Analyzing Multi-Labeled Data Based on the Roll of a Concept against a Semantic Range

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Abstract—Classifying data hierarchically is an efficient approach to analyze data. Data is usually classified into multiple categories, or annotated with a set of labels. To analyze multi-labeled data, such data must be specified by giving a set of labels as a semantic range. There are some certain purposes to analyze data. This paper shows which multi-labeled data should be the target to be analyzed for those purposes, and discusses the role of a label against a set of labels by investigating the change when a label is added to the set of labels. These discussions give the methods for the advanced analysis of multi-labeled data, which are based on the role of a label against a semantic range.

Keywords—Classification Hierarchies, Data Analysis, Multi-labeled Data, Orders of Sets of Labels

I. INTRODUCTION

For the purpose of getting more competitive advantages in economic competition, companies and governments have now needed information systems which can analyze collected data to support their decision making. With rapidly increasing data, including numerical data, texts data, image data, and audio data, it is becoming more important to organize collected data properly. Classifying data hierarchically is one of the efficient approach to organize collected data [1] [8]. Individual data is classified into various categories, or annotated with their categories which are used to specify a set of data to be analyzed.

Data is usually assumed to be classified into one category and annotated with the label of the category [1] [11]. For example, each news document in Newsgroups data set is classified into only one category [9]. Although specifying such data is straightforward if the data has only one label, there is data which should be classified into multiple categories. If such data is classified into only one category, it is not specified by the labels of the other classes into which the data should also be classified.

Multiple classifications give much more information for analysis because the data can be specified by several labels. For example, data, which is about the comparison between Japan and U.S.A, can be analyzed as the data related to both categories Japan and U.S.A, if the data is classified into those two categories. Such data is usually classified into multiple categories and annotated with multiple labels of the categories [5] [6]. For example, the data mentioned above is annotated with multiple labels Japan, U.S.A.

To analyze multi-labeled data, such data must be specified by giving a set of labels as a semantic range. There are some current researches about classification on data annotated with multiple labels. However, the set of data specified by a set of labels are usually the result of the intersection or union of the data specified by each label of the set of labels [2] [7] [8].

What data is expressed by a set of labels has been discussed in [4] by introducing the orders between the set of labels. The previous researches do not refer to the analysis of the multi-labeled data from such detailed discussions. This paper discusses the semantic ranges of data to be analyzed and the correspondence between the ranges and the orders precisely. And then, the role of a label against a set of labels is shown by investigating the change when a label is added to the set of labels. This discussion gives methods for the advanced analysis of multi-labeled data, which are based on the role of a label against a set of labels.

This paper is organized as follows. The orders for sets of labels in order to express multi-labeled data are introduced in Section 2. Section 3 discusses the semantic ranges of data to be analyzed and the correspondence between the ranges and the orders. Sections 4 and 5 show the analysis methods for multi-labeled data which are based on the role of a label against a set of labels. Section 6 concludes the paper.

II. INTRODUCING ORDERS FOR SETS OF LABELS

Individual data or an object is classified by certain types of characteristic, which are called attributes. For example, an object is classified into countries, states, cities, etc., where the attribute is region. While there is classification for multiple attributes, this paper discusses one specific attribute for simplicity, and assumes that a classification hierarchy of the attribute is given in advance and objects will be classified based on the hierarchy.

Let \( o \) be an object and \( L \) be a label which is used to classify objects. Let \( \overline{L} \) be the set of the objects expressed by \( L \), and \( \overline{o} \) be the label of \( o \) for the classification attribute. An object is usually classified into the lowest category (or categories in multi-label classification) related to the object in a given classification hierarchy [5] [6]. \( \overline{\partial} \) is the label (or the set of labels) of the category (or the categories) into which \( o \) is classified. Objects may be classified into intermediate categories, which are not leaves in the hierarchy [3] [10]. For example, the label of an object on Japan is on the country
level, which is not a label of a leaf category if the hierarchy has still city level categories.

For labels $L_1$ and $L_2$, $L_2$ is higher than $L_1$ ($L_1$ is lower than $L_2$) if the category of $L_2$ is a higher concept of the category of $L_1$, denoted by $L_1 < L_2$. $L_1 \subseteq L_2$ denotes that $L_1$ is higher than or equal to $L_1$. The membership of single-labeled objects to $L$ is decided by the label of the objects as $L = \{o \mid \bar{o} \preceq L\}$. When an object is classified into more than one category, the label of this object is a set of labels, called set-label.

