Filteristic Soft Lattice Implication Algebras

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Abstract—Applying the idea of soft set theory to lattice implication algebras, the novel concept of (implicative) filteristic soft lattice implication algebras which related to (implicative) filter( for short, $(IFF)$-soft lattice implication algebras) are introduced. Basic properties of $(IFF)$-soft lattice implication algebras are derived. Two kinds of fuzzy filters (i.e., $(\in \vee q_k)((E, E \cup \bar{E}))$-fuzzy (implicative) filter) of $L$ are introduced, which are generalizations of fuzzy (implicative) filters. Some characterizations for a soft set to be a $(IFF)$-soft lattice implication algebra are provided. Analogously, this idea can be used in other types of filteristic lattice implication algebras (such as fantastic (positive implicative) filteristic soft lattice implication algebras).

Keywords—Soft set; (implicative) filteristic lattice implication algebras; fuzzy (implicative) filters; $((\in, \in \vee q_k)) (E, E \cup \bar{E})$-fuzzy (implicative) filters.

I. INTRODUCTION

In order to research the many-valued logical system whose propositional value is given in a lattice, in 1993, Xu[1] firstly established the lattice implication algebras by combining lattice and implication algebras, and investigated many useful structures[2], [3], [4]. This logical algebra has been extensively investigated by several researchers, and many elegant results are obtained, collected in the monograph[4]. Because of various uncertainties typical for complicated problems in economics, engineering and environment, one can’t be successfully solved by existing theories such as theory of (intuitionistic) fuzzy sets, theory of vague sets, theory of interval mathematics, and theory of rough sets. However, all of these theories have their own difficulties which are pointed out in [12]. Molodtsov[12] suggested that one reason for these difficulties may be due to the inadequacy of the parametrization tool of the theory. To overcome these difficulties, Molodtsov(1999) introduced a novel concept called soft sets as a new mathematical tool for dealing with uncertainties. The soft set theory is free from many difficulties that has been troubled the usual theoretical approaches. Molodtsov pointed out several directions for the applications of soft sets. Research works on soft sets are very active and progressing rapidly in these years. Maji[16] discussed the application of soft set theory to a decision-making problems. They also investigated some operations on the theory of soft sets. In 2001, Maji[14] et al. investigated the fuzzification of a soft set and obtained many useful results on fuzzy soft set. Aktas and Cagman[12] related soft sets to groups, they defined soft groups, derive some basic properties, and showed that soft groups extended fuzzy groups. Jun[18], [19] applied the soft set theory to the $BCK$-algebras, investigated soft $BCK$-subalgebras and soft ideals, introduced the notion of $\in$-soft set and $q$-soft set, and gave characterizations for subalgebras and ideals. Furthermore, Feng et al.[17] applied soft set theory to the study of semirings and initiated the notion called soft semirings. Zhan et al.[22] applied soft set to $BLS$-algebras, initiated the notion (implicative) filteristic soft $BLS$-algebras.

The concept of fuzzy set was introduced by Zadeh[1965][5]. Since then this idea has been applied to other algebraic structures such as groups, semigroups, rings, modules, vector spaces and topologies. Some scholars[8], [20], [21] applied this fuzzification to the filter in lattice implication algebras, too. They further to introduce relative fuzzy filter such as fuzzy (positive) implicative filter, fuzzy fantastic filter and investigated some properties. The idea of fuzzy point and ‘belongingness’ and ‘quasi-coincidence’ with a fuzzy set were given by Pu et al.[6]. A new type of fuzzy subgroup (viz $(\in, \in \vee q)-fuzzy$ subgroup) was introduced in[9]. In fact, $(\in, \in \vee q)-fuzzy$ subgroup is an important and useful generalization of Rosenfeld’s fuzzy subgroup. The idea of fuzzy point and ‘belongingness’ and ‘quasi-coincidence’ with a fuzzy set have been applied some important algebraic system[10], [11]. Liu [7], [8] investigate the interval-valued $(\in, \in \vee q)$-fuzzy lattice implication subalgebras and fuzzy filters, respectively.

The aim of this paper is to apply the idea of soft set theory to lattice implication algebras, and introduce the (implicative) filteristic soft lattice implication algebras which related to (implicative) filter (for short, $(IFF)$-soft lattice implication algebras). Basic properties of $(IFF)$-soft lattice implication algebras are investigated. We introduce the notion of $(\in, \in \vee q_k)((E, E \cup \bar{E}))-fuzzy$ (implicative) filters, which are generalizations of fuzzy (implicative) filter. We provide characterizations for a soft set to be an $(IFF)$-soft lattice implication algebra. Analogously, this idea can be used in other types lattice implication algebras such as fantastic filteristic lattice implication algebras, positive implicative filteristic soft lattice implication algebras. We hope that it will be of great use to provide theoretical foundation to design intelligent information processing systems.

