Effects of Double Delta Doping on Millimeter and Sub-millimeter Wave Response of Two-Dimensional Hot Electrons in GaAs Nanostructures

N. Basanta Singh, Sanjoy Deb, G. P Mishra and Subir Kumar Sarkar

Abstract—Carrier mobility has become the most important characteristic of high speed low dimensional devices. Due to development of very fast switching semiconductor devices, speed of computer and communication equipment has been increasing day by day and will continue to do so in future. As the response of any device depends on the carrier motion within the devices, extensive studies of carrier mobility in the devices has been established essential for the growth in the field of low dimensional devices. Continuous evolution in technology over the years has shrunk power consumption and size of devices and circuits. Small-signal ac transport of degenerate two-dimensional hot electrons in GaAs quantum wells is studied here incorporating deformation potential acoustic, polar optic and ionized impurity scattering in the framework of heated drifted Fermi-Dirac carrier distribution. Delta doping is considered in the calculations to investigate the effects of double delta doping on millimeter and sub-millimeter wave response of two dimensional hot electrons in GaAs nanostructures. The inclusion of delta doping is found to enhance considerably the two dimensional electron density which in turn improves the carrier mobility (both dc and ac) values in the GaAs quantum wells thereby providing scope of getting higher speed devices in future.

Keywords—Carrier mobility, Delta doping, Hot carriers, Quantum wells.

I. INTRODUCTION

Since the beginning of the seventies, the microelectronics industry has followed Moore’s law, doubling processing power every 18 months [1]. This performance increase has been achieved by increasing the speed and decreasing both the power consumption and size of devices and circuits. Continuous evolution in technology over the years has shrunk devices and systems so much that these dimensions are now less than a 1/10th of a micron. The term ‘nanoelectronics’ is therefore used instead of ‘microelectronics’.

High-mobility two dimensional electron gas in modulation doped low dimensional structures such as quantum wells has attracted much attention because of the possibility of realization of high speed nano-devices. In such structure, mobility of the two-dimensional electron gas is enhanced considerably at low temperature by the so-called modulation doping technique [2]. This is due to spatial separation of electrons form donor impurities. The carrier mobility in GaAs quantum well (QW) is further enhanced by placing an undoped AlGaAs spacer layer between the doped AlGaAs and undoped GaAs layers [3]. The spacer layer increases the separation between the carriers and ionized donors thereby increasing the electron mobility because of less Coulomb interaction [4], [5]. The movement of charge carriers in a QW device depends on various factors like doping, carrier concentration, lattice temperature, applied field and carrier scattering etc. A lot of work is still going on to increase the mobility of carrier in a QW. There have been extensive studies in the nineties in the area of electronic state modulation followed by delta doping with an aim to enhance the carrier mobility [6], [7].

Polar-optical-phonon scattering is the most important scattering mechanism that limits electron mobility in a quantum well at room temperature [8], [9]. Therefore, for practical use of quantum well structures for very high speed application, it is desirable to suppress electron-phonon scattering which is dominant in modulation doped quantum well structures at temperature above 100K. Electronic-state modulation or wave function modulation is an attempt towards the achievement of such goal. In wave function modulation, thin barrier layers are inserted in the quantum well. Scattering strength in such type of modified heterostructure is reduced due to a change in the form factor for the interaction with phonon determined by the electron wave function along the direction perpendicular to the layer [10]. Several research works have suggested that the introduction of thin AlAs barriers in rectangular QWs leads to suppression of scattering by optical phonons, which in turn increases the electron mobility [10]-[13].
Due to structural limitations such as 3D solubility limit of Si and Si impurity levels, the carrier density in the GaAs channel takes values less than 1x10^{13} cm^{-2} in single heterojunctions and somewhat higher but not exceeding 2 x 10^{12} cm^{-2} in quantum wells [14]. This carrier density limit in QW can be surpassed in delta doped heterostructures [15]. Delta doping is an important doping technique widely used in a number of semiconductor devices for extracting extraordinary features of the structures. Delta-doping, spike-doping and pulse-doping profiles are examples of extremely narrow but well defined doping profiles. Such profiles are required in semiconductor structures scaled to their practical and theoretical limits. As the spatial extent of the dopant distribution is much smaller than the de Broglie wavelength, the dopant distribution in delta doping can be represented by Dirac’s delta function [16]. The delta doped quantum wells have the following advantages. The first advantage arises from those of the delta-doping technique, including higher mobility, greater electron density and uniform electron distribution [17]. The second advantage is the reduction of size of the quantum well [17]. Since the spatial distribution of dopants is well controlled and confined to a single atomic layer, the size of the quantum well is reduced.