For an object $o$ with a set-label, $L$ is defined by introducing an order between the set-label $\bar{o}$ and a label $L$. A set of labels $L$ is usually interpreted as conjunction or disjunction of the objects described by the labels in $L$. The orders for these interpretations are as follows:

1) Conjunction: For a label $L$ and a set of labels $L_1$, $L$ is lower than or equal to $L$ if every label of $L$ is lower than or equal to $L$, denoted by $L \preceq L$.

2) Disjunction: For a label $L$ and a set of labels $L_1$, $L$ is lower than or equal to $L$ if some label of $L$ is lower than or equal to $L$, denoted by $L \preceq L$.

A label used to express objects is extended to a set of labels. Let $\bar{L}$ be the set of the objects expressed by a set of labels $L$, and conjunction and disjunction interpretations of a set of labels for a single label are extended to for a set of labels. Generally, a set of labels $L$ is interpreted as the intersection or the union of the objects of labels expressed by the labels of $L$. Let $\bar{L}^{CI} = \bigcap_{L \in \bar{L}} L$ and $\bar{L}^{DU} = \bigcup_{L \in \bar{L}} L$ be the intersection and the union of the sets of objects expressed by the labels in $L$ for conjunction, respectively, and $\bar{L}^{DI} = \bigcap_{L \in \bar{L}} L$ and $\bar{L}^{CU} = \bigcup_{L \in \bar{L}} L$ for disjunction, respectively.

Since the set of objects expressed by a label is decided by the order of $\bar{o}$ and $L$, orders for sets of labels have to be introduced. The orders corresponding to $\bar{L}^{CI}$, $\bar{L}^{CU}$, $\bar{L}^{DI}$, and $\bar{L}^{DU}$ are defined as follows.

**Definition 1** For sets of labels $L_1$ and $L_2$, $L_1 \preceq L_2$ if $\forall L_1 \in L_1, L_2 \in L_2$, $L_1 \preceq L_2$, $L_1 \preceq L_2$ if $\exists L_1 \in L_1, L_2 \in L_2$, $L_1 \preceq L_2$, $L_1 \preceq L_2$ if $\exists L_1 \in L_1, L_2 \in L_2$, and $L_1 \preceq L_2$ if $\exists L_1 \in L_1, L_2 \in L_2$.

**Theorem 1** [4] For a set of labels $L$, $\bar{L}^{CI} = \{o \mid \bar{o} \preceq CI L\}$, $\bar{L}^{CU} = \{o \mid \bar{o} \preceq CU L\}$, $\bar{L}^{DI} = \{o \mid \bar{o} \preceq DI L\}$, and $\bar{L}^{DU} = \{o \mid \bar{o} \preceq DU L\}$.

There can be, on the other hand, the extension of these orders for a set of labels and a single-labeled object. There are two interpretations of a set of labels for single-labeled objects, intersection and union, which are formally expressed as $\bigcap_{L \in \bar{L}} L$ and $\bigcup_{L \in \bar{L}} L$, respectively.

Suppose that interpretation of $L$ is intersection. For a single-labeled object $o$ and $L' = \bar{o}$, $L'$ is lower than or equal to every label in $L$, and $L' \subseteq \bigcap_{L \in \bar{L}} L = \bigcap_{L \in \bar{L}} L \{o \mid \bar{o} \preceq L\}$. Thus the set of multi-labeled objects expressed by $L$ with conjunction is $\bigcap_{L \in \bar{L}} L \{o \mid \bar{o} \preceq L\}$. Hence, objects whose labels are lower than or equal to $L$, the orders corresponding to $\bar{L}^{CI}$, $\bar{L}^{DU}$, $\bar{L}^{CU}$, and $\bar{L}^{DU}$ are introduced.

**Definition 2** For sets of labels $L_1$ and $L_2$, $L_1 \preceq IC L_2$ if $\forall L_1 \in L_1, L_2 \in L_2$, $L_1 \preceq ID L_2$ if $\exists L_1 \in L_1, L_2 \in L_2$, $L_1 \preceq UC L_2$ if $\forall L_1 \in L_1, L_2 \in L_2$, $L_1 \preceq UD L_2$ if $\exists L_1 \in L_1, L_2 \in L_2$, and $L_1 \preceq DI L_2$ if $\exists L_1 \in L_1, L_2 \in L_2$. $L_1 \preceq DI L_2$ if $\exists L_1 \in L_1, L_2 \in L_2$.

