generalization of the notion of fuzzy sets. Pu and Liu[12] introduced the notion of fuzzy points. In [15], X.P. Wang, Z.W. Mo, W.J. Liu, in [18] Y.H. Yon and in [7] K.H. Kim characterized fuzzy ideals as fuzzy points of semigroups. Y.B. Jun and S.Z. Song introduced the notion of intuitionistic fuzzy points[6]. In this paper, we consider the semigroup S of the intuitionistic fuzzy points of a semigroup S, and discuss some relations between the fuzzy subsemigroups[fuzzy bi-ideals, fuzzy interior ideals, fuzzy ideals, fuzzy prime ideals, fuzzy semiprime ideals] of S and the subsets of S. Among other results we obtain some characterization theorems of regular and intra-regular semigroups in terms of intuitionistic fuzzy points.

II. PRELIMINARIES

In this section we discuss some elementary definitions that we use in the sequel.

Definition II.1. [11] If (X, +) is a mathematical system such that ∀a, b, c ∈ X, (a + b) + c = a + (b + c), then + is called associative and (X, +) is called a semigroup.

Definition II.2. [11] A subsemigroup of a semigroup S is a non-empty subset I of S such that I^2 ⊆ I.

Definition II.3. [11] A subsemigroup I of a semigroup S is a called an interior ideal of S if IS ⊆ I.

Definition II.4. [11] A subsemigroup I of a semigroup S is a called a bi-ideal of S if ISI ⊆ I.

Definition II.5. [11] A left (right) ideal of a semigroup S is a non-empty subset I of S such that S I ⊆ I (IS ⊆ I). If I is both a left and a right ideal of a semigroup S, then we say that I is an ideal of S.

Definition II.6. [11] Let S be a semigroup. Then an ideal I of S is said to be (i) prime if for ideals A, B of S, AB ⊆ I implies that A ⊆ I or B ⊆ I, (ii) semiprime if for an ideal A of S, A^2 ⊆ I implies that A ⊆ I.

Definition II.7. [1] [2] The intuitionistic fuzzy sets defined on a non-empty set X as objects having the form

A = {< x, μ_A(x), ν_A(x) > : x ∈ X},

where the functions μ_A : X → [0, 1] and ν_A : X → [0, 1] denote the degree of membership and the degree of non-membership of each element x ∈ X to the set A respectively, and 0 ≤ μ_A(x) + ν_A(x) ≤ 1 for all x ∈ X.

For the sake of simplicity, we shall use the symbol A = (μ_A, ν_A) for the intuitionistic fuzzy subset A = {< x, μ_A(x), ν_A(x) > : x ∈ X}.

Definition II.8. [6] Let α, β ∈ [0, 1] with α + β ≤ 1. An intuitionistic fuzzy point, written as x_{α, β}, is defined to be an intuitionistic fuzzy subset of S, given by

x_{α, β}(y) = \begin{cases} 
(α, β) & \text{if } x = y \\
(0, 1) & \text{otherwise}
\end{cases}

Definition II.9. [14] A non-empty intuitionistic fuzzy subset A = (μ_A, ν_A) of a semigroup S is called an intuitionistic fuzzy subsemigroup of S if (i) μ_A(xy) ≥ min{μ_A(x), μ_A(y)} ∀x, y ∈ S, (ii) ν_A(xy) ≤ max{ν_A(x), ν_A(y)} ∀x, y ∈ S.

Definition II.10. [14] An intuitionistic fuzzy subsemigroup A = (μ_A, ν_A) of a semigroup S is called an intuitionistic fuzzy
interior ideal of $S$ if (i) $\mu_A(xay) \geq \mu_A(a) \forall x, a, y \in S$, (ii) $\nu_A(xay) \leq \nu_A(a) \forall x, a, y \in S$.

**Definition II.11.** [14] An intuitionistic fuzzy subsemigroup $A = (\mu_A, \nu_A)$ of a semigroup $S$ is called an intuitionistic fuzzy bi-ideal of $S$ if (i) $\mu_A(xxy) \geq \min(\mu_A(x), \mu_A(y)) \forall x, y \in S$, (ii) $\nu_A(xxy) \leq \max\{\nu_A(x), \nu_A(y)\} \forall x, y \in S$.