Delta doping allows us to localize impurities within one monolayer of the host semiconductor crystal [18] giving rise to quantization in a V shaped quantum well [19]. Very high electron concentrations exceeding 10^{13} cm^{-2} have been reported in delta doped GaAs [20].

\[
N(x) = n^{2D} \delta(x - x_n) \quad (1)
\]

\[
V(x) = \begin{cases} 
1 \frac{e n^{2D}}{2k} (x - x_n) & \text{for } x \leq x_n \\
1 \frac{e n^{2D}}{2k} (x - x_n) & \text{for } x \geq x_n
\end{cases} \quad (2)
\]

\[
\varepsilon_n = \frac{3\pi}{4} (n + \frac{1}{2})^{2/3} \left( \frac{e^2 \hbar^2}{2m^*} \frac{en^{2D}}{2k} \right)^{1/3} , n = 0,1,\ldots \quad (3)
\]

Where \(n^{2D}\) is the donor density, \(e\) is the electronic charge, \(k\) is the dielectric constant, \(m^*\) the effective mass and \(\hbar\) is Planck’s constant \(h\) divided by \(2\pi\).

It has been reported that the sheet density in delta doped structures is nearly doubled with respect to the densities obtained in modulation doped heterostructures [24], [25]. The growth of delta doped single heterojunction has been reported in [24]. Delta doped QW with 2D electron density of 4 x 10^{12} cm^{-2} in the GaAs channel has been reported in [25]. However, most of the work in this area is concentrated in finding the carrier density and mobility in single sided delta doping heterostructures and quantum wells. All these have motivated us to study the effects of double sided delta doping on the carrier concentration and the small-signal mobility of the two dimensional hot electrons in a quantum well of GaAs/AlGaAs.

The delta doping technique represents the ultimate technological limit of impurity profiles and this technique has resulted in a series of novel electronic and photonic devices [21], [22]. When a semiconductor is delta doped, the ionized donor atoms create a continuously positive sheet of charge which bends the conduction band to form a V-shaped potential well. The electrons remain close to their parent ionized donors as a result of the electrostatic attraction. When the dimension of the V-shaped potential well is comparable to the de Broglie wavelength of free electrons, the electron energies for motion perpendicular to the delta doped plane are quantized into a number of discrete bands and a quasi-two dimensional electron gas (2DEG) is formed in the V-shaped potential well. The dopant distribution, the potential well and the subband structure of a Si delta doped layer are shown in Figs. 1(a), (b) and (c). The equations describing dopant concentration, the V-shaped potential well \(V(x)\) and energies \(\varepsilon_n\) are given in (1), (2) and (3), respectively [23].

Fig. 1 Schematic diagrams of (a) the dopant distribution, (b) the V-shaped potential well and (c) the subband structure of a Si delta doped layer in GaAs.

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\]
studied.

