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Stability Analysis of a Class of Nonlinear Systems Using Discrete Variable Structures and Sliding Mode Control

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***Abstract*—**This paper presents the application of discrete-time variable structure control with sliding mode based on the 'reaching law' method for robust control of a 'simple inverted pendulum on moving cart' - a standard nonlinear benchmark system. The controllers designed using the above techniques are completely insensitive to parametric uncertainty and external disturbance. The controller design is carried out using pole placement technique to find state feedback gain matrix , which decides the dynamic behavior of the system during sliding mode. This is followed by feedback gain realization using the control law which is synthesized from 'Gao's reaching law'. The model of a single inverted pendulum and the discrete variable structure control controller are developed, simulated in MATLAB-SIMULINK and results are presented. The response of this simulation is compared with that of the discrete linear quadratic regulator (DLQR) and the advantages of sliding mode controller over DLQR are also presented

***Keywords*—**Inverted pendulum, Variable Structure, Sliding mode control, Discrete-time systems, Nonlinear systems.

I. INTRODUCTION

THE *single inverted pendulum on moving cart* (SIPMC) is often used as a benchmark problem for a class of control systems. This inherently open-loop unstable system is highly nonlinear and hence controlling the system is difficult. The objective is to bring the cart to a desired position while balancing the pendulum, so that it is always in up-right equilibrium point. The single inverted pendulum consists of rod of negligible weight and a mass mounted on the top, making its centre of gravity to lie on the top and pivoted at the bottom on a freely moving motor driven cart. On a stationary cart if the rod is exactly centered and if there is no disturbance then it will remain balanced infinitely, which is practically unachievable. Any disturbance that shifts the rod from the equilibrium point will further push it down and finally rest it on the ground. Hence, the control task is to maintain the pendulum on up-right equilibrium point along with bringing the cart to the set reference point by moving the cart back and forth within the limit at required speed.

The controllers based on PID techniques, Neural networks, Fuzzy logic, linear quadratic optimal controllers are available, but they are found to be less robust for parameter perturbations and external disturbances. In this paper a more robust controller, based on discrete variable structures and sliding mode is considered.

Variable Structures with sliding mode Control, simply called as Variable Structure Control (VSC) or Sliding Mode Control (SMC), is a type of robust control technique that is applicable to linear, nonlinear, MIMO, discrete-time and stochastic systems. The theory of variable structure systems has been developed in 1950s by Emelyanov *et.al*, and further by Utkin [1]. The detailed survey and fundamental developments on variable structure control with sliding mode can be found in [2] and [3]. A novel approach on the continuous control of nonlinear systems was suggested by Gao *et.al* in [4]. The concepts of discrete SMC (DSMC) were dealt by Gao *et.al* and the conditions for the existence of quasi-sliding mode (QSM) control were also proposed [6]. The comprehensive guide on sliding mode control for control engineers is given in [5].

The paper has been organized into six Sections. Section 2 gives a complete overview of variable structures and sliding mode control along with its discrete counterpart. Section 3 explains the, *reaching law method* established by GAO. Section 4 presents mathematical model of the inverted pendulum. Design of discrete-time sliding mode controller for inverted pendulum model using reaching law approach has been discussed in Chapter 5. The responses of the simulated model with the sliding mode controller under different initial conditions are discussed in section 5 and Section 6 provides the conclusion and future work.

II. OVERVIEW OF VARIABLE STRUCTURES AND SLIDING MODE CONTROL

In variable structure control the system is allowed to vary its structure by properly changing the sign and / or magnitude of the input, so that it is made to move towards sliding mode in finite time. The desired dynamics of the system during the sliding mode are achieved by pole placement technique. Hence, once the system enters in to the sliding surface it will definitely reach the steady state, the inherent phenomenon of the variable structures with sliding mode control. The

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controller based on the above technique is simulated using MATLAB-Simulink and the simulation results are presented. The performance of sliding mode controller is compared with that of a linear quadratic controller and the robustness of the former is also observed.

The main advantage of SMC is its insensitivity to parameter variations, external disturbances and modeling errors [3]. In this technique the structure of the system to be controlled is deliberately changed with a discontinuous control which drives the phase trajectory to a stable manifold or switching surface from any initial condition. The new useful properties thus achieved are not present in any of the structures used [2]. The switching surface decides the closed loop dynamics of the system and easily controlled by the designer.

In discrete sliding mode (DSM) the control input is also discrete and hence it is applied only at certain sampling instants. Hence, when the states reach the switching surface, the subsequent control would be unable to keep the states to be confined to the surface. As a result, DSM can undergo only quasi-sliding mode, i.e., the system states would approach the sliding surface but would generally be unable to stay on it [3].