**Definition II.12.** [14] A non-empty intuitionistic fuzzy subset $A = (\mu_A, \nu_A)$ of a semigroup $S$ is called an intuitionistic fuzzy left(right) ideal of $S$ if (i) $\mu_A(xy) \geq \mu_A(y)(\text{resp. } \mu_A(x)) \forall x, y \in S$, (ii) $\nu_A(xy) \leq \nu_A(y)(\text{resp. } \nu_A(x)) \forall x, y \in S$.

**Definition II.13.** [14] A non-empty intuitionistic fuzzy subset $A = (\mu_A, \nu_A)$ of a semigroup $S$ is called an intuitionistic fuzzy two-sided ideal or an intuitionistic fuzzy ideal of $S$ if it is both an intuitionistic fuzzy left and an intuitionistic fuzzy right ideal of $S$.

Alternative definition of Definition 2.13, is as follows.

**Definition II.14.** [14] A non-empty intuitionistic fuzzy subset $A = (\mu_A, \nu_A)$ of a semigroup $S$ is called an intuitionistic fuzzy ideal of $S$ if (i) $\mu_A(xy) \geq \max\{\mu_A(x), \mu_A(y)\} \forall x, y \in S$, (ii) $\nu_A(xy) \leq \min\{\nu_A(x), \nu_A(y)\} \forall x, y \in S$.

**Definition II.15.** [14] An intuitionistic fuzzy ideal $A = (\mu_A, \nu_A)$ of a semigroup $S$ is called an intuitionistic fuzzy prime ideal of $S$ if (i) $\mu_A(x) \geq \mu_A(x^2) \forall x \in S$, (ii) $\nu_A(x) \leq \mu_A(x^2) \forall x \in S$.

**Definition II.16.** [14] An intuitionistic fuzzy ideal $A = (\mu_A, \nu_A)$ of a semigroup $S$ is called an intuitionistic fuzzy prime ideal of $S$ if (i) $\mu_A(xy) = \max\{\mu_A(x), \mu_A(y)\} \forall x, y \in S$, (ii) $\nu_A(xy) = \min\{\nu_A(x), \nu_A(y)\} \forall x, y \in S$.

### III. MAIN RESULTS

Let $\mathcal{I}(S)$ be the set of all intuitionistic fuzzy subsets of a semigroup $S$. For each $A = (\mu_A, \nu_A), \ B = (\mu_B, \nu_B) \in \mathcal{I}(S)$, the product of $A$ and $B$ is an intuitionistic fuzzy subset $A \circ B$ defined as follows:

$$A \circ B = \{x \in S : (\mu_A \circ \mu_B)(x), (\nu_A \circ \nu_B)(x) > x \in S\}$$

where $\mu_A \circ \mu_B(x) = \sup_{x \in S} \{\min\{\mu_A(u), \mu_B(v)\} : u, v \in S\}$ and $\nu_A \circ \nu_B(x) = \inf_{x \in S} \{\max\{\nu_A(u), \nu_B(v)\} : u, v \in S\}$.

It is clear that $(A \circ B) \circ C = A \circ (B \circ C)$, and that if $A \subseteq B$, then $A \circ C \subseteq B \circ C$ and $C \circ A \subseteq C \circ B$ for any $A, B, C \in \mathcal{I}(S)$. Thus $\mathcal{I}(S)$ is a semigroup with the product $\circ$.

Let $S$ be the set of all intuitionistic fuzzy points in a semigroup $S$. Then $x_{(\alpha, \beta), (\gamma, \delta)} = (xy)_{(\alpha \wedge \gamma, \beta \wedge \delta)} \in S$, where $\alpha \wedge \gamma = \min\{\alpha, \gamma\}$ and $\beta \wedge \delta = \max\{\beta, \delta\}$. For any $x_{(\alpha, \beta), (\gamma, \delta)} \in S$, $\alpha \wedge \gamma \leq \alpha$ and $\beta \wedge \delta \leq \beta$. Thus $S$ is a subsemigroup of $\mathcal{I}(S)$.

For any $A = (\mu_A, \nu_A) \in \mathcal{I}(S)$, let $\mathcal{A}$ denote the set of all intuitionistic fuzzy points contained in $A = (\mu_A, \nu_A)$, that is, $\mathcal{A} = \{x_{(\alpha, \beta)} : \mu_A(x) \geq \alpha \text{ and } \nu_A(x) \leq \beta\}$. If $x_{(\alpha, \beta)} \in S$, then $\alpha > 0$ and $\beta < 1$.

