Abstract—In production planning (PP) periods with excess capacity and growing demand, the manufacturers have two options to use the excess capacity. First, it could do more changeovers and thus reduce lot sizes, inventories, and inventory costs. Second, it could produce in excess of demand in the period and build additional inventory that can be used to satisfy future demand increments, thus delaying the purchase of the next machine that is required to meet the growth in demand. In this study we propose an enhanced supply chain planning model with flexible planning capability. In addition, a 3D supply chain planning system is illustrated.

Keywords—Supply chain, capacity expansion, inventory management, planning system.

I. INTRODUCTION

PRODUCTION scheduling is concerned with the allocation of production resources while production planning is concerned with the determination of the level of production resources over time [2]. An extensive literature exists in production planning, often referred to as aggregate planning, and in production scheduling, which has further developed into the lot-sizing and machine scheduling literature for closed and open shops, respectively [2]. MacCarthy and Liu [3] provide reviews of the machine scheduling literature. Nam and Logendran [4] provide a survey of models and methodologies in the aggregate planning literature. Recent papers in the aggregate planning area [1], [7] generally do not consider equipment capacity issues, even though they are relevant in environments such as the one described earlier. Although some researchers have analyzed capacity expansion along with inventory and aggregate planning [6], they restrict themselves to a single product environment and do not consider lot-sizing issues.

Rajagopalan and Swaminathan [5] propose a production planning model with capacity expansion and inventory management. While demand growth is gradual, capacity expansion is discrete. Periods following a machine purchase are characterized by excess machine capacity. The conventional approach in the operational management used the excess capacity to conduct more changeovers and reduce lot sizes and inventories. However, this approach ignores an alternative use of the excess capacity in demand growth environments. In periods with excess capacity, the firm has two options for using the excess capacity. First, it could do more changeovers and thus reduce lot sizes, inventories, and inventory costs. Alternatively, it could produce in excess of demand in the period and build additional inventory that can be used to satisfy future demand increments, thus delaying the purchase of the next machine that is required to meet the growth in demand.

The remaining of this paper is organized as follows. In section II, we review the coordinated production planning model introduced by Rajagopalan and Swaminathan [5]. Section III proposes an enhanced coordinated production planning model. The objective of this work is to maximize the profits under the constraints of forecast demand and capacity availability. Section IV introduces the proposed methods. In section V a system framework of the supply chain management system is introduction. The conclusions will be given in the final section.

II. COORDINATED PRODUCTION PLANNING MODEL

This section defines the notations consistent with Rajagopalan and Swaminathan’s coordinated production planning model (PP) with capacity expansion and inventory management [5]. Consider a scenario with $M$ items, $i = 1, \ldots, M$, in $T$ periods, $t = 1, \ldots, T$. The PP model is formulated as below:

\[
\begin{align*}
\min & \sum_{t=1}^{T} g_t Y_t + \sum_{i=1}^{M} \left( h_{it} (I_{it} + Q_{it}/2) + p_{it} X_{it} \right) \\
\text{s.t.} & \quad X_{it} - I_{it} = d_{it}, \forall i, t
\end{align*}
\]
\begin{align}
\sum_{i=1}^{M} (\alpha_i X_{it} + \beta_i X_{it}/Q_{it}) &\leq C_t, \forall t \tag{3} \\
C_t - C_{t-1} - bY_t = 0, \forall t \tag{4} \\
Y_t &\in \{0,1\}, \forall t, t \tag{5} \\
X_{it} &\geq 0, I_{it} \geq 0, Q_{it} \geq 0, C_t \geq 0, \forall i, t \tag{6}
\end{align}

where inputs and parameters are:

\text{\textit{g}}_i: \text{Cost of capacity purchase at time } t.
\text{\textit{b}}_{it}: \text{Cost of holding inventory of item } i \text{ at } t.
\text{\textit{d}}_{it}: \text{Demand for item } i \text{ at } t.
\text{\textit{a}}_i: \text{Processing time for producing unit item } i.
\text{\textit{\beta}}_i: \text{Set-up (or changeover) time for producing unit item } i.
\text{\textit{b}}: \text{Purchase capacity in increments of size (b)}.