**Theorem 2** [4] For a set of labels $L$, $\bar{L}^{IC} = \{o \mid \bar{o} \preceq IC L\}$, $\bar{L}^{ID} = \{o \mid \bar{o} \preceq ID L\}$, $\bar{L}^{UC} = \{o \mid \bar{o} \preceq UC L\}$, and $\bar{L}^{UD} = \{o \mid \bar{o} \preceq UD L\}$.

There are eight kinds of orders which can be integrated to three kinds according to their definitions. Since the definitions of $\bar{L}^{IC}$ and $\bar{L}^{UD}$ are equal to $\bar{L}^{IC}$ and $\bar{L}^{UD}$, respectively, $\bar{L}^{IC}$ and $\bar{L}^{UD}$ are excluded. It is obvious that $\bar{L}^{ID}$ is a special case of $\bar{L}^{ID}$, and both $\bar{L}^{IC}$ and $\bar{L}^{UD}$ are special cases of $\bar{L}^{IC}$ by the definitions of those orders. Orders $\bar{L}^{ID}$, $\bar{L}^{IC}$, and $\bar{L}^{UD}$ are excluded from our considerations. Thus the orders are summarized to three kinds which are the orders $\bar{L}^{ID}$, $\bar{L}^{IC}$, and $\bar{L}^{UD}$.

While the set of objects described by a set of labels $L$ is decided by the order of $\bar{o}$ and $L$, there may exist some labels in $L$ and $\bar{o}$ which do not affect this membership.

**Example 1** Suppose $L_1$ and $\bar{o}_1$ are $\{Japan, U.S.A\}$ and $\{Tokyo, New York, Shanghai\}$, respectively. $o_1$ is in $L_1^{DI}$ because there is a lower label in $L_1$ for each label in $L_1$. $\bar{o}_1$ in $L_1$ does not affect this membership. Although there must be a label in $\bar{o}_1$ for each label of $L_1$, $\bar{o}_1$ may include labels unrelated to $L_1$. On the other hand, the labels of object $o_2$ labeled $\{Tokyo, Kyoto\}$ in $L_1$ are not lower than or equal to label U.S.A in $L_1$. Object $o_2$ labeled $\{Tokyo, Shanghai\}$ is in $L_1^{DU}$, where U.S.A in $L_1$ and Shanghai in $\bar{o}_2$ do not affect the membership of $o_2$ to $L_1^{DU}$. All. Fig. 1 illustrates these memberships.
which is a restriction on the higher set \( L_2 \). In the same way, \( L_1 \preceq_{UC} L_2 \) has the restriction on the lower set \( L_1 \). There is no restriction for \( L_1 \preceq_{DU} L_2 \). Thus \( \preceq_{DI}, \preceq_{UC}, \) and \( \preceq_{DU} \) are renamed to \( \preceq_{RU}, \preceq_{RL}, \) and \( \preceq_{RN} \), respectively.

**Definition 3** For sets of labels \( L_1 \) and \( L_2 \),

\[
\begin{align*}
L_1 \preceq_{RU} L_2 & \text{ if } \forall L_2, \exists L_1 \in L_1, L_1 \preceq L_2, \\
L_1 \preceq_{RL} L_2 & \text{ if } \forall L_1, \exists L_2 \in L_2, L_1 \preceq L_2, \\
L_1 \preceq_{RN} L_2 & \text{ if } \exists L_1 \in L_1, \exists L_2 \in L_2, L_1 \preceq L_2.
\end{align*}
\]

Let \( \mathcal{L}^{\preceq_{RU}}, \mathcal{L}^{\preceq_{RL}}, \) and \( \mathcal{L}^{\preceq_{RN}} \) be the sets of the objects expressed by a set of labels \( L \) with orders \( \preceq_{RU}, \preceq_{RL}, \) and \( \preceq_{RN} \), respectively, then \( \mathcal{L}^{\preceq_{RU}} = \mathcal{L}^{\preceq_{DI}}, \mathcal{L}^{\preceq_{RL}} = \mathcal{L}^{\preceq_{UC}}, \) and \( \mathcal{L}^{\preceq_{RN}} = \mathcal{L}^{\preceq_{DU}} \).