**Proposition III.1.** Let $A = (\mu_A, \nu_A)$ and $B = (\mu_B, \nu_B)$ be two intuitionistic fuzzy subsets of a semigroup $S$. Then

(i) $A \cup B = A \cup B$.

(ii) $A \cap B = A \cap B$.

(iii) $A \circ B = A \circ B$.

Proof: (i) Let $x_{(\alpha, \beta)} \in A \cup B \iff \{x \in S : (\mu_A \cup \mu_B)(x) \geq \alpha \text{ and } (\nu_A \cup \nu_B)(x) \leq \beta\} = \{x \in S : \mu_A(x) \geq \alpha \text{ or } \mu_B(x) \geq \alpha\}$ and $\{x \in S : (\mu_A \cap \mu_B)(x) \geq \beta\} = \{x \in S : \mu_A(x) \geq \beta \text{ and } \mu_B(x) \geq \beta\}$.

(ii) Let $x_{(\alpha, \beta)} \in A \cap B \iff \{x \in S : (\mu_A \cap \mu_B)(x) \geq \alpha\}$ and $\{x \in S : (\nu_A \cap \nu_B)(x) \leq \beta\} = \{x \in S : \mu_A(x) \geq \alpha \text{ and } \nu_B(x) \leq \beta\}$.

(iii) $A \circ B = \{x \in S : (\mu_A \circ \mu_B)(x) \geq \alpha \} \text{ and } \{x \in S : (\nu_A \circ \nu_B)(x) \leq \beta\} = \{x \in S : \mu_A(x) \geq \alpha \text{ and } \nu_B(x) \leq \beta\}$.

**Theorem III.2.** Let $A = (\mu_A, \nu_A)$ be a non-empty intuitionistic fuzzy subsemigroup of $S$. Then following conditions are equivalent:

(i) $A = (\mu_A, \nu_A)$ is an intuitionistic fuzzy subsemigroup of $S$.

(ii) $A$ is a subsemigroup of $S$.

Proof: (i) $\Rightarrow$ (ii): Let $A = (\mu_A, \nu_A)$ be an intuitionistic fuzzy subsemigroup of $S$. Let $x_{(\alpha, \beta), (\gamma, \delta)} \in A$. Then $\mu_A(x) \geq \alpha > 0$, $\nu_A(x) \geq \gamma > 0$ and $\nu_A(x) \leq \beta < 1$, $\nu_A(x) \leq \delta < 1$. Since $\mu_A(xy) \geq \min\{\mu_A(x), \mu_A(y)\} \geq \alpha \wedge \gamma$ and $\nu_A(xy) \leq \max\{\nu_A(x), \nu_A(y)\} \leq \beta \vee \delta$. Consequently, $x_{(\alpha, \beta) \circ (\gamma, \delta)} = (xy)_{(\alpha \wedge \gamma, \beta \wedge \delta)} \in A$. This implies that $A^2 \subseteq A$. Hence $A$ is a subsemigroup of $S$. 

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(ii) \( \Rightarrow (i) \): Let us suppose that \( A \) is a subsemigroup of \( S \). Let \( x, y \in S \). If \( \mu_A(x) = \mu_A(y) = 0 \) and \( \nu_A(x) = \nu_A(y) = 1 \), then \( \min(\mu_A(x), \mu_A(y)) = 0 \leq \mu_A(xy) \) and \( \max(\nu_A(x), \nu_A(y)) = 1 \geq \nu_A(xy) \). If \( \mu_A(x) = \mu_A(y) \neq 0 \) and \( \nu_A(x) = \nu_A(y) < 1 \), then \( x \in \mu_A(x, \nu_A(y)) \) and \( y \in \mu_A(y, \nu_A(x)) \). Since \( A \) is a subsemigroup of \( S \), so we have \( (xy) \in \mu_A(x, \nu_A(y)) \cap \mu_A(y, \nu_A(x)) \). Hence \( A \) is a subsemigroup of \( S \).

Theorem III.3. Let \( A = (\mu_A, \nu_A) \) be a non-empty intuitionistic fuzzy subset of a semigroup \( S \). Then following conditions are equivalent:

(i) \( A = (\mu_A, \nu_A) \) is an intuitionistic fuzzy bi-ideal of \( S \).