GAO introduced a new '*reaching law approach'* to design the controller for a discrete time system. This approach is found satisfactory when compared to the other method proposed in [8]. The required constrains of Discrete-time Variable Structure Control (DVSC) are satisfied in this method and the switching function $s(k)$ is effectively controlled to meet the required dynamics. This followed by the derivation of the control law in conjunction with the known plant model and parameter variations.

Chattering phenomenon is a major disadvantage of SMC, which is due to the discontinuous switching control applied to the plant, which excites the unmodelled high frequency dynamics of the system. This problem may be minimized by two approaches, *viz.,* the continuous method and the reaching law method [4]. Out of these the reaching law method is simpler and directly deals with the reaching process and makes it easy to obtain the control law.

III. GAO'S REACHING LAW

Consider a discrete-time system specified by the standard format

$$\mathbf{x}(k) = \mathbf{A}x(k) + \mathbf{b}u(k) \tag{1}$$

where the matrices $\mathbf{A}$ and $\mathbf{b}$ are of appropriate dimensions.
For the above mentioned system to reach and remain in the steady state it should posses the following attributes [6].

a1. Starting from any initial state, the trajectory will move monotonically toward the switching plane and cross it in finite time.

a2. Once the trajectory has crossed the switching plane the first time, it will cross the plane again in every successive sampling period, resulting in a zigzag motion about the switching plane.

a3. The size of each successive zigzagging step is non-increasing and the trajectory stays within a specified band.

A. Reaching law

The reaching law for a continuous VSC is [6]

$$\dot{s}(t) = -\varepsilon \operatorname{sgn}(s(t)) - qs(t)$$

where $\varepsilon > 0, q > 0.$

Similarly, for a VSC of a discrete system, an equivalent form of the reaching law is

$$s(k+1) - s(k) = -qTs(k) - \varepsilon T \operatorname{sgn}(s(k)) \tag{2}$$

where, $T > 0$ is the sampling period, $\varepsilon > 0, q > 0$ are constants and ,

Attribute *a1* is satisfied if the inequality for T holds and hence restricted. The *signum* i.e. sign of $s(k)$ term guarantees attributes *a2* and *a3*.

The dynamics of a discrete VSC system which satisfies the attributes *a1* and *a2* is called as *quasi-sliding mode* (QSM) and the band that contains the QSM is known as *quasi-sliding mode band* (QSMB) and given by

$$\{x \mid -\Delta < s(x) < +\Delta\} ,$$

where 2Δ is the width of the band. Obviously for an ideal case the bandwidth $\Delta = 0$. If a discrete system posses all the three attributes *a1, a2* and *a3* then it is said to satisfy the reaching condition.

B. Merits of reaching law

1) All the three attributes *a1* to *a3* are satisfied.
2) With the proper choice of k and q a desirable reaching mode response can be achieved.
3) The width of the quasi-sliding mode band (QSMB) can conveniently be calculated.
4) As T is one of the parameters of the reaching law, the effect of it on VSC may easily be calculated.
5) It is easier to extend the control law for MIMO systems using (2).
6) The control laws is in equality form as (2) is also in the same format

C. Design of switching function

Consider a linear switching plane

$$s(x) = \mathbf{C}^T \mathbf{x} = 0 \tag{3}$$

Upon extending this to ideal discrete quasi-sliding, mode the same is represented as

$$s(k+1) = s(k) = 0, \qquad k = 0,1,2.... \tag{4}$$

From (4), (3), and (1),

$$\mathbf{C}^T \mathbf{x}(k) + \mathbf{C}^T \mathbf{b}u(k) = s(k) = 0$$

Solving for u, an equivalent control signal is given by

$$u_e = -(\mathbf{C}^T \mathbf{b})^{-1} \mathbf{C}^T \mathbf{A}\mathbf{x}(k) \tag{5}$$

where $\mathbf{C}^T \mathbf{b} \neq 0$ has been assumed, implying the controllability of the VSC system. In fact, $\mathbf{C}^T \mathbf{b} = 1$ which can easily be proved [7]. As the control (5) is linear in $\mathbf{x}$, the dynamical equation of the quasi-sliding mode is also linear and given by

$$\mathbf{x}(k+1) = [\mathbf{I} - \mathbf{b}(\mathbf{C}^T\mathbf{b})^{-1}\mathbf{C}^T]\mathbf{A}\mathbf{x}(k)$$
$$\mathbf{C}\ \ \mathbf{x}(k) = 0 \tag{6}$$

D. Alternative approach

Using linear transformation system (1) is transformed in to normal form, which leads to a more direct way to analyze the sliding mode dynamics. This approach also provides a convenient way for designing the vector $\mathbf{C}$ of the switching function.