There may be other orders defined as that a set of labels \( L_1 \) are lower than or equal to a set of labels \( L_2 \) if \( L_1 \preceq_{x} L_2 \) and \( L_1 \preceq_{y} L_2 \), \( (x, y \in \{RN, RU, RL\}) \). The orders except the order defined with \( x = RU \) and \( y = RL \) are either \( \preceq_{x} \) or \( \preceq_{y} \). For example, the order defined with \( x = RN \) and \( y = RU \) is \( \preceq_{RU} \).

The order where \( x = RU \) and \( y = RL \) has restrictions of \( \preceq_{RU} \) and \( \preceq_{RL} \). Such order is denoted by \( \preceq_{RB} \), where \( \preceq_{RB} \) restricts both of higher and lower sets of labels. Let \( \mathcal{L}^{\preceq_{RB}} \) be the set of objects expressed by a set of labels \( L \) with order \( \preceq_{RB} \). Since \( \mathcal{L}^{\preceq_{RB}} \) is expressed as \( \mathcal{L}^{\preceq_{RB}} = \{o | \o \preceq_{RB} L \} = \{o | \o \preceq_{RU} L, \o \preceq_{RL} L \} \), \( \preceq_{RB} \) is defined as follows.

**Definition 4** For sets of labels \( L_1 \) and \( L_2 \),

\[
\begin{align*}
L_1 \preceq_{RB} L_2 & \text{ if } \forall L_2, \exists L_1 \in L_1, L_1 \preceq L_2, \\
L_1 \preceq_{RB} L_2 & \text{ if } \forall L_1, \exists L_2 \in L_2, L_1 \preceq L_2, \\
L_1 \preceq_{RB} L_2 & \text{ if } \exists L_1 \in L_1, \exists L_2 \in L_2, L_1 \preceq L_2.
\end{align*}
\]

### III. Semantic Ranges of the Target Objects

If objects are annotated with a single-label, it is easy to specify the target objects to be analyzed. On the other hand, if objects are annotated with set-labels, it is usually supposed that there are several kinds of the relations between the set-labels of the objects and a given set of labels. This section shows the target objects to be changed with these interpretations and the correspondence between the objects and the orders proposed in Section 2.

The target objects are decided by the conditions for their set-labels against a given set of labels. There are two kinds of conditions. One is related to the labels which are included in \( L \), and the other is related to the labels which are not included in \( L \).

The condition which is related to the labels included in \( L \):

There are two ways to decide the target objects, for the labels included in \( L \):

1. The set-label of an object is only related to some of the labels in \( L \).
2. The set-label of an object is related to all labels of \( L \). The object described by (1) is the object with the set-label which has the label lower than or equal to some label of \( L \), namely the range of the target objects is specified by \( L \).

The object described by (2) is the object with the set-label, which has the label that is lower than or equal to \( L \) for each label \( L_1 \) of \( L \), and each label of \( L \) is higher than or equal to some label of the set-label. It can be used to analyze the connection of \( L \).

The condition between the set-label and the labels which are not included in \( L \):

There are two kinds of relations between the set-label and the labels which are not included in \( L \):

1. The set-label of an object may be related to the labels not included in \( L \).
2. The set-label of an object must not be related to the labels not included in \( L \).

The objects described by (a) are the objects with the set-labels which may have a label lower than or equal to a label not in \( L \), namely, the target objects are not limited within \( L \).

The objects described by (b) are the objects with the set-labels which do not have a label lower than or equal to any other labels not in \( L \), namely, the objects to be analyzed are limited within \( L \).

By combining these two conditions, there are four kinds of target objects which are annotated with set-labels.

1. The set-label of an object is related to some label of \( L \) and may be related to labels not included in \( L \). For this kind of objects, the target of the objects is specified by \( L \) and the objects are not limited within \( L \).
2. The set-label of an object is related to some label of \( L \) and must not be related to the labels which are not included in \( L \). For this kind of objects, the target of the objects is specified by \( L \) and the objects are limited within \( L \).
3. The set-label of an object is related to all labels of \( L \) and may be related to the labels which are not included in \( L \). For this kind of objects, the connection of \( L \) is analyzed and the objects are not limited within \( L \).
4. The set-label of an object is related to all labels of \( L \) and must not be related to the labels which are not included in \( L \). For this kind of objects, the connection of \( L \) is analyzed and the objects are limited within \( L \).

The following shows that the four kinds of objects correspond to the orders proposed in Section 2 by discussing the objects expressed with each order.