(ii) \( A \) is a bi-ideal of \( S \).

Proof: (i) \( \Rightarrow (ii) \): Let \( A = (\mu_A, \nu_A) \) be an intuitionistic fuzzy bi-ideal of \( S \). Then \( A = (\mu_A, \nu_A) \) is an intuitionistic fuzzy subsemigroup of \( S \). Hence by Theorem 3.2, \( A \) is a subsemigroup of \( S \). Let \( x, y, z \in S \). If \( \mu_A(x) = \mu_A(z) = 0 \) and \( \nu_A(x) = \nu_A(z) = 1 \), then \( \min(\mu_A(x), \mu_A(z)) = 0 \leq \mu_A(xy) \) and \( \max(\nu_A(x), \nu_A(z)) = 1 \geq \nu_A(xy) \). If \( \mu_A(x) = \mu_A(z) \neq 0 \) and \( \nu_A(x) = \nu_A(z) < 1 \), then \( x \in \mu_A(x, \nu_A(z)) \) and \( z \in \mu_A(z, \nu_A(x)) \). Since \( A \) is a subsemigroup of \( S \), so we have \( xy \in \mu_A(x, \nu_A(z)) \cap \mu_A(z, \nu_A(x)) \). Hence \( A \) is a subsemigroup of \( S \).

Theorem III.4. Let \( A = (\mu_A, \nu_A) \) be a non-empty intuitionistic fuzzy subset of a semigroup \( S \). Then following conditions are equivalent:

(i) \( A = (\mu_A, \nu_A) \) is an intuitionistic fuzzy ideal of \( S \).

(ii) \( A \) is an interior ideal of \( S \).

Proof: (i) \( \Rightarrow (ii) \): Let \( A = (\mu_A, \nu_A) \) be an intuitionistic fuzzy ideal of \( S \). Then \( A = (\mu_A, \nu_A) \) is an intuitionistic fuzzy subsemigroup of \( S \). Hence by Theorem 3.2, \( A \) is a subsemigroup of \( S \). Let \( x, y, z \in S \). If \( \mu_A(y) = 0 \) and \( \nu_A(y) = 1 \), then \( \mu_A(xy) = 0 \leq \mu_A(xy) \) and \( \nu_A(xy) = 1 \geq \nu_A(xy) \). If \( \mu_A(y) \neq 0 \) and \( \nu_A(y) < 1 \), then \( y \notin \mu_A(y, \nu_A(x)) \) and \( \nu_A(y, \mu_A(x)) \notin A \). Since \( A \) is an interior ideal of \( S \), so we have \( (xy) \in \mu_A(y, \nu_A(x)) \). Hence \( A \) is an interior ideal of \( S \).

Theorem III.5. Let \( A = (\mu_A, \nu_A) \) be a non-empty intuitionistic fuzzy subset of a semigroup \( S \). Then following conditions are equivalent:

(i) \( A = (\mu_A, \nu_A) \) is an intuitionistic fuzzy (right, left) ideal of \( S \).

(ii) \( A \) is a right(left) ideal of \( S \).

Proof: (i) \( \Rightarrow (ii) \): Let \( A = (\mu_A, \nu_A) \) be an intuitionistic fuzzy right ideal of \( S \). Let \( x, y, z \in S \). Since \( \mu_A(x) \geq 0 \) and \( \nu_A(x) \leq 1 \), then \( \mu_A(xy) \geq \mu_A(x) \) and \( \nu_A(xy) \leq \nu_A(x) \). Hence \( A \) is a right ideal of \( S \).

(ii) \( \Rightarrow (i) \): Let us suppose that \( A \) is a right ideal of \( S \). Let \( x, y \in S \). If \( \mu_A(x) = 0 \) and \( \nu_A(x) = 1 \), then \( \mu_A(xy) \leq \mu_A(xy) \) and \( \nu_A(xy) = 1 \geq \nu_A(xy) \). If \( \mu_A(x) \neq 0 \) and \( \nu_A(x) < 1 \), then \( x \notin \mu_A(x, \nu_A(y)) \) and \( y \notin \mu_A(y, \nu_A(x)) \). Hence \( A \) is a right ideal of \( S \).

Remark 1. It is clear that any ideal of a semigroup \( S \) is an interior ideal of \( S \). It is also clear that any intuitionistic fuzzy ideal of a semigroup \( S \) is an intuitionistic fuzzy interior ideal of \( S \).