II. BASIC RESULTS ON LATTICE IMPLICATION ALGEBRAS

Definition 2.1: [1] Let $(L, \vee, \wedge, O, I)$ be a bounded lattice with an order-reversing involution $\prime$, the greatest element $I$ and the smallest element $O$, and

$$\rightarrow: L \times L \rightarrow L$$

be a mapping. $L = (L, \vee, \wedge, \prime, \rightarrow, O, I)$ is called a lattice implication algebra if the following conditions hold for any $x, y, z \in L$:

\[(I_1) \ x \rightarrow (y \rightarrow z) = y \rightarrow (x \rightarrow z),\]

\[(I_2) \ x \rightarrow (y \rightarrow z) = (x \rightarrow y) \rightarrow (x \rightarrow z),\]

\[(I_3) \ x \rightarrow (y \rightarrow z) = (x \rightarrow y) \rightarrow (x \rightarrow z).\]
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(1) The basic intersection of two soft sets $(F, A)$ and $(G, B)$ is defined as the soft set $(H, C) = (F, A) \cap (G, B)$, where $C = A \times B$ and $H(a, b) = F(a) \cap G(b)$ for any $(a, b) \in A \times B$.

(2) The intersection of soft sets $(F, A)$ and $(G, B)$ over a common universe $U$ is defined as the soft set $(H, C) = (F, A) \cap (G, B)$, where $C = A \cap B$, and $H(c) = F(c) \cap G(c)$ for any $c \in C$.

(3) The union $(H, C)$ of two soft sets $(F, A)$ and $(G, B)$ is defined as the soft set $(H, C) = (F, A) \cup (G, B)$, where $C = A \cup B$, and $H(c) = F(c) \cup G(c)$ when $c \in A \cup B$.

Definition 2.3: Let $(F, A)$ be a nonempty soft set over a lattice implication algebra $L = (L, \vee, \wedge, ^{\prime}, \rightarrow, O, I)$. Then

(1) $(F, A)$ is a called a F-soft lattice implication algebra

(2) $(F, A)$ is called an IF-soft lattice implication algebra

Example 3.1: Let $L = \{O, a, b, c, d, I\}$, the Hasse diagram of $L$ and its implication operator $\rightarrow$ and negation operator $^{\prime}$ be defined in EXAMPLE 2.1.4 in [4] Then $L = (L, \vee, \wedge, ^{\prime}, \rightarrow, O, I)$ is a lattice implication algebra.

(1) Let $(F, A)$ be a soft set over $L$, where $A = \{I, a, b\}$ and the set-valued function $F : A \rightarrow P(L)$ defined by $F(t) = \{x \in L \mid x \rightarrow t = I\}$. Then $F(I) = L, F(a) = \{I, b\}, F(b) = \{a\}$ are all filters of $L$. Therefore $(F, A)$ is a F-soft lattice implication algebra over $L$.

(2) Let $(F, A)$ be a soft set over $L$, where $A = \{c, d\}$ and $F : A \rightarrow P(L)$ the set-valued function defined by $F(t) = \{y \in L \mid y \rightarrow t \in \{a, b\}\}$. Then $F(c) = \{O, a, d\}, F(d) = \{O, c\}$ are filters of $L$. Therefore $(F, A)$ is not a F-soft lattice implication algebra of $L$.

Example 3.2: Let $L = \{O, a, b, I\}$, its implication operator $\rightarrow$ and negation operator $^{\prime}$ be defined as Table 2. Then $L = (L, \vee, \wedge, ^{\prime}, \rightarrow, O, I)$ is a lattice implication algebra.

Let $(F, A)$ be a soft set over $L$, where $A = \{0.1\}$ and $F : A \rightarrow P(L)$ the set-valued function defined by $F(t) = \{y \in L \mid y \rightarrow t \in \{a, c\}\}$, then $F(c) = \{a, I\}, F(d) = \{b, d\}$ are filters of $L$. Therefore $(F, A)$ is a F-soft lattice implication algebra of $L$.
Proof: Let \((F, A) \cup (G, B) = (H, C)\) and \(A \cap B = \emptyset\), where \(C = A \cup B = \emptyset\). We have \(c \in A \cup B\) for any \(c \in C\). If \(c \in A \cup B\), then \(H(c) = F(c)\), it follows that \(H(c)\) is a \(F\)-soft lattice implication algebra over \(\mathcal{L}\). Similarly, we have \(H(c)\) is a \(F\)-soft lattice implication algebra over \(\mathcal{L}\). That is \((H, C)\) is a \(F\)-soft lattice implication algebra over \(\mathcal{L}\).