Let the system (1) be in normal form in which

$$\mathbf{A} = \begin{bmatrix} \mathbf{A} & \mathbf{A} \\ \mathbf{A} & \end{bmatrix}, \quad \mathbf{x} = \begin{bmatrix} \mathbf{x}_1 \\ \mathbf{x}_2 \end{bmatrix}$$

$$\mathbf{b}^T = [0,....,0,1], \quad \mathbf{C}^T = [\overline{\mathbf{C}}^T, 1] \tag{7}$$

The dimensions of $\mathbf{A}_{11} = (n-m)\times(n=-m)$, $\mathbf{A}_{12} = (n-m)\times 1$, $\mathbf{A}_{21} = 1\times(n-m)$, and x_2 are scalars. Then when the dynamics of (1) are restricted on the surface $s(k) = 0$ it can be expressed as

$$\mathbf{x}_1(k+1) = \mathbf{A}_{11}\mathbf{x}_1(k) + \mathbf{A}_{12}\mathbf{x}_2(k)$$
$$\mathbf{x}_2(k) = -\overline{\mathbf{C}}^T\mathbf{x}_1(k) \tag{8}$$

Eliminating $\mathbf{x}_2$ gives

$$\mathbf{x}_1(k+1) = [\mathbf{A}_{11} - \mathbf{A}_{12}\overline{\mathbf{C}}^T]\mathbf{x}_1(k) \tag{9}$$

This is the equation of *ideal quasi-sliding mode*. It can easily be proved that if the pair $(\mathbf{A}, \mathbf{b})$ is *controllable* then $(\mathbf{A}_{11}, \mathbf{A}_{12})$ is also *controllable*. Then this becomes regulator problem and hence $\mathbf{C}^T$ can be obtained by arbitrarily assigning the poles of the $[\mathbf{A}_{11} - \mathbf{A}_{12}\overline{\mathbf{C}}^T]$, such that the system is stable.

E. Derivation of Control Law

From (3),

$$s(k+1) - s(k) = \mathbf{C}^T\mathbf{x}(k+1) - \mathbf{C}^T\mathbf{x}(k)$$
$$= \mathbf{C}^T\mathbf{A}\mathbf{x}(k) + \mathbf{C}^T\mathbf{b}u(k) - \mathbf{C}^T\mathbf{x}(k) \tag{10}$$

Referring the equations (2) and (10),

$$s(k+1) - s(k) = \varepsilon T\,\mathrm{sgn}(s(k)) - qTs(k)$$
$$= \mathbf{C}^T\mathbf{A}\mathbf{x}(k) + \mathbf{C}^T\mathbf{b}u(k) - \mathbf{C}^T\mathbf{x}(k) \tag{11}$$

Solving (11) for $u(k)$ the control law is given by

$$u(k) = -(\mathbf{C}^T\mathbf{b})^{-1}[\mathbf{C}^T\mathbf{A}\mathbf{x}(k) - \mathbf{C}^T\mathbf{x}(k) + qT\mathbf{C}^T\mathbf{x}(k) + \varepsilon T\,\mathrm{sgn}(\mathbf{C}^T\mathbf{x}(k))] \tag{12}$$

Substituting (12) into (1) gives the response of the discrete VSC system as

$$\mathbf{x}(k+1) = \mathbf{A}\mathbf{x}(k) - \mathbf{b}(\mathbf{C}^T\mathbf{b})^{-1}[\mathbf{C}^T\mathbf{A}\mathbf{x}(k) - \mathbf{C}^T\mathbf{x}(k) + qT\mathbf{C}^T\mathbf{x}(k) + \varepsilon T\,\mathrm{sgn}(\mathbf{C}^T\mathbf{x}(k))] \tag{13}$$

F. Control Law for Robust Control

With parameter variation $\mathbf{\Delta A}$ and external disturbance $\mathbf{f(k)}$, the system represented by (1) is rewritten as

$$\mathbf{x}(k+1) = \mathbf{A}\mathbf{x}(k) + \mathbf{\Delta A}\mathbf{x}(k) + \mathbf{b}u(k) + \mathbf{f}(k) \tag{14}$$