Definition III.6. [9] A semigroup \( S \) is called regular if, for each element \( a \in S \), there exists an element \( x \in S \) such that \( a = axa \). A semigroup \( S \) is called intra-regular if, for each element \( x \in S \), there exist elements \( a, b \in S \) such that \( x = axa = b \).

Theorem III.7. Let \( A = (\mu_A, \nu_A) \) be a non-empty intuitionistic fuzzy subset of a regular semigroup \( S \). Then following conditions are equivalent:

(i) \( A = (\mu_A, \nu_A) \) is an intuitionistic fuzzy (right, left) ideal of \( S \).

(ii) \( A \) is an interior ideal of \( S \).

Proof: (i) \( \Rightarrow (ii) \): Follows easily from Remark 1 and Theorem 3.4.

(ii) \( \Rightarrow (i) \): Let us suppose that \( A \) is an interior ideal of \( S \). Let \( x \in S \). Then there exists an element \( a \in S \) such that \( x = axa \). If \( \mu_A(x) = 0 \) and \( \nu_A(x) = 1 \),
then $\mu_A(x) = 0 \leq \mu_A(xy)$ and $\nu_A(x) = 1 \geq \nu_A(xy)$. If $\mu_A(x) \neq 0$ and $\nu_A(x) < 1$, then $x(\mu_A(x),\nu_A(x)) = A$ and $y(\mu_A(x),\nu_A(x)) = S$. Since $A$ is an interior ideal of $S$, we have $(x\gamma y)(\mu_A(x),\nu_A(x)) = (x(\mu_A(x),\nu_A(x)),y(\mu_A(x),\nu_A(x))) = (x(\mu_A(x),\nu_A(x)) \circ y(\mu_A(x),\nu_A(x))) \in A$.

Conversely, let $S$ be an intuitionistic fuzzy left ideal and $a \in S$. Then for any $\alpha, \beta \in [0,1]$, there exist elements $x(\gamma,\delta), y(\eta,\zeta) \in S$ such that $a(\alpha,\beta) = x(\gamma,\delta) \circ a(\alpha,\beta) = y(\eta,\zeta)$. This implies that $x = ax^2$ and $y \in S$. Hence $S$ is intra-regular.

**Theorem III.11.** A semigroup $S$ is regular if and only if the semigroup $S$ is regular.

**Proof:** Let $a(\alpha,\beta) \in S$ and $a \in S$. Then there exists an element $x \in S$ such that $a = axa$ (since $S$ is regular). So $x(\alpha,\beta) \in S$. Then $a(\alpha,\beta) \circ x(\alpha,\beta) = (axa)(\alpha,\beta,\delta) = (axa)(\alpha,\beta,\delta)$. Hence $S$ is regular.

**Lemma III.12.** [9] For a semigroup $S$, the following conditions are equivalent:

(i) $S$ is intra-regular.

(ii) $S \subseteq SL \subseteq LR$ holds for every left ideal $L$ and right ideal $R$ of $S$.

**Lemma III.13.** [14] For a semigroup $S$, the following conditions are equivalent:

(i) $S$ is intra-regular.

(ii) $A \cap B \subseteq A \cap B$ for every intuitionistic fuzzy left ideal $A = (\mu_A, \nu_A)$ and intuitionistic fuzzy right ideal $B = (\mu_B, \nu_B)$ of $S$.

**Theorem III.14.** For a semigroup $S$, the following conditions are equivalent:

(ii) $S$ is intra-regular.

(iii) $A \cap B \subseteq A \cap B$ for every intuitionistic fuzzy left ideal $A = (\mu_A, \nu_A)$ and intuitionistic fuzzy right ideal $B = (\mu_B, \nu_B)$ of $S$.

**Proof:** (ii) $\Rightarrow$ (iii) : Let $S$ be an intra-regular semigroup and $A = (\mu_A, \nu_A), B = (\mu_B, \nu_B)$ be respectively intuitionistic fuzzy left and intuitionistic fuzzy right ideal of $S$. By Theorem 3.10, $S$ is an intra-regular semigroup. By Theorem 3.5, $A$ and $B$ are respectively left and right ideal of the semigroup $S$. Hence by Lemma 3.12, $A \cap B \subseteq A \cap B$.