**Theorem 3.3:** Let \((F, A)\) and \((G, B)\) be two \(F\)-soft lattice implication algebras over \(\mathcal{L}\). Then \((F, A) \cap (G, B)\) is also a \(F\)-soft lattice implication algebra over \(\mathcal{L}\).

Proof: Let \((H, C) = (F, A) \cap (G, B)\), where \(C = A \times B\) and \(H(x_1, x_2) = F(x_1) \cap F(x_2)\), \((x_1, x_2) \in A \times B\). Now, \(F(x_1)\) and \(F(x_2)\) are two filters of \(\mathcal{L}\), so \(F(x_1) \cap (x_2)\) is also a filter of \(\mathcal{L}\). Hence \((H, C)\) is a \(F\)-soft lattice implication algebra over \(\mathcal{L}\).

**Definition 3.4:** Let \((F, A)\) be a \(F\)-soft lattice implication algebra over \(\mathcal{L}\).

1. \((F, A)\) is called the trivial \(F\)-soft lattice implication algebra over \(\mathcal{L}\) if \(F(x) = \{1\}\) for any \(x \in A\).
2. \((F, A)\) is called the whole \(F\)-soft lattice implication algebra over \(\mathcal{L}\) if \(F(x) = \{0\} \cup \{1\}\) for any \(x \in A\).

**Example 3.3:** In Example 3.1, let \((F, A)\) be a soft set over \(\mathcal{L} = \{L, \lor, \land, \rightarrow, O, I\}\). Let \(A = \{1\}\) and \(F : A \rightarrow P(L)\) be the set-valued function defined by \(F(x) = \{y \in L | x \leq y\}\), then \(F(I) = \{1\}\) is a filter of \(\mathcal{L}\). Hence \((F, A)\) is a \(F\)-soft lattice implication algebra over \(\mathcal{L}\).

**Theorem 3.4:** Let \(\mathcal{L}_1 = (L_1, \lor, \land, \rightarrow, O, I)\) and \(\mathcal{L}_2 = (L_2, \lor, \land, \rightarrow, O, I)\) be two lattice implication algebras and \(f : \mathcal{L}_1 \rightarrow \mathcal{L}_2\) a mapping of lattice implication algebras. If \((F_1, A)\) and \((F_2, B)\) are soft sets over \(\mathcal{L}_1\) and \(\mathcal{L}_2\), respectively. Then \(\{f(F_1)(I)\}\) is a soft set over \(\mathcal{L}_2\) where \(f(F_1)(I) = \{f(F(x)) | x \in A\}\) is a filter of \(\mathcal{L}_2\) and \(f^{-1}(F(I))\) is defined by \(f^{-1}(F(I)) = \{f^{-1}(F_2(y)) | y \in B\}\) for any \(y \in B\).

**Theorem 4.1:** Let \(\mathcal{L}_1, \mathcal{L}_2, \mathcal{L}_3\) be lattice implication algebras and \(f : \mathcal{L}_1 \rightarrow \mathcal{L}_2\) be a lattice implication homomorphism. If \((F_2, B)\) is a \(F\)-soft lattice implication algebra over \(\mathcal{L}_2\), then \((f^{-1}(F_2), B)\) is a \(F\)-soft lattice implication algebra over \(\mathcal{L}_1\).

Proof: Since \((F, B)\) is a \(F\)-soft lattice implication algebra over \(\mathcal{L}_2\), then \(f^{-1}(F_2)(y)\) is a filter for any \(y \in B\) and so \(f(I) = I^* \in F_2(y)\), where \(I, I^*\) are the greatest elements of \(\mathcal{L}_1\) and \(\mathcal{L}_2\), respectively. It follows that \(I = f^{-1}(I) \in f^{-1}(F_2(y))\). If \(x_1, x_2 \in F_2(y)\) for any \(y \in B\), then \(f(x_1) = f(x_2) = F_2(y)\). Since \(F_2(y)\) is a \(F\)-soft lattice implication algebra over \(\mathcal{L}_2\), \(f\) is a lattice implication homomorphism, we have \(f(y_1) \in F_2(y)\), that is, \(y_1 \in f^{-1}(F_2(y))\). Therefore, \(f^{-1}(F_2(y))\) is a filter of \(\mathcal{L}_2\) and so \((f^{-1}(F_2), B)\) is a \(F\)-soft lattice implication algebra over \(\mathcal{L}_1\).