For a given set of labels \( L \), \( \mathcal{L}^{\preceq_{RN}} \) and \( \mathcal{L}^{\preceq_{RL}} \) are the union of the objects expressed by the labels of \( L \), and \( \mathcal{L}^{\preceq_{RU}} \) and \( \mathcal{L}^{\preceq_{RB}} \) are the intersection of the objects expressed by the labels of \( L \). \( \mathcal{L}^{\preceq_{RN}} \) and \( \mathcal{L}^{\preceq_{RB}} \) include objects with labels which are not related to \( L \), while \( \mathcal{L}^{\preceq_{RU}} \) and \( \mathcal{L}^{\preceq_{RB}} \) do not. In the other words, the labels of the objects in \( \mathcal{L}^{\preceq_{RU}} \) and \( \mathcal{L}^{\preceq_{RB}} \) are within the range of \( L \).

**Example 2** For set of labels \( L = \{Japan, U.S.A\} \), \( \mathcal{L}^{\preceq_{RN}} \) and \( \mathcal{L}^{\preceq_{RL}} \) are the union of the objects expressed by the labels of \( L \), which include objects labeled \{Tokyo\}, \{Tokyo, New York\}, and \{Tokyo, New York, Shanghai\} for \( \mathcal{L}^{\preceq_{RN}} \), and \{Tokyo, New York\} for \( \mathcal{L}^{\preceq_{RB}} \). The objects in \( \mathcal{L}^{\preceq_{RU}} \) and \( \mathcal{L}^{\preceq_{RB}} \) are the intersection, which include the objects labeled \{Tokyo, New York\} and \{Tokyo, New York, Shanghai\} for \( \mathcal{L}^{\preceq_{RU}} \), and \{Tokyo, New York\} for \( \mathcal{L}^{\preceq_{RB}} \). While objects of \( \mathcal{L}^{\preceq_{RN}} \) and \( \mathcal{L}^{\preceq_{RU}} \) may include label Shanghai which is related to neither Japan nor
The labels which are not contained in $L$ \\
\begin{tabular}{|c|c|c|}
\hline
(Interpretation of a set of labels) & Some labels (Union) & Not Included (Exists) \\
\hline
\multicolumn{3}{|c|}{(Range of a Set of Labels)} \\
\hline
\multicolumn{3}{|c|}{Included (Not Exists)} \\
\hline
\hline
All labels (Intersection) & 1-a (RN) & 2-a (RU) \\
\hline
\hline
\hline
\hline
\end{tabular} \\
\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{fig2.png}
\caption{Objects to be Analyzed and Orders of Sets of Labels}
\end{figure}

\section{IV. ANALYSIS METHODS BY ADDING A LABEL TO A SET OF LABELS}

This section shows the role of a label against a set of labels by discussing the change of objects expressed by a set of labels when a new label is added into the original set of labels. It gives the analysis method, which is based on the role of a label against a set of labels for multi-labeled objects.

In this section, it’s assumed that $L_i \neq L_j$ for all $i \neq j$ for a set of labels, and each set $L_i$ is called exclusive. In section 5, the discussion is extended to a set of labels where $L_i \leq L_j$ may appear. Since deleting a label is the opposite operation to adding a label, only the case of the addition is discussed.

The objects expressed by adding a label are shown as Lemma 1, where Power ($L$) is the power set of a set of labels $L$.

\begin{lemma}

For a set of labels $L$ and a label $L'$, 
\begin{equation}
L \cup \{L'\} \subseteq L \cup \{L\}
\end{equation}
\end{lemma}

\begin{proof}

Since an object $o$ in $L \cup \{L'\}$ has the set-label whose lower bound is the same as that of $L'$. Thus, $o$ is included in $L \cup \{L\}$.
\end{proof}

\begin{example}

If label \{\text{China}\} is added to set of labels \{\text{Japan, U.S.A.}\}, the result of the union of \{\text{China}\} is \{\text{Japan, U.S.A., China}\}, and \{\text{Japan, U.S.A., China}\} are newly added to \{\text{Japan, U.S.A.}\}.
\end{example}

\begin{theorem}

The objects expressed by $L \cup \{L'\}$ with $\leq$ are totally different from the objects expressed by $L$. Since it is impossible to compare mutually, $\leq$ is not discussed.
\end{theorem}

\begin{proof}

Since the range of a label is the same as that of $L'$, the objects expressed by $L \cup \{L'\}$ are the objects expressed by $L \cup \{L\}$.
\end{proof}