The standard matching conditions $\mathbf{\Delta A} = \mathbf{b}\tilde{\mathbf{A}}$ and $\mathbf{f} = \mathbf{b}\tilde{\mathbf{f}}$ is assumed here where $\tilde{\mathbf{A}}$ is a row vector and $\tilde{\mathbf{f}}$ is a scalar. Rearranging (14)

$$\mathbf{x}(k+1) = \mathbf{A}\mathbf{x}(k) + \mathbf{b}[u(k) + \tilde{\mathbf{A}}\mathbf{x}(k) + \tilde{\mathbf{f}}(k)] \tag{15}$$

It can be easily proved that the *ideal quasi-sliding mode* (9) is unchanged [6]. Hence the system is free from the effect of parameter variation or external disturbance. But, the control becomes

$$u(k) = -((\mathbf{C}^T\mathbf{b})^{-1})[\mathbf{C}^T\mathbf{A}\mathbf{x}(k) - \mathbf{C}^T\mathbf{x}(k) + qT\mathbf{C}^T\mathbf{x}(k) + \varepsilon T\,\mathrm{sgn}(\mathbf{C}^T\mathbf{x}(k)) - \tilde{\mathbf{A}}\mathbf{x}(k) + \tilde{\mathbf{f}}(k)] \tag{16}$$

Substituting (16) in (15)

$$\mathbf{x}(k+1) = \mathbf{A}\mathbf{x}(k) + \mathbf{b}(\mathbf{C}^T\mathbf{b})^{-1}[\mathbf{C}^T\mathbf{A}\mathbf{x}(k) - \mathbf{C}^T\mathbf{x}(k) + qT\mathbf{C}^T\mathbf{x}(k) + \varepsilon T\,\mathrm{sgn}(\mathbf{C}^T\mathbf{x}(k))$$

which is similar to (13) excluding the negative terms of the disturbances

G. The Quasi-sliding Mode Band

To determine the bandwidth 2Δ within which the trajectory stays in once it enters into the band, multiply both sides of (13) by $\mathbf{C}^T$ and from (3)

$$s(k+1) = (1-qT)s(k) - \varepsilon T\,\mathrm{sgn}(s(k), (1-qT) > 0 \tag{17}$$

This ensures that the sign of the first term on the left side of (17) is that of $s(k)$ and the sign of second term is the negation of sign of $s(k)$. It is clear that the signs of $s(k+1)$ and $s(k)$ are opposite to each other and thus the region of QSMB is given by

$$\left\{\mathbf{x} \mid s(\mathbf{x}) < \frac{\varepsilon T}{1-qT}\right\}$$

and hence, the width of QSMB is

$$2\Delta = 2\frac{\varepsilon T}{1-qT} \tag{18}$$

It is clear from (18) that the width of the band decreases with higher sampling frequencies.

IV. Inverted Pendulum Modeling

The schematic diagram of a SIPMC is shown in fig.1. The SIPMC is unstable in that it may fall over any time in any direction unless a suitable control force is applied to the cart. The objective of this paper is to design a discrete sliding mode controller such that, given any initial conditions (caused by the disturbances), the pendulum can be brought back to the reference position $\theta = 0$ as quickly as possible and without overshoot.

Using the equations of motions the dynamics of SIPMC are represented by the following mathematical model [9], [10].

$$M_c l\ddot{\theta} = (M_c + M_p)g\theta - u \tag{19}$$

$$M_c\ddot{x} = u = M_p g\theta \tag{20}$$

where

- M_c mass of the cart,
- l length of the pendulum rod,
- M_p mass of the pendulum rod,
- θ rotation of the pendulum rod about the point P,
- g center of gravity,
- u control force applied to the cart.

On defining the state variables $x_1 = \theta$, $x_2 = \dot{\theta}$, $x_3 = x$ and $x_4 = \dot{x}$, the state model of the system described by (19) and (20) may be written as

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \\ \dot{x}_4 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ \dfrac{M_c + M_p}{M_g l}g & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ -\dfrac{M_p}{M_C}g & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} + \begin{bmatrix} 0 \\ -\dfrac{1}{M_c l} \\ 0 \\ \dfrac{1}{M_c} \end{bmatrix} \tag{21}$$

$$y = \begin{bmatrix} 1 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} \tag{22}$$

V. Simulation of DSMC for an SIPMC

The parameters of the inverted pendulum considered for simulation are $l = 0.5m$, $M_c = 2kg$ and $M_p = 0.1kg$. By substituting the given numerical values for M_c, M_p and l in the equation (3), it becomes