(ii) $\Rightarrow$ (i) : Let $A = (\mu_A, \nu_A)$ be an intuitionistic fuzzy left ideal and intuitionistic fuzzy right ideal of $S$. The proof is similar in case of intuitionistic fuzzy left ideal and intuitionistic fuzzy right ideal.

**Theorem III.10.** A semigroup $S$ is intra-regular if and only if the semigroup $S$ is intra-regular.

**Proof:** Let $a(\alpha,\beta) \in S$ and $a \in S$. Then there exist elements $x, y \in S$ such that $a = ax^2y$ (since $S$ is intra-regular). So $x(\alpha,\beta), y(\alpha,\beta) \in S$. Then $x(\alpha,\beta) \circ a(\alpha,\beta) \circ y(\alpha,\beta) = x(\alpha,\beta) \circ (ax^2y)(\alpha,\beta,\delta) \circ y(\alpha,\beta) = x(\alpha,\beta) \circ (ax^2y)(\alpha,\beta,\delta) \circ y(\alpha,\beta) = (ax^2y)(\alpha,\beta,\delta)$. Hence $S$ is intra-regular.
Lemma III.15. [14] Let $A = (\mu_A, \nu_A)$ be an intuitionistic fuzzy right ideal and $B = (\mu_B, \nu_B)$ be an intuitionistic fuzzy left ideal of a semigroup $S$. Then $A \circ B \subseteq A \cap B$.

Theorem III.16. For a semigroup $S$, the following conditions are equivalent:

(i) $S$ is regular.
(ii) $B \cap A = B \circ A$ for every intuitionistic fuzzy left ideal $A = (\mu_A, \nu_A)$ and intuitionistic fuzzy right ideal $B = (\mu_B, \nu_B)$ of $S$.

Proof: (i) $\Rightarrow$ (ii): Let $S$ be a regular semigroup and $A = (\mu_A, \nu_A)$, $B = (\mu_B, \nu_B)$ be respectively intuitionistic fuzzy left and intuitionistic fuzzy right ideal of $S$. By Theorem 3.11, $S$ is a regular semigroup. By Theorem 3.5, $A$ and $B$ are respectively left and right ideal of the semigroup $S$. Hence by Lemma 3.12, $A \cap B = A \circ B$.

(iii) $\Rightarrow$ (i): Form (ii) $\Rightarrow$ (i) we have $A \circ B \subseteq A \cap B$. Again by Lemma 3.15, $A \circ B \subseteq A \cap B$. Consequently, $A \circ B = A \cap B$ and hence by Lemma 3.13, $S$ is regular.

Theorem III.17. Let $A = (\mu_A, \nu_A)$ be a non-empty intuitionistic fuzzy subset of a semigroup $S$. Then the following are equivalent:

(i) $A = (\mu_A, \nu_A)$ is an intuitionistic fuzzy semiprime ideal of $S$.
(ii) $A$ is a semiprime ideal of $S$.

Proof: (i) $\Rightarrow$ (ii): Let $A = (\mu_A, \nu_A)$ be an intuitionistic fuzzy semiprime ideal of $S$. Then $\mu_A(x) \geq \mu_A(x^2)$ and $\nu_A(x) \leq \nu_A(x) \forall x \in S$. Let $x_{(a,b)} \circ x_{(a,b)} \in A$, i.e., $(x_{(a,b)}^2_{(a,b)}) \in A$. Then $\mu_A(x^2) \geq \alpha$ and $\nu_A(x^2) \leq \beta$. Since $A = (\mu_A, \nu_A)$ is an intuitionistic fuzzy semiprime ideal of $S$, so $\mu_A(x) \geq \mu_A(x^2) \geq \alpha$ and $\nu_A(x) \leq \nu_A(x^2) \leq \beta$, which implies that $x_{(a,b)} \in A$. Hence $A$ is a semiprime ideal of $S$.

(ii) $\Rightarrow$ (i): Let $A$ be a semiprime ideal of $S$. Let $\mu_A(x^2) = \alpha$ and $\nu_A(x^2) = \beta$. Then $(x_{(a,b)}^2_{(a,b)}) \in A$, i.e., $x_{(a,b)} \circ x_{(a,b)} \in A$ implies that $x_{(a,b)} \in A$(since $A$ is a semiprime ideal of $S$). Then $\mu_A(x) \geq \alpha$ and $\nu_A(x) \leq \beta$ which implies that $\mu_A(x) \geq \mu_A(x^2)$ and $\nu_A(x) \leq \nu_A(x^2)$. Hence $A = (\mu_A, \nu_A)$ is an intuitionistic fuzzy semiprime ideal of $S$.

