The Reconstruction New Agegraphic and Gauss- Bonnet Dark Energy Models with a Special Power Law Expansion

V. Fayaz, F. Felegary

Abstract—Here, in this work we study correspondence the energy density New agegraphic and the energy density Gauss- Bonnet models in flat universe. We reconstruct $\Omega_\Lambda$ and $\omega_\Lambda$ for them with $a(t) = a_0 t^h$.

Keywords—dark energy, new age graphic, gauss- bonnet, late time universe

I. INTRODUCTION

NOWADAYS it is strongly believed that the universe is undergoing an accelerated expansion. These observations are those which are obtained by type Ia Supernova (SNIa) [1], Large Scale Structure (LSS) [2] and Cosmos Microwave Background (CMB) [3]. This acceleration is triggered by more than 70% of dark energy (DE) which is an exotic energy with negative pressure, but the nature of dark energy is still unknown. Among different candidates for probing the nature of dark energy, The most obvious theoretical candidate of DE is the cosmological Constant, $\Lambda$, which has the equation of state (EoS) $\omega = -1$ [4,5].

Recently, the origin agegraphic dark energy (OADE) model was proposed by Cai [6]. Cai [6] proposed the OADE model to explain the accelerated expansion of the universe based on the uncertainty relation of quantum mechanics as well as the gravitational effect in general relativity. The OADE model had some difficulties. In particular, it could not justify the matter dominated era[6]. This motivated Wei and Cai [7] to propose the New agegraphic dark energy (NADE) model, in which the time scale is chosen to be the conformal time instead of the age of the universe. The evolution behavior of the NADE is similar to that of the holographic dark energy [8]. But some essential differences exist between them.

Nojiri et.al [9] proposed the Gauss- Bonnet dark energy model. They suggest that current acceleration may be significantly influenced by string effects and the current acceleration of the universe may be caused by a mixture of scalar phantom. It is expected that the combination of fermionic field and Gauss- Bonnet term will produce significant effects in cosmological evolution [10]. The Gauss- Bonnet term may lead to some interesting results in cosmology. The conditions under which a correspondence between holographic and Gauss- Bonnet models of dark energy have been investigated in [11].

In this paper, we will study New agegraphic and Gauss- Bonnet models of dark energy. Then we will perform correspondence between New agegraphic and Gauss- Bonnet dark energy models for the positive- $h_0$ case in $a(t) = a_0 t^h$.

II. NEW AGEGRAPHIC MODEL

The energy density of the New agegraphic dark energy is given by [12]

$$\rho_\Lambda = \frac{3 n^2 M^2 \rho^2}{\eta^2} \quad (1)$$

where the numerical factor $3n^2$ is introduced to parameterize some uncertainties, such as the species of quantum fields in the universe, the effect of curved spacetime and so on. The astronomical data for the NADE gives the best-fit value (with $1\sigma$ uncertainty) $n = 2.716^{+0.111}_{-0.109}$ [12]. Also $\eta$ is conformal time of the Friedmann- Robertson- Walker (FRW) universe:

$$\eta = \int \frac{dt}{a} = \int \frac{da}{H a^\eta} \quad (2)$$

Note that in the energy density of the OADE model, the age of the universe is appeared in Eq. (1) instead of $\eta$. Also, the dark energy density Eq. (1) has the same form as the holographic dark energy, but the conformal time stands instead of the future event horizon distance of the universe [8]. From definition of dimensionless dark energy, we get

$$\Omega_\Lambda = \frac{\rho_\Lambda}{3 M^2 \rho H^2} = \frac{n^2}{\eta^2 H^2} \quad (3)$$

Thus, the Eq. (3) can be written:

$$\eta = \frac{n}{H \sqrt{\Omega_\Lambda}} \quad (4)$$

We consider a dark energy dominated universe:

$$\dot{\rho}_\Lambda + 3 H (1 + \omega_\Lambda) \rho_\Lambda = 0 \quad (5)$$

Taking the time derivative of Eq.(1), using $\dot{\eta} = 1/a$ and Eq. (4) yields

$$\dot{\rho}_\Lambda = -\frac{2 H \sqrt{\Omega_\Lambda}}{n a} \rho_\Lambda \quad (6)$$
Inserting Eq. (6) in (5), we obtain the equation of state (EoS) parameter of the New agegraphic dark energy model as
\[ \omega_\Lambda = -1 + \frac{2 \sqrt{\Omega_\Lambda}}{3 n a} \]  
(7)

The Eq. (7) shows that \( \omega_\Lambda \) is always larger than -1 and cannot cross the phantom divide.