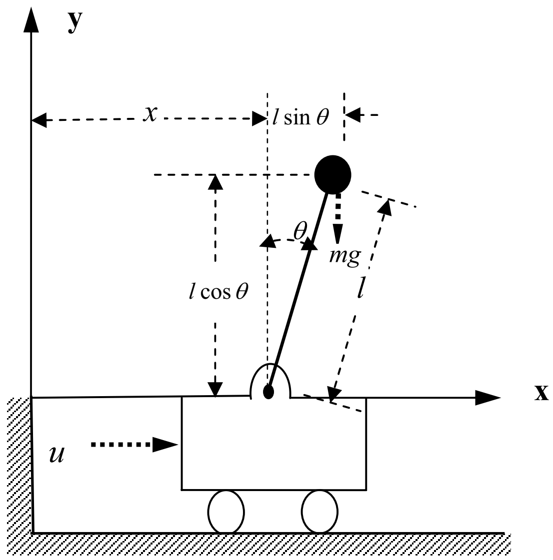

Fig. 1 Schematic model of SIPMC

$$\dot{x}(t) = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 20.601 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ -0.4905 & 0 & 0 & 0 \end{bmatrix} x(t) + \begin{bmatrix} 0 \\ -1 \\ 0 \\ 0.5 \end{bmatrix} u(t) \tag{23}$$

Discretizing the system with zero-order hold and a sampling period of $T = 0.1\sec$ gives

$$x(k+1) = \begin{bmatrix} 1.1408 & 0.1035 & 0 & 0 \\ 2.1316 & 1.1048 & 0 & 0 \\ -0.0025 & -0.0001 & 1 & 0.1 \\ 0.0508 & -0.0025 & 0 & 0 \end{bmatrix} x(t) + \begin{bmatrix} -0.0051 \\ -0{,}1035 \\ 0.0025 \\ 0.0501 \end{bmatrix} u(k) \tag{24}$$

Transforming the system (24) to a controllable canonical form and rearranging using (7) gives

$$\mathbf{A}_{11} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix}, \qquad \mathbf{A}_{12} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix},$$

$$\mathbf{A}_{21} = \begin{bmatrix} -1 & 4.2096 & -6.4191 \end{bmatrix}, \ \mathrm{A}_{22} = 4.2096$$

By choosing $-0.1, -0.2, -0.3$ as desired eigen-values in such a way that they are restricted inside unit circle for stability, and using pole placement technique, the $\mathbf{C}$ matrix is $\overline{\mathbf{C}}^T = \begin{bmatrix} 0.006 & 0.11 & 0.6 \end{bmatrix}$ and $\mathbf{C}^T = \begin{bmatrix} 0.006 & 0.11 & 0.6 & 1 \end{bmatrix}$

Hence, the system is represented as

$$x_1(k+1) = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix} x_1(k) + \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} x_2(k)$$

$$x_2(k+1) = \begin{bmatrix} -1 & 4.2096 & -6.4191 \end{bmatrix} x_1(k) + 4.2096 x_2(k) + u(k) \tag{25}$$

By using the transformation matrix

$$T_2 = \begin{bmatrix} \mathbf{I} & \\ -\mathbf{C}^T & 1 \end{bmatrix}$$

and (25) becomes

$$x_1(k+1) = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -0.006 & -0.11 & -0.6 \end{bmatrix} x_1(k) + \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} s(k)$$

$$s(k+1) = \begin{bmatrix} -1.0289 & 3.6865 & -9.1949 \end{bmatrix} x_1(k) + 4.8096 s(k) + u(k) \tag{26}$$

when $s(k) = 0$ first equation of (26) becomes

$$x_1(k+1) = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -0.006 & -0.11 & -0.6 \end{bmatrix} x_1(k)$$

Equating (17) and the second equation of (26) and solving for $u(k)$ the control law obtained is

$$u(k) = [s(k) - \varepsilon T \operatorname{sgn}(s(k)) - qTs(k) \\ - [-1.0289 \quad 3.6865 \quad -9.1949] x_1(k) = 4.8906 s(k)] \tag{27}$$

The parameters εT and qT are chosen as 0.005 and 0.5 respectively for the control law to implement the DSMC for SIPMC and simulated using MATLAB – Simulink with different initial conditions and with disturbance. The responses of all the four states with respect to time are given in fig.2 to fig.5.

VI. Results and Discussion

The responses of the SIPMC controlled by DSMC and DLQR are shown in fig.2 to fig.5. From fig.2 with no initial movement of the cart, i.e. $x_4 = 0,$ it is observed that the settling time of all the four states with the DSMC is far better than that of the DLQR. Though the pendulum angle θ has higher overshoot with DSMC as observed from fig.2, the settling time is better than DLQR. It is observed from fig.2 that there in no overshoot in the remaining three states where as all these states overshoot in case of DLQR.