**Theorem 3.5:** Let \(f : \mathcal{L}_1 \rightarrow \mathcal{L}_2\) be an \(F\)-soft lattice implication homomorphism and \((F, A)\) and \((G, B)\) are \(F\)-soft lattice implication algebras over \(\mathcal{L}_1\) and \(\mathcal{L}_2\), respectively.

1. If \(F(x) = D - \ker(f)\) for any \(x \in A\), then \((f(F), A)\) is a \(F\)-soft lattice implication algebra over \(\mathcal{L}_2\).
2. If \(f\) is onto and \((F, A)\) is whole, then \((f(F), A)\) is a \(F\)-soft lattice implication algebra over \(\mathcal{L}_2\).
3. If \(G(y) = F(L_1)\) for any \(y \in B\), then \((f^{-1}(G), B)\) is \(F\)-soft lattice implication algebra over \(\mathcal{L}_1\).
4. If \(f\) is injective and \((G, B)\) is trivial, then \((f^{-1}(G), B)\) is a \(F\)-soft lattice implication algebra over \(\mathcal{L}_1\).

Proof: (1) Assume that \(f(x) = D - \ker(f)\) for any \(x \in A\), then \(f(F(x)) = f(F(x)) = \{I\}\) for any \(x \in A\), where \(I\) is the greatest element of \(\mathcal{L}_2\). Obviously, \(f(F(x))\) is a filter of \(\mathcal{L}_2\) for any \(x \in A\), that is \((f(F), A)\) is a \(F\)-soft lattice implication algebra over \(\mathcal{L}_2\).

2. Assume that \(f\) is onto and \((F, A)\) is whole, then \(f(F(x)) = f(F(x)) = \{I\}\) for any \(x \in A\) and \(f(F(x)) = f(F(x)) = \{I\}\). Obviously, \(f(F(x)) = f(F(x)) = \{I\}\) is a filter of \(\mathcal{L}_2\) for any \(x \in A\), that is \((f(F), A)\) is a \(F\)-soft lattice implication algebra over \(\mathcal{L}_2\).

3. Suppose that \(G(y) = f(L_1)\) for any \(y \in B\). Then \(f^{-1}(G)(y) = f^{-1}(G)(y) = f^{-1}(f(L_1))\) for any \(y \in B\). Hence \((f^{-1}(G), B)\) is \(F\)-soft lattice implication algebra over \(\mathcal{L}_1\).

4. Let \(f\) be injective and \((G, B)\) is trivial, then \(G(y) = I\) for any \(y \in B\) and so \(f^{-1}(G)(y) = f^{-1}(G)(y) = f^{-1}(I) = D - \ker(f) = \{I\}\) for any \(y \in B\). Therefore \(f^{-1}(G)(y)\) is \(F\)-soft lattice implication algebra over \(\mathcal{L}_1\).

**IV. (IF)-F-Soft Lattice Implication Algebras in Fuzzy Context**

In this section, firstly, we discuss the relations between \((IF)\)-\(F\)-soft lattice implication algebras and fuzzy (implicative) filters. Secondly, we generalize fuzzy filters of \(\mathcal{L}\), initiating the notion of \((\varepsilon, \in, \forall, \alpha)-\)fuzzy (implicative) filters for \(\mathcal{L}\), their equivalent characterizations are derived. At last, we discuss the relations between \((IF)\)-\(F\)-soft lattice implication algebras and \((\varepsilon, \in, \forall, \alpha)-\)fuzzy (implicative) filters.
y ∈ F(t). Therefore F(t) is a filter of L for any t ∈ (0, 1]. Hence (F, (0, 1]) is a F-soft lattice implication algebra over L.

Conversely, assume that (F, (0, 1]) is a F-soft lattice implication algebra over L. If there exists a ∈ L such that μ(I) < μ(a), then we can choose t ∈ (0, 1] such that μ(I) < μ(a) and so I ∈ F(t), contradiction. Thus μ(I) ≥ μ(x) for any x ∈ L. Suppose that there exist a, b ∈ L such that μ(b) < min{μ(a → b), μ(a)}, we choose t ∈ (0, 1] such that μ(b) < t < min{μ(a → b), μ(a)}. It follows that a → b ∈ F(t) and a ∈ F(t), but b ∉ F(t), which contradicts with F(t) is a filter of L. Hence μ(y) ≥ min{μ(x → y), μ(x)} for any x, y ∈ L. Therefore μ is a fuzzy filter of L.