\begin{corollary}

If the target objects are already added to $L \cup \{L'\}$, then the result of the union of $L \cup \{L\}$ is $L \cup \{L'\}$. Thus, $L \cup \{L\}$ is the objects which are different from the objects expressed by $L$. Since it is impossible to compare mutually, $\leq$ is not discussed.
\end{corollary}

\begin{proof}

Since the range of a label is the same as that of $L'$, the objects expressed by $L \cup \{L'\}$ are the objects expressed by $L \cup \{L\}$.
\end{proof}

\begin{table}[h]
\centering
\begin{tabular}{|c|c|c|}
\hline
& Included (Not Exists) & Not Included (Exists) \\
\hline
\hline
\end{tabular}
\end{table}
of all of the labels in the classification hierarchy. For a set of labels \( L \), the set of labels whose values are not lower than or equal to any labels of \( L \) are formally expressed as 
\[ L^C = \{ L \mid L \in \Omega, \forall L' \in L(L' \neq L), L \not\preceq L' \} \]

**Theorem 3** For a set of labels \( L \) and a label \( L' \), 
\[ L \cup \{ L' \} - L^{RN} = \bigcup_{L' \in \text{Power}(L)} L \cup \{ L' \}^{RB}, \]
\[ L^{RU} - L \cup \{ L' \}^{RU} = \bigcup_{L' \in \text{Power}(L)} L^{RU} - L \cup \{ L' \}^{RU}, \]
\[ L \cup \{ L' \}^{RL} - L^{RL} = \bigcup_{L' \in \text{Power}(L)} L \cup \{ L' \}^{RL}. \]

**Proof:** By Lemma 1, \( L \cup \{ L' \}^{RN} - L^{RN} = \bigcup_{L' \in \text{Power}(L)} L \cup \{ L' \}^{RN} - L^{RN} \). An object \( o \) in \( \{ L \}^{RN} - L^{RN} \) is an object related to \( L \) which may be related to some other labels than \( L \). Such objects are the union of the objects expressed by each element of the power set of \( L^C \) and \( L \) with order \( \preceq \).

By Lemma 1, \( L^{RU} - L \cup \{ L' \}^{RU} = \bigcup_{L' \in \text{Power}(L)} L^{RU} - L \cup \{ L' \}^{RU} \). An object \( o \) in \( L^{RU} - \{ L \}^{RU} \) is the object related to \( L \) and may be related to other labels than \( L \). Such objects are the union of the objects expressed by each element of the power set of \( L^C \) and \( L \) with order \( \preceq \).

It is obvious \( L \cup \{ L \}^{RL} = \bigcup_{L' \in \text{Power}(L)} L \cup \{ L' \}^{RL} \) by Lemma 1. Q.E.D.

When a label \( L \) is added to a set of labels \( L \), the objects related to \( L \), and the objects related to \( L \) and some other labels than \( L \) are newly added to the objects expressed by \( L \) if \( L \) with order \( \preceq \). If the objects related to \( L \) are \( \{ L \}^{RU} \) (\( = \{ L \}^{RU} \) \) are deleted from \( L \cup \{ L \}^{RN} \), the resulted objects show the connection of \( L \) with some labels except \( L \).

In the case of \( \preceq \), the objects without the label which is lower than or equal to \( L \) are excluded from the objects expressed by a set of labels \( L \) if \( L \) and \( \{ L \}^{RU} \). The objects related to \( L \) and some labels of \( L \) are newly added to the objects expressed by \( L \).

In the case of \( \preceq \), the objects related to \( L \) and the objects related to \( L \) and some labels of \( L \) are newly added to the objects expressed by \( L \).

**Example 4** For set of labels \( L = \{ \text{Japan}, \text{U.S.A} \} \) and label \( L = \text{China} \), the objects expressed by \( L \cup \{ L \} \), and \( \text{U.S.A} \) are compared with the objects expressed by \( L \) with \( \preceq \), \( \preceq \), and \( \preceq \).

If the aggregation values of \( \text{Japan}, \text{U.S.A}, \text{China} \) and \( \text{China} \), \( \text{U.S.A} \) are compared with the objects expressed by \( L \) with \( \preceq \), \( \preceq \), and \( \preceq \).

The reason why the objects expressed by a set of labels \( L \) with \( \preceq \) and \( \preceq \) are used to specify the range of the target objects and the higher concept labels cover the lower concept labels. Since \( \preceq \) is used to analyze the connection of \( L \) by the lower concept labels have harder connection than the higher concept labels, the objects expressed by \( L \) with \( \preceq \) agree with the objects expressed by \( L \).