With initial movement in the cart i.e. $x_4 \neq 0,$ it is found that the DLQR gives lesser overshoot for all the four states but for the states x and $\dot{x}$, there exist a steady state error, which is undesirable. The settling time is better for all the states in DSMC in spite of the initial overshoots. This can be observed from fig.3 and fig.4 for two different initial movements of the cart. In the first case the cart movement is towards the left-side and in the second case the cart moves towards the right-side, form the desired position.

The dynamics discussed above are under the assumption that there is no disturbance. In the presence of the disturbance the dynamics of the two controllers plotted in fig.5. It is found that the DSMC is insensitive to the disturbance and hence, the robustness of the DSMC is ensured. But, though very small, there is a definite jolt in the responses in all the states and there is a steady state error in state x_3 in case of DLQR.

Further, it may be noticed that for DSMC the width of the QSMB reduces with the reduction in εT [6] and the parameters q and k decide the transient response of the controller. Due to the high-frequency switching of the control signal the presence of chattering is unavoidable on the sliding surface, the inherent phenomenon of SMC.

VII. Conclusion

The DSMC is designed for SIPMC and the dynamics of it under differrent environments have been studied. The ensured robustness of the SMC is well utilized for this benchmark problem. A simple pendulum is considered in this paper and the design technique may be extended for other types of problems like double inverted pendulum, mobile inverted pendulum etc. As the design is for the known perturbations, it may be extended so that the controller is made robust even for unknown disturbances.

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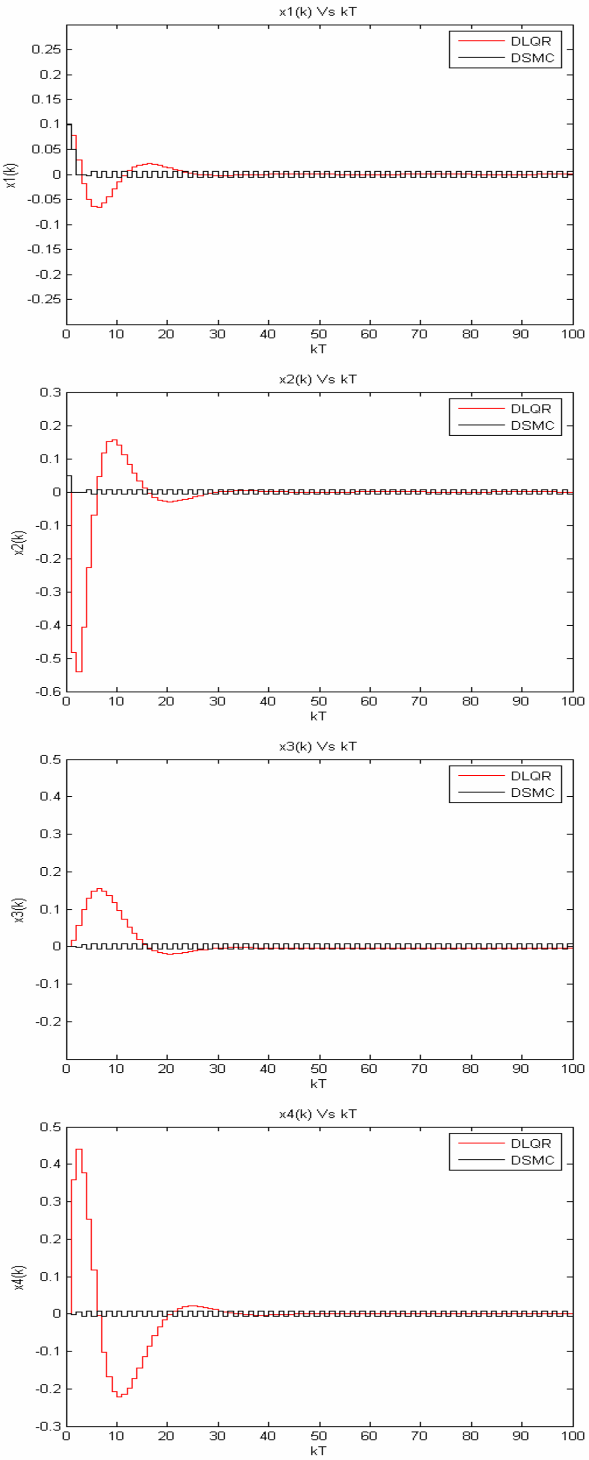

Fig. 2. Response of the SIPMC for DSMC and DLQR with $x_0 = [0.1 \quad 0.5 \quad 0 \quad 0]$, *i.e.* without initial movement of the cart and without disturbance.