(2) The case for (2) can be similarly proved.

**Theorem 4.2:** Let μ be a fuzzy set of L and (F_q, (0, 1]) be a soft set over L. Then

1. (F_q, (0, 1])[(F_q(t) ≠ Φ, t ∈ (0, 1]) is a F-soft lattice implication algebra if and only if μ is a fuzzy filter of L.
2. (F_q, (0, 1])[(F_q(t) ≠ Φ, t ∈ (0, 1]) is a 1F-soft lattice implication algebra if and only if μ is a fuzzy implicative filter of L.

**Proof:** (1) Assume that μ is a fuzzy filter of L and let t ∈ (0, 1] such that F_q(t) ≠ Φ. If I ∉ F_q(t), i.e. I ∉ G_q and so μ(I) + t < 1. It follows that μ(x) + t ≤ μ(I)[t + t < 1 for any x ∈ L, so that F_q(t) = Φ. This is a contradiction and so I ∈ F_q(t). Let x, y ∈ L be such that x → y ∈ F_q(t) and x ∈ F_q(t), then (x → y) ∨ μ y and x ∨ μ y, i.e. μ(x → y) + t > 1 and μ(y) + t > 1. Hence μ(y) + t ≥ min{μ(x → y), μ(y)} + t = min{μ(x → y), t, μ(y) + t} > 1. We have y ∈ F_q(t) for any t ∈ (0, 1]. Therefore F_q(t) is a filter of L for any t ∈ (0, 1], i.e. (F_q, (0, 1]) is a F-soft lattice implication algebra over L.

Conversely, assume that (F_q, A) is a F-soft lattice implication algebra over L. If there exists a ∈ L such that μ(I) < μ(a), then μ(I) + t < 1 < μ(a) + t for some t ∈ (0, 1]. Thus α_q μ and F_q(t) ≠ Φ. We have F_q(t) is a filter of L for any t ∈ (0, 1]. Hence I ∈ F_q(t) and I ∉ G_q, i.e. (μ(I) + t) > 1, is impossible, and so μ(I) ≥ μ(x) for any x ∈ L. Suppose that there exist a, b ∈ L such that μ(b) < min{μ(a → b), μ(a)}, then μ(a) + s ≤ 1 < min{μ(a → b), μ(a)} + s for some s ∈ (0, 1]. It follows that (a → b) ∨ μ and α_q μ, that is, a → b ∈ F_q(a) and a ∈ F_q(b). It follows from F_q(a) is a filter that μ(b) ∈ F_q(t). So μ(a) + s > 1, contradiction. Therefore μ(x) ≥ min{μ(x → y), μ(x)} for any x, y ∈ L. Hence μ is a fuzzy filter of L.

(2) The case for (2) can be similarly proved.

In what follows, let k denote an arbitrary element of [0, 1) unless otherwise specified. To say x_q μ(y) we mean μ(x → y) + k > 1. To say x ∈ y or x_q μ(y), we mean x ∈ μ(y) or x ∨ μ(y). For α ∈ (e, e_q), to say x ∨ μ(y), we mean x ∨ α_q μ(y).

**Definition 4.1:** A fuzzy set μ in L is said to be an (e, e_q)-fuzzy filter of L if it satisfies the following:

1. x_q μ(x) implies I_q μ(x) for any x, y ∈ L and t, r ∈ (0, 1].

**Definition 4.2:** A fuzzy set μ in L is said to be an (e, e_q)-fuzzy implicative filter of L if it satisfies the following:

1. x_q μ(x) implies I_q μ(x) for any x, y ∈ L and t, r ∈ (0, 1].
2. (x → (y → z))_t ∈ μ and (x → y)_r ∈ μ imply (x → z)_min{t,r} ∈ μ for any x, y ∈ L and t, r ∈ (0, 1].

**Example 4.1:** In Example 3.1, we define a fuzzy set μ of L as following:

μ(I) = 0.45, μ(a) = 0.8, μ(b) = μ(c) = μ(d) = μ(O) = 0.3. It is routine to verify that μ is an (∈, e_q)-fuzzy filter.

**Theorem 4.3:** A fuzzy set μ in L is an (e, e_q)-fuzzy filter of L if and only if it satisfies the following, for any x, y ∈ L.