II. THEORY

Let us consider the energy band diagrams of a delta doped heterostructure (δDH) of GaAs/AlGaAs and a conventional homogeneously doped heterostructure (HDH) of the same material. The energy band diagrams of such structures are shown in Figs. 2(a) and 2(b) [18]. All donor impurities are localized in a plane at a distance $L_s$ from the GaAs/AlGaAs interface in δDH. This localization results in a V-shaped quantum well. The conduction band offset $\Delta e_c$ can be written as

$$\Delta e_c = e_o + e_{F_c} - e_o + e_F - e_o$$

(4)

Where $e_o$ is the lowest subband energy in the well, $(e_F - e_o)$ is the degeneracy, $\Delta e_c$ is the conduction band offset, $e_o^1$ is the lowest energy state in the V-shaped quantum well and $F$ is the electric field in the spacer layer. Assuming that there are no mobile carriers in the V-shaped well in AlGaAs i.e $(e_F - e_o^1) = 0$, (4) reduces to

$$\Delta e_c = e_o + e_{F_c} - e_o + e_F - e_o$$

(5)

For the homogeneously doped heterostructure, $\Delta e_c$ can be written as

$$\Delta e_c = e_o + (e_F - e_o) + e_{F_D} + e_{V_D}$$

(6)

Where, $V_D$ is the potential drop in the deletion region. It is clear from (5) and (6) that in δDH there is no term for potential drop in the depletion region. This is due to localization of donor impurities in the delta doped AlGaAs.

Secondly, the $e_o^1$ adds up to the barrier height $\Delta e_c$, thereby enhancing the effective conduction band discontinuity as compared to the HDH. The energy terms of (5) and (6) can be replaced by the following equations which are taken from [18] and the two dimensional electron gas concentrations in the quantum wells can be calculated.

$$e_o = \frac{1}{2} \left( n+1 \right)^{2/3} \left( e^2 2 \pi m^2 e^{n_{2D}} / k \sqrt{m^2} \right)^{3/2} \cdot 2^{1/3}$$

(7)

$$F = e / k_b T$$

(8)

$$e_{V_D} = e^2 2 \pi m^2 / 2 k_B T$$

(9)

$$e_{V_D} = e^2 2 \pi m^2 / 2 k_B T$$

(10)

In the above expressions, $k_1$ and $k_2$ are the dielectric constants in the well and barrier, $k_B$ is the Boltzmann’s constant, $T$ is the temperature in Kelvin, $\Delta e_g$ is the change in band gap, $n_{2D}$ is the carrier concentration in the well, $n_{2D}$ and $n_0$ are the donor densities for δDH and HDH respectively.

Now, let us consider a GaAs/AlGaAs quantum well of width $L_x$. The AlGaAs barriers are symmetrically delta-doped at a distance $L_s$ from the quantum well. A schematic diagram and a simplified energy band diagram of the delta doped GaAs quantum well is shown in Figs. 4 and 5. The total areal electron density in a delta doped QW well is known and is equal to the introduced doping density [15]. From the value of doping density and imposing charge neutrality, the value of total areal electron density is given by [15].

$$N = \frac{1}{e} \int_{-x}^{x} \rho_e(x) dx = 2n^{2D}$$

(13)

Where

$$\rho_e(x) = \frac{e}{2} \sum \frac{f(e_F - e_e)}{\sqrt{\hbar^2 m^2 \sqrt{2m^2} / k_B T}}$$

is the local electron density, $n^{2D}$ is the doping density on each side and $e_e$ is the Fermi energy which is self consistently determined. The electron density can also be calculated from the energy balance equation [18]. The electron density in the QW due to single sided doping is a little more than one-half the values for the corresponding double sided doping [26]. Quantum size effects in Quantum well prevent the electron density from being exactly one half the electron density $N_x$ of the double sided QW.

Fig. 3 A Schematic diagram of a delta doped GaAs/AlGaAs Quntum well
Improved carrier concentration and reduced ionized impurity scattering in the QW establish a strong electron-electron interaction, favouring a heated drifted Fermi-Dirac distribution function for the carriers characterized by an electron temperature, $T_e$.

The net field is $F$ applied parallel to the heterojunction interface. The carrier distribution function characterized by an electron temperature and the drift momentum $T_\gamma$.