### III. GAUSS-BONNET MODEL

The action for Gauss-Bonnet dark energy model reads
\[ S = \int d^4x \sqrt{-g} \left[ \frac{1}{2k^2} R - \frac{\gamma}{2} \partial_\mu \phi \partial_\mu \phi - V(\phi) + f(\phi) G \right] \]  
(8)

where \( k^2 = 8\pi G \) and \( \gamma = \pm 1 \). For a canonical scalar field \( \gamma = 1 \) and for phantom behavior \( \gamma = -1 \), the model is extended to \( \gamma = -1 \). In Eq. (8) \( G \) is Gauss-Bonnet invariant:
\[ G = R^2 - 4R_{\mu\nu}R^{\mu\nu} + R_{\mu\nu\rho\sigma}R^{\mu\nu\rho\sigma} \]  
(9)

Then the FRW metric for the flat universe is considered to be:
\[ ds^2 = -dt^2 + a(t)^2 \left( dr^2 + r^2 d\Omega^2 \right) \]  
(10)

thus we impose \( k=0 \) in Eq. (8). The using Eq. (8) can result [9]
\[ \rho_\Lambda = \frac{\gamma}{2} \partial_\mu \phi \partial_\mu \phi H^2 + 16f'(\phi)H^2 \frac{\dot{\phi}}{\dot{a}} + \left( f'(\phi) \phi'' + f''(\phi) \phi' \right) H^2 \]  
(11)

And
\[ \gamma \left[ \partial_\mu \partial^\mu H + \frac{V'(\phi)}{\gamma} \right] = 24 f'(\phi) H^2 \frac{\dot{\phi}}{\dot{a}} \]  
(12)

and
\[ \rho_\Lambda = \frac{\gamma}{2} \dot{\phi}^2 + V(\phi) - 24 f'(\phi) H^2 \]  
(13)

where \( \rho_\Lambda \) and \( \rho_x \) are the pressure and energy density due to the scalar field and the Gauss-Bonnet interaction [14].

### IV. CORRESPONDENCE BETWEEN NEW AGEGRAPHIC AND GAUSS-BONNET DARK ENERGY MODELS FOR THE POSITIVE-CONSTANT CASE

Here like [11] a Correspondence between New agegraphic and Gauss-Bonnet Dark Energy models in the flat universe is suggested. In this case \( f(\phi) \) is given as [13]
\[ f(\phi) = f_\phi e^{\frac{2\phi}{\phi_0}} \]  
(14)

where \( a = a(t)^{h_0} \) is obtained just like [9]. We will consider when \( h_0 > 0 \), \( a = a(t)^{h_0} \) [11].

Thus can be written [9] when \( h_0 > 0 \)
\[ H = \frac{h_0}{t}, \quad \phi = \phi_0 \ln \frac{t}{t_i} \]  
(15)

If we establish a correspondence between New agegraphic Dark Energy model and Gauss-Bonnet Dark Energy model, then using correspondence Eqs. (3) and (13), together with expressions (15) we can get:
\[ V(\phi) = e^{\frac{2\phi}{\phi_0}} \left[ 3 M^2 \frac{1}{\phi_0^2} \Omega_\Lambda - \frac{\gamma}{2} \phi_0^2 + 48 f_\phi h_0^2 \right] \]  
(16)

Also we find that the equation of motion for \( \Omega_\Lambda \) given by
\[ \frac{d\Omega_\Lambda}{d\phi} = \frac{d\Omega_\Lambda}{dt} \frac{\dot{\phi}}{\dot{\phi}_0} = \frac{d\Omega_\Lambda}{dt} \frac{\dot{t}}{t_i} e^{\frac{\phi}{\phi_0}} \]  
(17)

where
\[ \frac{d\Omega_\Lambda}{d\phi} = \Omega_\Lambda H \left[ \frac{2 \sqrt{\Omega_\Lambda}}{n a} (\Omega_\Lambda - 1) \right] \]  
(18)

Now, consider a universe with power law expansion \( a = a(t)^{h_0} \). Then Eqs. (3), (4) and (7) can be rewritten as:
\[ \eta = \frac{t}{a(1 - h_0)} \]  
(19)

\[ \Omega_\Lambda = \frac{a^2 n^2 (1 - h_0)^2}{h_0^2} \]  
(20)

and
\[ \omega_\Lambda = -1 + \frac{2(1 - h_0)}{3 h_0} \]  
(21)

which indicates that the phantom EOS \( \omega_\Lambda < -1 \) can be obtained when \( h_0 > 1 \) and \( h_0 \neq 1 \) and that the acceleration universe EOS \( \omega_\Lambda < -1/3 \) can be obtained when \( h_0 > 1/2 \) and \( h_0 \neq 1 \). Note that for the late-time universe, i.e. \( a \rightarrow \infty \), \( \Omega_\Lambda = 1 \). Eq. (21) shows that \( \omega_\Lambda = -1 \). This means that NADE in Gauss-Bonnet mimics a cosmological constant.

### V. CONCLUSION

In this paper, we considered the New agegraphic and Gauss-Bonnet models of dark energy. We reconstructed \( \Omega_\Lambda \) and \( \omega_\Lambda \) for them to lead to an accelerating universe under special conditions.

### REFERENCES