1. μ(I) ≥ min{μ(x, 1/2k)}
2. μ(I) ≥ min{μ(x → y), μ(x, 1/2k)}

**Proof:** Let μ be an (∈, e_q)-fuzzy filter. Assume that μ(I) < min{μ(x, 1/2k)}. Then μ(I) < r < min{μ(x, 1/2k)} for some r ∈ (0, 1/2k). If μ(I) < 1/2k, then μ(I) < r ≤ μ(x). Therefore μ(I) ≥ 1/2k, then μ(I) < r ≤ 1/2k, hence x_q μ and I ∉ G_q. Furthermore, μ(I) + r + r = 2r < 1 − k, that is, I_q μ and I ∉ G_q, contradiction. If μ(x) ≥ 1/2k, then μ(I) < r < 1/2k, hence x_q μ and I_q μ. Therefore μ(I) + 1/2k ≤ 1 − 1/k, that is, I_q μ = μ_q. Therefore, y_q μ ∈ μ_q. contradiction. Hence μ(I) ≥ min{μ(x, 1/2k)} for any x, y ∈ L.

Assume that (2) doesn’t hold, then there exist x, y ∈ L such that μ(x) < min{μ(x → y), μ(x → y)} + k. If min{μ(x → y), μ(x → y)} < 1/2k, then μ(y) < min{μ(x → y), μ(x)} + k. Hence μ(y) < t < min{μ(x → y), μ(x)} for some t ∈ (0, 1/2k]. It follows that (x → y → x)_t ∈ L and x ∈ L, but y ∉ μ_q. Moreover, μ(y) + t + 2t < 1 − 1/k and so y_q μ and G_q μ_q, contradiction. If min{μ(x → y), μ(x)} > 1/2k, then μ(x) ≥ 1/2k and μ(y) ≥ 1/2k. It follows that x_q μ and (x → y)_r ≤ μ. Since μ is an (∈, e_q)-fuzzy filter of L, we have y_q μ ∈ μ_q and μ_q < min{μ(x → y), μ_q(x)}, μ_q(x) = 1/2k. And so y_q μ ∉ μ_q. Also μ(y) + 1/2k < 1/2k + 1/2k = 1 − 1/k. Hence y_q μ ∈ μ q. contradiction. Therefore μ(x) ≥ min{μ(x → y), μ(x, 1/2k)} for any x, y ∈ L.

Conversely, let μ be a fuzzy set in L satisfying two conditions. Let x, y ∈ L and t, r ∈ (0, 1/2k] be such that x_q μ and μ(x) ≥ t and so μ(I) ≥ min{μ(x, 1/2k)} ≥ min{t, 1/2k}. If t ≤ 1/2k, then μ(I) ≥ t and I_q μ and μ(I) > 1/2k, then μ(I) t + k > t + 1, i.e. I_q μ_q, we have I_q μ_q. Let μ_q satisfying μ_q(y) ≥ min{μ(x → y), μ(x, 1/2k)} for any x, y ∈ L. Let x, y ∈ L and t, r ∈ (0, 1/2k] be such that x_q μ and (x → y)_t ∈ μ, then μ(x) ≥ t, (x → y)_r ∈ μ. We have μ_q(y) ≥ min{μ(x → y), μ(x, 1/2k)} ≥ min{t, r, 1/2k}. If min{t, r} > 1/2k, then μ_q(y) ≥ min{t, r, 1/2k}, it follows that y_q μ_q(x) ∈ μ_q and so y_q μ_q(x) ∈ μ_q. If min{t, r} > 1/2k.
i.e. \( t \geq \frac{1}{k} b \) and \( r \geq \frac{k-1}{k} \), we have \( \mu(y) > \frac{1}{k} b \), \( \mu(y) + \min(t, r) > \frac{1}{k} b + \frac{k-1}{k} = 1 \). k. It follows that \( y_{\min(t,r)} \in \forall \mu (y) \). Therefore \( \mu \) is an \((\varepsilon, \in \forall \mu)\)-fuzzy filter of \( \mathcal{L} \). 