Lemma III.18. [14] For a semigroup $S$ the following conditions are equivalent:

(i) $S$ is an inregular semigroup.
(ii) Every intuitionistic fuzzy ideal $A = (\mu_A, \nu_A)$ of $S$ is an intuitionistic fuzzy semiprime ideal of $S$.

Theorem III.19. For any non-empty intuitionistic fuzzy subset $A = (\mu_A, \nu_A)$ of an irregular semigroup $S$ the following conditions are equivalent:

(i) $A = (\mu_A, \nu_A)$ is an intuitionistic fuzzy ideal of $S$.
(ii) $A$ is a semiprime ideal of $S$.

Proof: (i) $\Rightarrow$ (ii): Let $A = (\mu_A, \nu_A)$ be an intuitionistic fuzzy ideal of an irregular semigroup $S$. Then by Lemma 3.18, $A = (\mu_A, \nu_A)$ is an intuitionistic fuzzy semiprime ideal of $S$.

(ii) $\Rightarrow$ (i): Let $A$ be a semiprime ideal of $S$. Then by Theorem 3.17, $A = (\mu_A, \nu_A)$ is an intuitionistic fuzzy semiprime ideal of $S$ and hence $A = (\mu_A, \nu_A)$ is an intuitionistic fuzzy ideal of $S$.

Theorem III.20. Let $A = (\mu_A, \nu_A)$ be a non-empty intuitionistic fuzzy subset of a semigroup $S$. Then the following are equivalent:

(i) $A = (\mu_A, \nu_A)$ is an intuitionistic fuzzy prime ideal of $S$.
(ii) $A$ is a prime ideal of $S$.

Proof: (i) $\Rightarrow$ (ii): Let $A = (\mu_A, \nu_A)$ be an intuitionistic fuzzy prime ideal of $S$. Then $\mu_A(xy) = \max\{\mu_A(x), \mu_A(y)\}$ and $\nu_A(xy) = \min\{\nu_A(x), \nu_A(y)\} \forall x, y \in S$. Let $x_{(a,b)} \circ y_{(a,b)} \in A$, i.e., $(xy_{(a,b)})_{(a,b)} \in A$. Then $\mu_A(xy) \geq \alpha$ and $\nu_A(xy) \leq \beta$ which implies that $\max\{\mu_A(x), \mu_A(y)\} \geq \alpha$ and $\min\{\nu_A(x), \nu_A(y)\} \leq \beta$. Then $(\mu_A(x) \geq \alpha$ or $\mu_A(y) \geq \alpha)$ and $(\nu_A(x) \leq \beta$ or $\nu_A(y) \leq \beta$) $\Rightarrow$ $(\mu_A(x) \geq \alpha$ or $\nu_A(y) \leq \beta$) and $\nu_A(x) \leq \beta$. Hence $A$ is a prime ideal of $S$.

(ii) $\Rightarrow$ (i): Let $A$ be a prime ideal of $S$. Let $\mu_A(xy) = \alpha$ and $\nu_A(xy) = \beta$. Then $x_{(a,b)} \circ y_{(a,b)} = (xy_{(a,b)})_{(a,b)} \in A$ implies that $x_{(a,b)} \in A$ or $y_{(a,b)} \in A$(since $A$ is a prime ideal of $S$). Then $\mu_A(x) \geq \alpha$ and $\nu_A(x) \leq \beta$ or $\mu_A(y) \geq \alpha$ and $\nu_A(y) \leq \beta$ or $\mu_A(x) \geq \alpha$ and $\nu_A(y) \leq \beta$ or $\nu_A(y) \leq \beta$, i.e., $\max\{\mu_A(x), \mu_A(y)\} \geq \alpha$ and $\min\{\nu_A(x), \nu_A(y)\} \leq \beta$, i.e., $\max\{\mu_A(x), \mu_A(y)\} \geq \mu_A(xy)$ and $\min\{\nu_A(x), \nu_A(y)\} \leq \nu_A(xy)$. Hence $A$ is a prime ideal of $S$.

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