The energy and momentum balance equations obeyed by the carrier are:

$$\langle dE/dt \rangle_{scat} = \frac{d\langle E \rangle}{dt}$$

and

$$eF + \langle dp/dt \rangle_{scat} = \frac{dp_d}{dt}$$

Where $-\langle dp/dt \rangle_{scat}$ and $\langle dE/dt \rangle_{scat}$, respectively, the average momentum and energy loss due to scatterings and $\langle E \rangle$ depicts the average energy of a carrier with charge $e$. In the present model the effects of delta doping is included in the energy and momentum loss calculations to give more accurate results.

We insert (17) and (18) in (19) and (20), retain terms up to the linear in alternating components and equate the steady parts and the coefficients of $\sin \omega t$ and $\cos \omega t$ on the two sides of the resulting equations following the procedure adopted in [28]. For a given electric field $F_0$, we solve for $p_d$ and $Te$. The dc mobility $\mu_{dc}$ and ac mobility $\mu_{ac}$ are then expressed as:

$$\mu_{dc} = \frac{p_o}{m^* F_0}$$

(21)

$$\mu_{ac} = \frac{\sqrt{p_o^2 + p_{1r}^2}}{m^* F_0}$$

(22)

The phase lag $\phi$, the resulting alternating current lags behind the applied field is expressed as

$$\phi = \tan^{-1}\left( -\frac{p_{1r}}{p_o}\right)$$

(23)

### III. RESULTS AND DISCUSSION

Numerical calculations are performed for delta doped GaAs/AlGaAs quantum wells with the parameters given in Table I.

#### TABLE I

<table>
<thead>
<tr>
<th>Parameters employed in the present calculation for GaAs</th>
<th>Values</th>
</tr>
</thead>
<tbody>
<tr>
<td>Electron effective mass $m_e$ (Kg)</td>
<td>0.61033x10^{-11}</td>
</tr>
<tr>
<td>Longitudinal elastic constant, $C_L$ (N.m^{-2})</td>
<td>14.03x10^{10}</td>
</tr>
<tr>
<td>Static dielectric constant, $K_s$</td>
<td>12.52</td>
</tr>
<tr>
<td>Optic dielectric constant, $K_o$</td>
<td>10.82</td>
</tr>
<tr>
<td>LO phonon angular frequency $\omega_o$ (rad/s)</td>
<td>5.37x10^{15}</td>
</tr>
<tr>
<td>Lattice Constant $a$(Å)</td>
<td>3.2</td>
</tr>
<tr>
<td>Acoustic deformation potential, $E_{1}(J)$</td>
<td>17.6x10^{12}</td>
</tr>
<tr>
<td>Background ionized impurity concentration, $n_i$ (m^{-3})</td>
<td>6.0 x 10^{19}</td>
</tr>
<tr>
<td>Longitudinal acoustic velocity, $u_t$ (m.s^{-1})</td>
<td>5.11 x 10^{5}</td>
</tr>
</tbody>
</table>
The variation of 2D electron density $N_s$ with quantum well width $L_x$ for different values of $L_s$ is shown in Fig. 5. Curves $a$, $b$, $c$ and $d$ are for $L_s$ values of 5 nm, 7 nm, 30 nm, and 50 nm respectively. The 2D electron density increases with increasing $L_s$ and with decreasing $L_s$. Since larger spacer width implies larger potential decay $V_s$ in the undoped layer, $N_s$ decreases with increasing $L_s$. An increase of quantum well width with the same value of $L_s$ results in an increase of 2D electron concentration. The dots (1 & 2) in the figure show results obtained in [15]. Our calculated results are found to agree with the previous theoretical work of [15]. The high concentration of two dimensional electron gas is due to the quantum size effect in the delta doped region and spatial localization of donor impurities [18]. The nature of variation can be explained in a manner similar to the work reported in [26].