When such a label \( L \) is added to a set of labels \( L \), that \( L \) is a higher concept than any other labels of \( L \), the inclusion relations between the objects expressed by \( L \) are shown as Theorem 4.

**Theorem 4** For a set of labels \( L \) and a label \( L \) such that \( \exists L' \in L \), \( L' \preceq L \), 
\[ L^{RN} \subseteq L \cup \{ L \}^{RN} \]
\[ L^{RU} = L \cup \{ L \}^{RU}, \]
\[ T^{RL} \subseteq L \cup \{L \}^{RL} \quad \text{and} \quad T^{RB} \subseteq L \cup \{L \}^{RB}. \]

**Proof:** For a set of labels \( L \) and a label \( L' \) such that \( \exists L' \in L, L' \leq L \), \( T^{RU} = L \cup \{L \}^{RU} \) because of Lemma 2.

An object \( o \) in \( T^{RL} \) is included in \( L \cup \{L \}^{RL} \) because there exists such label of \( o \) that the label is lower than or equal to a label of \( L \), and \( T^{RL} \subseteq L \cup \{L \}^{RL} \). In the same way, it is proved that \( T^{RN} \subseteq L \cup \{L \}^{RN} \).

An object \( o \) in \( T^{RB} \) is included in \( L \cup \{L \}^{RB} \) because for each label \( L' \) of \( L \cup \{L \} \) there is a label of \( o \) that is lower than or equal to \( L' \) and every label of \( o \) is lower than or equal to a label of \( L \cup \{L \} \). Thus \( T^{RB} \subseteq L \cup \{L \}^{RB} \). \( Q.E.D. \)

By Theorem 4, the objects expressed by \( L \) with \( \leq RU \) are not changed even \( L \) is added, which means that adding such labels does not effect the analysis. For such the lowest label \( L' \) in \( L \) that \( L' \leq L \), it is easy to prove that \( L \cup \{L \}^{RN} = L \cup \{L \} - \{L \}^{RN} = \{L \}^{RN} \). Since the same objects as \( L \cup \{L \}^{RN} \) can be obtained with \( L' \), it is not necessary to compare the objects expressed by \( L \) and the objects expressed by \( L \cup \{L \} \) even if the objects expressed with \( \leq RN \) are changed by adding \( L \). The objects expressed with \( \leq RL \) are in the same situation. On the other hand, \( \leq RB \) is used analyze the connection of \( L \) with labels within the range of \( L \) because \( L \cup \{L \}^{RB} - L^{RB} \) are the objects related to \( L \) and labels lower than or equal to \( L \).

**Example 5** For set of labels \( L = \{Japan, China\} \) and label \( L = Asia \), suppose the objects expressed by \( L \cup \{L \} \) are compared with the objects expressed by \( L \) with \( \leq RU \).

If the aggregation values of \( \{Japan, China, Asia\}^{RB} - \{Japan, China\}^{RB} \) such as the number of the target objects is, the more important for the connection Tokyo is. As the same as the \( \leq RU \), the smaller the aggregation values of \( \{Japan, U.S.A\}^{RB} - \{Japan, U.S.A, Tokyo\}^{RB} \) is, the more important Tokyo is. \( Q.E.D. \)

**VI. Conclusions**

This paper discussed the semantic ranges of objects to be analyzed precisely, and showed the correspondence of the objects to be analyzed and the objects expressed with orders. The objects to be analyzed are specified with \( \leq RN, \leq RU, \leq RL, \) and \( \leq RB \). Based on the role of a label \( L \) against a set of labels \( L \), this paper proposed some analysis methods for multi-labeled objects. The following three orders are used to analyze the connection about \( L \) and \( \leq RN, \leq RU, \) and \( \leq RL \) are used to analyze the connection of \( L \) with labels except \( L \), all labels of \( L \), and same labels of \( L \), respectively. For a set of labels which is not exclusive, \( \leq RB \) can be used to analyze the connection of \( L \) in detail. The discussions of this paper are effective for the advanced analysis for multi-labeled objects. Economy globalization makes the connection among different countries more complicated such as China against Japan and U.S.A. With the results of this paper, the connection of China with Japan and U.S.A can be analyzed more precisely, for example.

**REFERENCES**