**Corollary 4.1:** (see [21]) Let \( \mu \) be a fuzzy subset of \( \mathcal{L} \). Then \( \mu \) is an \((\varepsilon, \in \forall \mu)\)-fuzzy filter of \( \mathcal{L} \) if and only if \( \mu \) satisfies following

1. \( (\forall x \in L)(\mu(I) \geq \min\{\mu(x), 0.5\}) \)
2. \( (\forall x,y \in L)(\mu(y) \geq \min\{\mu(x), \mu(x \to y), 0.5\}) \)

**Theorem 4.4:** A fuzzy set \( \mu \) in \( \mathcal{L} \) is an \((\varepsilon, \in \forall \mu)\)-fuzzy implicative filter of \( \mathcal{L} \) if and only if it satisfies the following, for any \( x, y, z \in L \)

1. \( (\mu(I) \geq \min\{\mu(x), \frac{1}{k} b\}) \)
2. \( (\mu(x \to z) \geq \min\{\mu(x \to (y \to z)), \mu(x \to y), \frac{1}{k} b\}) \)

**Proof:** It is similarly proved as Theorem 4.3. 

**Corollary 4.2:** (see [21]) Let \( \mu \) be a fuzzy subset of \( \mathcal{L} \). Then \( \mu \) is an \((\varepsilon, \in \forall \mu)\)-fuzzy implicative filter of \( \mathcal{L} \) if and only if \( \mu \) satisfies following

1. \( (\forall x \in L)(\mu(I) \geq \min\{\mu(x), 0.5\}) \)
2. \( (\forall x,y \in L)(\mu(y) \geq \min\{\mu(x), \mu(x \to y), 0.5\}) \)

**Theorem 4.5:** Let \( \mu \) be a fuzzy set of \( \mathcal{L} \) and \( (F, (0, \frac{1}{k} b)) \) be a soft set. Then \( (F, (0, \frac{1}{k} b)) \) is a \( F \)-soft lattice implication algebra over \( \mathcal{L} \) if and only if \( \mu \) is an \((\varepsilon, \in \forall \mu)\)-fuzzy filter of \( \mathcal{L} \).

**Proof:** Let \( \mu \) be an \((\varepsilon, \in \forall \mu)\)-fuzzy filter of \( \mathcal{L} \). For any \( x \in F(t) \), we have \( \mu(I) \geq \min\{\mu(x), \frac{1}{k} b\} \) for any \( x \in F(t) \). Hence \( \mu(I) \geq \min\{\mu(x), \frac{1}{k} b\} \geq \min(t, \frac{1}{k} b) = t \), which implies \( I_t \in \mu \) and so \( I_t \in F(t) \). 

**Example 4.2:** In Example 3.1, we define a fuzzy set \( \mu \) as follows:

\( \mu(O) = 0.4, \mu(I) = \mu(b) = \mu(c) = 0.9, \mu(a) = \mu(d) = 0.3 \).

It is routine to verify that \( (\varepsilon, \in \forall \mu) \) is a \( (\varepsilon, \in \forall \mu) \)-fuzzy (implicative) filter of \( \mathcal{L} \).

**Theorem 4.7:** Let \( \mu \) be a fuzzy subset of \( \mathcal{L} \), then \( \mu \) is an \((\varepsilon, \in \forall \mu)\)-fuzzy filter of \( \mathcal{L} \) if and only if for any \( x, y \in L \),

\( \max\{\mu(I), \frac{1}{k} b\} \geq \min\{\mu(x), \mu(x \to y)\} \)

**Proof:** Assume that there exists \( x, y \in L \) such that \( \max\{\mu(I), \frac{1}{k} b\} < \mu(x \to y) \). Then \( t \in (\frac{1}{k} b, 1) \) and \( I_t \in \mu \). It follows that \( x \in \forall \mu \). Hence \( \mu(x) < t \) or \( \mu(x) + t + k \leq 1 \), we have \( t \leq \frac{1}{k} b \) for \( \mu(x) = t \), contradiction. Therefore, \( \max\{\mu(I), \frac{1}{k} b\} \geq \mu(x), (1) \) is valid.

Assume that there exist \( x, y \in L \) such that

\( \max\{\mu(I), \frac{1}{k} b\} \geq \min\{\mu(x), \mu(x \to y)\} = t \), then

\( \mu(x) < t \) and \( t \in (\frac{1}{k} b, 1) \). It follows that \( y \in \forall \mu \). But \( x \in \forall \mu \) and \( (x \to y) \in \forall \mu \). Hence \( x \in \forall \mu \) or \( (x \to y) \in \forall \mu \). If \( \mu(x) \geq t \) and \( \mu(x) + t + k \leq 1 \), we have that \( t \leq \frac{1}{k} b \), contradiction. Therefore, \( 2 \) holds.
Conversely, assume that there exist $x \in L$ and $t, r \in (\frac{1-t}{2}, 1)$ such that $I_t \vee \mu_t$, but $x \in \bar{\vee} \mu_t$, then $\mu(I) = \mu(x) \geq t$ and $\mu(x) + t + k \geq 1$. Therefore, $\mu(x) \geq \frac{t}{1-t}$. Thus $\max{\mu(I), \frac{t}{1-t}} < \max{\mu(I), \frac{t}{1-t}} \leq \max{\mu(x), t} = \mu(x)$, contradiction. That is, $I_t \vee \mu_t$ implies $x \in \bar{\vee} \mu_t$.