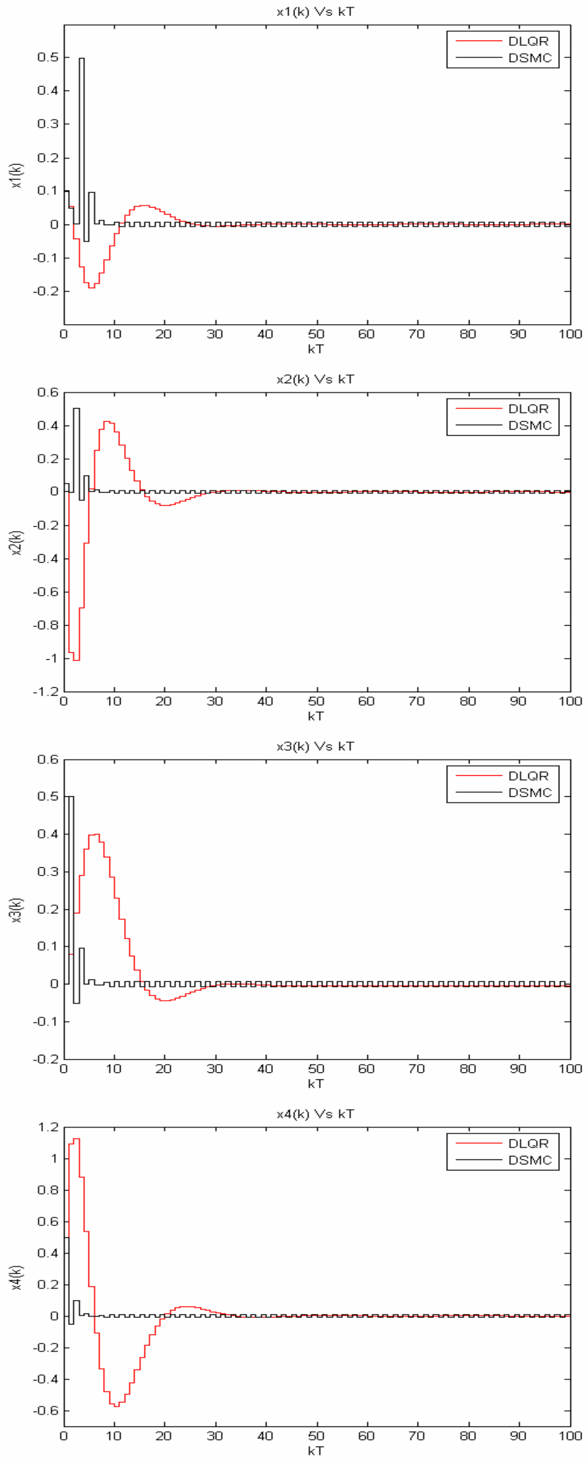

Fig. 3. Response of the SIPMC for DSMC and DLQR with $x_0 = [0.1 \quad 0.5 \quad 0 \quad 0.5]$, *i.e.* with initial movement of the cart on the left-hand side and without disturbance

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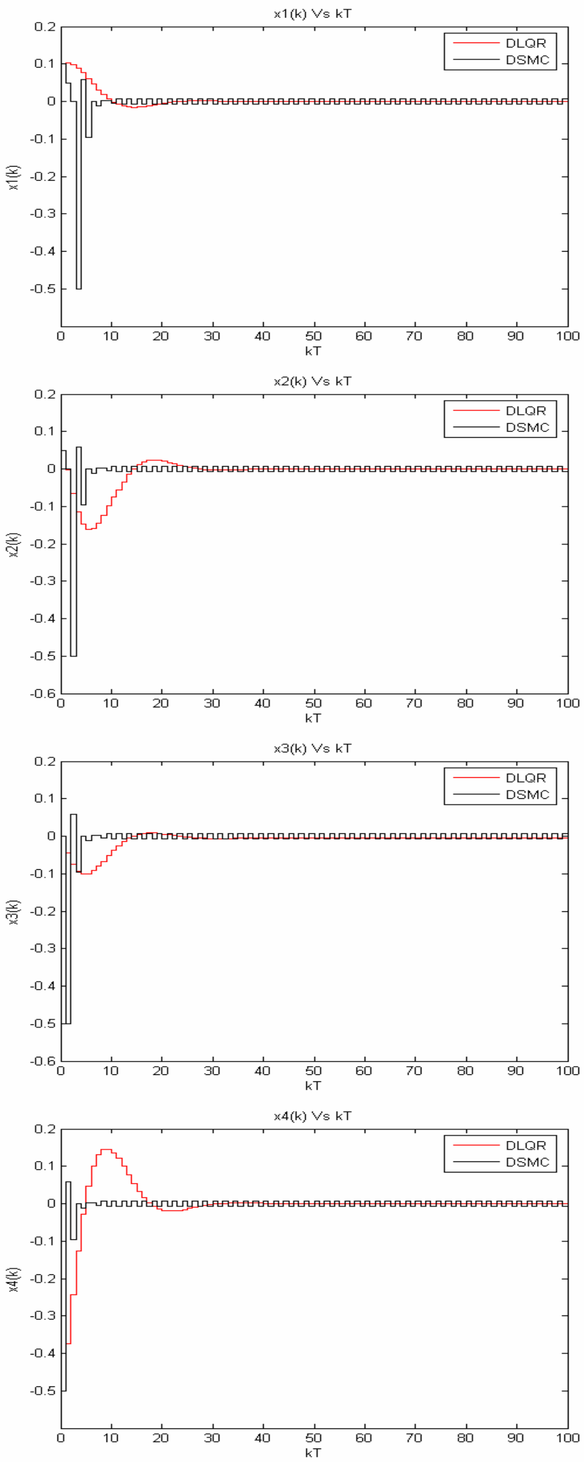