The variation of $\mu_{ac}$ and $\mu_{dc}$ with spacer layer thickness $L_s$ for a lattice temperature of 77K is shown in Fig. 7. Both $\mu_{ac}$ and $\mu_{dc}$ decreases with increasing $L_s$ due to increased potential decay in the undoped AlGaAs layer and reduction of the extent of carrier degeneracy as the influence of delta doping is decreased with the increase of $L_s$. The ac and dc mobility values are found to be higher for delta doped QW than the conventional doped QW due to pronounced carrier degeneracy and upward shift of Fermi-level [30].

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Fig. 8. Plot of $\mu_{ac}$ and $\mu_{dc}$ with doping density for $L_x=10$ nm, $N_i=6\times10^{21}$ m$^{-3}$ and $n=1\times10^{16}$ m$^{-2}$. Curves $a$ - $d$ have the same significance.
as in Fig. 7.

The variation of $\mu_{ac}$ and $\mu_{dc}$ with doping density at 77K is given in Fig. 8. The curves a - d have the same significance as in Fig. 7. Both $\mu_{ac}$ and $\mu_{dc}$ increases with increasing doping density due to increased 2D electron density in the well. The weakening of scattering at higher 2D electron density due to enhanced screening and upward shift of Fermi level accounts for this behaviour [30]. The $\mu_{ac}$ and $\mu_{dc}$ mobility values are found to be higher for delta doped QW than the conventional doped QW due to enhanced 2D carrier concentration in the well and upward shift of Fermi-level [30].

The variation of $\mu_{ac}$ and $\mu_{dc}$ with quantum well width $L_x$ is shown in Fig. 9. $\mu_{ac}$ and $\mu_{dc}$ for both type of quantum wells increases with increasing $L_x$ owing to reduced scattering for a higher channel width [30].

![Fig. 9. Variation of $\mu_{ac}$ and $\mu_{dc}$ with $L_x$ at 77K for $L_s=5$nm, $N_i=6\times10^{21}$m$^{-3}$ and $n_{2D}=1\times10^{16}$ m$^{-2}$. Curves a - d have the same significance as in Fig. 7.](image)

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![Fig. 10. Plot of $f_{3dB}$ with $L_s$ for 77K and 300K. Curves a and c are for conventional doping and b & d are for delta doping. The curves are plotted with $L_x=5$nm, $N_i=6\times10^{21}$m$^{-3}$ and $n_{2D}=1\times10^{16}$ m$^{-2}$.](image)

Fig. 10 gives the plot of 3dB cut-off frequency $f_{3dB}$ with channel width $L_x$ for 77K and 300K. The curves a and c are for conventional doped QW. The curves b and d are for delta doped QW. The 3dB cut-off frequency is found to decrease with increasing $L_x$ reflecting that the fall of $\mu_{ac}$ with frequency is sharper at a greater value of $L_x$ [27]. The values of 3dB cut-off frequency is found to be higher at 300K that at 77K.

![Fig. 11. Variation of $f_{3dB}$ with $L_x$ for 77K and 300K. The curves a - d have the same significance as in Fig. 10 and are plotted with $L_s=10$ nm, $N_i=6\times10^{21}$m$^{-3}$ and $n_{2D}=2\times10^{16}$ m$^{-2}$.](image)

Fig. 11 gives the plot of 3dB cut-off frequency with $L_x$ for 77K and 300K. The curves a - d have the same significance as in Fig. 10. The 3dB cut-off frequency is found to increase with increasing $L_s$. This behaviour is explained with the reduced carrier degeneracy due to increase in $L_s$ thereby reducing the influence of delta doping on surface states. The values of 3dB cut-off frequency $f_{3dB}$ are higher at 300K than at 77K.

![Fig. 12. Variation $f_{3dB}$ with doping density for 77K and 300K ($L_x=10$ nm, $L_s=5$ nm and $N_i=6\times10^{21}$m$^{-3}$). Curves a - d have the same significance as in Fig. 11.](image)

Fig. 12 gives the plot of 3dB cut-off frequency $f_{3dB}$ with doping density for 77K and 300K. The curves a - d have the same significance as in Fig. 11. The 3dB cut-off frequency is found to decrease with increasing doping density. This is because of the combined scattering mechanisms and increasing degeneracy of the carriers at higher 2D electron concentration [27]. The values of 3dB cut-off frequency are
higher at 300K than at 77K.