Assume (2) holds and let $y_{min(t,r)} \in \mu_t$, then $\mu(y) < min(t,r)$. There are two cases to be discussed.

(a) If $\mu(y) \geq \min{\mu(x), \mu(x-y)}$, then $\min(t,r) > \min{\mu(x), \mu(x-y)}$. It follows that $\mu(x) < t$ or $\mu(x-y) < r$, that is, $x \in \bar{\vee} \mu_t$ or $(y \in \bar{\vee} \mu_t)$. Of course, $x \in \bar{\vee} \mu_t$ or $(y \in \bar{\vee} \mu_t)$.

(b) If $\mu(y) < \min{\mu(x), \mu(x-y)}$, then $\frac{t}{1-t} \geq \min{\mu(x), \mu(x-y)}$. Assume that $x \in \bar{\vee} \mu_t$ and $(x \in \bar{\vee} \mu_t)$, then $\mu(x) \geq r$ and $\mu(x-y) + r + k \geq 1$. Therefore, $\mu(x) \geq \frac{t}{1-t}$ and $\mu(x-y) > \frac{t}{1-t}$. Hence $\min{\mu(x), \mu(x-y)} \geq \frac{t}{1-t}$, which contradicts with $\min{\mu(x), \mu(x-y)} \leq \frac{t}{1-t}$.

Therefore, $x \in \bar{\vee} \mu_t$ or $(y \in \bar{\vee} \mu_t)$.

**Theorem 4.8:** Let $\mu$ be a fuzzy subset of $\mathcal{L}$, then $\mu$ is an $(\bar{\vee} \mu_t, \bar{\vee} \mu_t)$-fuzzy implicative filter of $\mathcal{L}$ if and only if for any $x, y \in L$,

$$\begin{align*}
(1) & \max{\mu(I), \frac{t}{1-t}} \geq \mu(x), \\
(2) & \max{\mu(x-z), \frac{r}{1-r}} \geq \min{\mu(x-y), \mu(x-y-z)}.
\end{align*}$$

**Proof:** It is similarly proved as Theorem 4.7.

**Corollary 4.5:** Let $\mu$ be a fuzzy subset of $\mathcal{L}$, then $\mu$ is an $(\bar{\vee} \mu_t, \bar{\vee} \mu_t)$-fuzzy implicative filter of $\mathcal{L}$ if and only if for any $x, y \in L$,

$$\begin{align*}
(1) & \max{\mu(I), \frac{t}{1-t}} \geq \mu(x), \\
(2) & \max{\mu(x-z), \frac{r}{1-r}} \geq \min{\mu(x-y), \mu(x-y-z)}.
\end{align*}$$

**Theorem 4.9:** Let $\mu$ be a fuzzy subset of $\mathcal{L}$ and $(F, \frac{t}{1-t}, 1)$ be a soft set. Then $F, \frac{t}{1-t}, 1)$ is a $F$-soft lattice implication algebra over $\mathcal{L}$ if and only if $\mu$ is an $(\bar{\vee} \mu_t, \bar{\vee} \mu_t)$-fuzzy implicative filter of $\mathcal{L}$.

**V. CONCLUSION**

Soft sets are related to fuzzy sets and rough sets. It applied to some algebraic structures. In this paper, we introduce the (implicative) filteristic soft lattice implication algebras which related with (implicative) filter(for small). $(F)$-soft lattice implication algebras. Basic properties of $(F)$-soft lattice implication algebras are investigated. We introduce the notion of $(c, \in, \in \vee q_k)$-fuzzy soft lattice implication algebras, which are generalizations of fuzzy (implicative) filter. We provide characterizations for a soft set to be a $(F)$-soft lattice implication algebra. Analogously, this idea can be applied in other structures (such as positive implicative filters, ultrafilter, fantastic filter, and so on), analogously. It will be of great use to provide theoretical foundation to design intelligent information processing systems.

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**REFERENCES**


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