Fig. 4. Response of the SIPMC for DSMC and DLQR with $x_0 = [0.1 \quad 0.5 \quad 0 \quad -0.5]$, with initial movement of the cart on the right-hand side and without disturbance

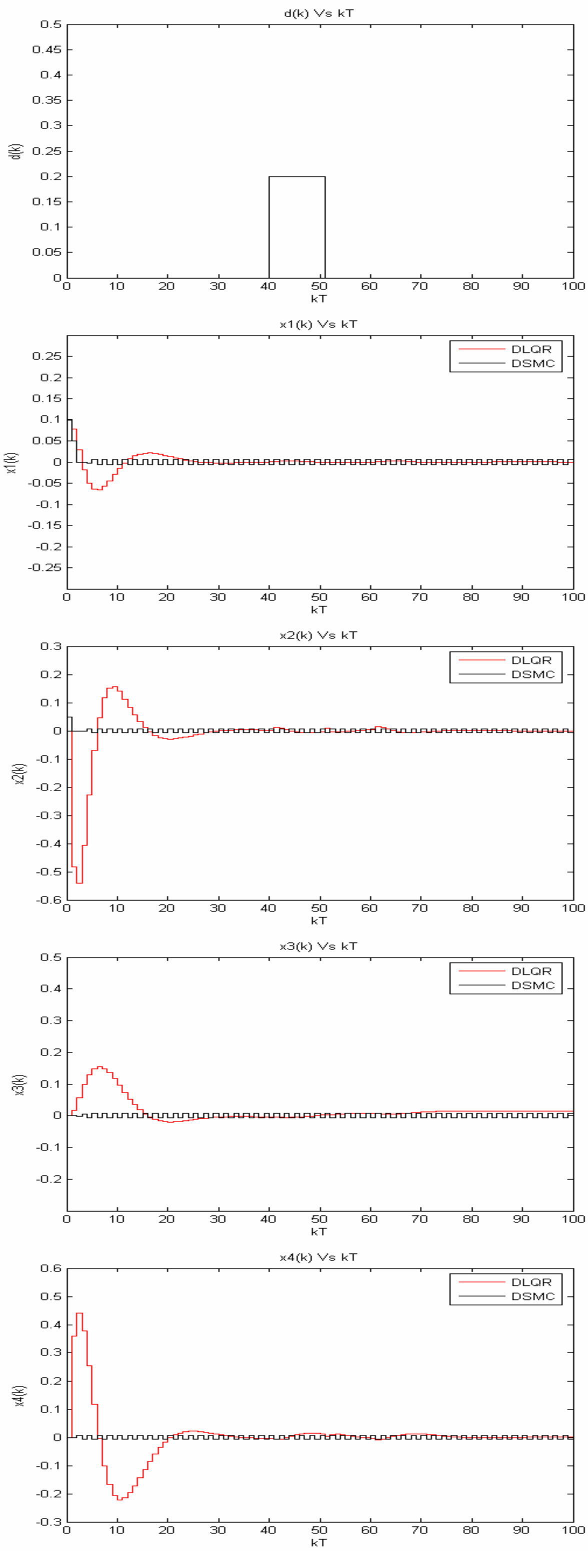

Fig. 5. Response of the SIPMC with disturbance *d(k)* for DSMC and DLQR with $x_0 = [0.1 \quad 0.5 \quad 0 \quad 0]$