IV. CONCLUSION

In this work we have investigated the effects of delta doping on the carrier concentration, dc mobility, cut-off frequency and the small-signal mobility of the two dimensional electrons in a GaAs/AlGaAs QW. Our calculation shows the basic effect of 2D electron concentration and mobility enhancement in GaAs QW with delta doping in the barrier AlGaAs layer. The observed increase in 2D electron concentration of the GaAs well in delta doping in the barrier AlGaAs layer. The observed concentration and mobility enhancement in GaAs QW with calculation shows the basic effect of 2D electron dimensional electrons in a GaAs/AlGaAs QW. Our frequency and the small-signal mobility of the two doping on the carrier concentration, dc mobility, cut-off wave function and potential in the QW are given by [26].

Calculation of 2D electron density in a delta doped quantum well

Assuming that only the lowest subband is occupied in the quantum well, the zero order approximation for the electronic wave function and potential in the QW are given by [26].

\[
\psi_n(x) = \left(\frac{2}{L_x}\right)^{1/2} \cos\left(\frac{nx}{L_x}\right)
\]  

(A1)

and

\[
V_n(x) = \frac{\epsilon^2 N_s}{k_i L_x} \left[1 - \cos\left(\frac{2\pi x}{L_x}\right)\right] \frac{L_x^2}{4\pi^2} + \frac{x^2}{2}
\]  

(A2)

The energy balance equation can be written as

\[
\Delta E_c = V_H + V_C + \epsilon_2 + V_S - \epsilon_1 - (\epsilon_2' - \epsilon_1')
\]  

(A3)

Assuming \((\epsilon_2 - \epsilon_1') = 0\), (A3) reduces to

\[
\Delta E_c = V_H + V_C + \epsilon_2 + V_S - \epsilon_1
\]  

(A4)

The value for confinement energy \(V_c\) from the first-order perturbation theory is

\[
V_c = \frac{\hbar^2}{8m^* L_x^2} - \frac{\epsilon^2 N_s}{24k_i} \left(1 - \frac{3}{\pi^2}\right)
\]  

(A5)

The values of the remaining terms are given below:

\[
V_H = \frac{\epsilon^2 N_s}{8k_i} \left(1 + \frac{4}{\pi^2}\right)
\]  

(A6)

\[
\epsilon_2 = \frac{m^*}{m} N_s
\]  

(A7)

\[
V_S = \frac{\epsilon^2 N_s}{2k_i}
\]  

(A8)

\[
\epsilon_1 = 2^{-7/3} (n+1)^{1/3} \left(\frac{\epsilon^2 2\pi m^*}{k_i \sqrt{m}}\right)^{2/3}
\]  

(A9)

\[
\Delta E_c = 0.66 \Delta E_g
\]  

(A10)

In the above expression \(k_i\) and \(k_s\) are the dielectric constants in GaAs and AlGaAs respectively, \(K_s\) is Boltzmann’s constant, \(\Delta E_g\) is the change in band gap, \(N_s\) is the carrier concentration in the well and \(n^{2D}\) is the donor density.

Inserting the above values in [A4], we derived [A11]

\[
\Delta E_c = \frac{\epsilon^2 N_s L_x}{8k_i} \left[1 + \frac{4}{\pi^2}\right] \frac{\hbar^2}{8m^* L_x^2} - \frac{\epsilon^2 N_s}{24k_i} \left(1 - \frac{3}{\pi^2}\right)
\]  

(A11)

\[
\frac{\hbar^2}{m} N_s \left\{\frac{\epsilon^2 N_s}{2k_i} - 2^{-7/3} \left(\frac{\epsilon^2 2\pi m^*}{k_i \sqrt{m}}\right)^{2/3}\right\}
\]  

(A12)

REFERENCES