**Decision variables:**

\text{\textit{X}}_{it}: \text{Production of item } i \text{ at time } t.
\text{\textit{I}}_{it}: \text{Inventory of item } i \text{ at the end of time } t.
\text{\textit{Q}}_{it}: \text{Lot size in which an item } i \text{ is produced at } t.
\text{\textit{C}}_t: \text{Capacity available at time } t.

\text{\textit{Y}}_t: \text{Y}_t \in \{0,1\}. \text{If } \text{Y}_t = 1, \text{capacity is purchased at } t. \text{Y}_t = 0, \text{otherwise.}

The objective (1) is to minimize the sum of costs which include capacity purchase costs, carrying costs associated with planning inventories \text{\textit{I}}_{it} \text{ carried between periods and the average cycle stock } \text{Q}_{it}/2 \text{ carried within a period due to set-ups.}

In (2), the demand balance constraint for each item \textit{i} at time \textit{t}. That is, “(Production of item \textit{i} at time \textit{t}) – (Inventory of item \textit{i} at time \textit{t}) + (Inventory of item \textit{i} at time (t - 1)) = Demand for item \textit{i} at time \textit{t}”. In (3), the capacity constraint in each period, which takes into account both production and set-up times. That is, “(Total production times for all items) + (Total set-up times for all items) \leq \text{Capacity available at time } t”. In (4), the capacity balance constraint that tracks capacity levels in each period. That is, “(Capacity available at time \textit{t}) – (Capacity available at time (t - 1)) = Purchase capacity at time \textit{t}’”, where \text{Y}_t = 1, \text{if capacity is purchased at time } t, \text{and } \text{Y}_t = 0, \text{otherwise.}

The integral nature of capacity acquisition (1) and the nonlinearity in lot sizing constraints of the Rajagopalan and Swaminathan’s model make the problem difficult to solve to optimality. They developed a lower bound with Lagrangian relaxation and heuristics approach to simulate \text{Q}_{it} \text{ to obtain the minimum. However, the usefulness of their methods is limited by the following difficulties:}

1) Unable to converge to an optimum: As the experimental results the model with Lagrangian relaxation can not converge to optimal lower bound due to the characteristic of non-convexity.

2) Unable to find a global optimum: The heuristics approach by Rajagopalan and Swaminathan can find a feasible solution but not guarantee to be a local optimum.

**III. AN ENHANCED COORDINATED PRODUCTION PLANNING MODEL**

In this section an enhanced coordinated production planning model is proposed. The objective of this work is to maximize the profits under the constraints of forecast demand and capacity availability.

Fig. 1 illustrates the order, transportation, and capacity purchase activities within a four-level supply chain model. The relevant costs include purchase costs, transportation costs, production costs, inventory costs of raw materials and products, shortage costs of products, capacity purchase cost, and set-up costs.

The objective of the extension model, as expressed in (7), is to maximize the company’s profits:

\[
\max \sum_t \left( \sum_c \sum_g \mu_{gct} \sum_w X_{gwct} - \text{Cost}_t \right) \tag{7}
\]

\[
s.t.,
\]

\[
\text{Cost}_t = \sum_{pf} m_{pf} \cdot X_{pf} + \sum_{pf} t_{pf} \cdot X_{pf} + \sum_{gft} t_{gft} \cdot X_{gft} + \sum_{gwt} t_{gwt} \cdot X_{gwt} + \sum_{gft} h_{gft} \cdot I_{gft} + \sum_{gwt} h_{gwt} \cdot I_{gwt} + \sum_{pf} p_{pf} \cdot X_{gft} + \sum_{gft} s_{gft} \cdot S_{gft} + \sum f gft \cdot Y_{gft} + \sum g gft \cdot V_{gft} \tag{8}
\]

\[
\sum g \left( \alpha_g X_{gft} + \beta_g X_{gft}/Q_{gft} \right) \leq C_{tf}, \forall f, t \tag{9}
\]

\[
C_{tf} - C_{f(t-1)} - bY_{ft} = 0, \forall f, t \tag{10}
\]

\[
X_{gft} \geq Q_{gft}, X_{gft} \geq V_{gft}, X_{gft} \geq 0, \forall g, f, t \tag{11}
\]

\[
Q_{gft} \geq 0, \forall g, f, t \tag{12}
\]

\[
Y_{ft} \in \{0,1\}, C_{ft} \geq 0, \forall f, t \tag{13}
\]

where inputs and parameters are:

\text{\textit{\mu}}_{gct}: \text{Unit price of goods } g \text{ to customer } c \text{ at time } t.
\text{\textit{m}}_{pf}: \text{Material cost of goods } p \text{ supplied by supplier } v \text{ to facility } f \text{ at } t.
\text{\textit{p}}_{gft}: \text{Production cost of goods } g \text{ produced by facility } f \text{ at } t.
\text{\textit{h}}_{gft}: \text{Inventory cost of material } p \text{ in facility } f \text{ at } t.
\text{\textit{h}}_{gwt}: \text{Inventory cost of goods } g \text{ in facilities } f \text{ at } t.
\text{\textit{t}}_{pf}: \text{Transportation cost of material } p \text{ from supplier } v \text{ to facility } f \text{ at } t.
\text{\textit{t}}_{gft}: \text{Transportation cost of goods } g \text{ from facility } f \text{ to warehouse } w \text{ at } t.
\text{\textit{t}}_{gwt}: \text{Transportation cost of goods } g \text{ from warehouse } w \text{ to customer } c \text{ at } t.
$s_{gct}$: Stockout costs of goods $g$ to customer $c$ at $t$.
$v_{gft}$: Set-up cost of goods $g$ in facility $f$ at $t$.
$g_{f}$: Capacity purchase cost in facility $f$ at $t$.
$\alpha_{f}$: Unit processing time for producing goods $g$.
$\beta_{g}$: Set-up (or changeover) time for producing goods $g$.
$b$: Purchase capacity in increments of size ($b$).

**Decision variables are:**

1. $\text{Cost}_{t}$: Total cost at time $t$.
2. $X_{gft}$: Amount of goods $g$ which facility $f$ produced at $t$.
3. $X_{spft}$: Amount of material $p$ transported from supplier $v$ to facility $f$ at $t$.
4. $X_{gfwt}$: Amount of goods $g$ transported from warehouse $w$ to customer $c$ at $t$.
5. $I_{gft}$: Inventory of material $p$ in facility $f$ at $t$.
6. $I_{gft}$: Inventory of goods $g$ in facility $f$ at $t$.
7. $S_{gct}$: Stockout of goods $g$ in customer $c$ at $t$.
8. $V_{gft}$: Set-up times for producing goods $g$ in facility $f$ at $t$.
9. $C_{f}$: Capacity available in facility $f$ at $t$. $Y_{f}$: Lot size in which a goods $g$ is produced in facility $f$ at $t$. $Y_{f} = 1$, if capacity is purchased in facility $f$ at $t$. $Y_{f} = 0$, otherwise.

The objective in (7) is to maximize the profits. In (8), costs included purchase, transportation, and inventory costs of materials, transportation, inventory, and stockout costs of products, capacity purchase costs, and set-up costs. In (9), the capacity constraint in each period, which takes into account both production and set-up times. That is, “Total production times for goods $g$ in facility $f$ + Total set-up times for goods $g$ in facility $f$ ≤ Capacity available in facility $f$ at time $t$”. In (10), the capacity balance constraint that tracks capacity levels in each period. That is, “(Capacity available in facility $f$ at time $t$ − Capacity available in facility $f$ at time $(t-1)$) = Purchase capacity in facility $f$ at time $t'$, where $Y_{f}$ = 1, if capacity is purchased in facility $f$ at time $t$, and $Y_{f}$ = 0, otherwise.

### IV. PROPOSED METHOD

The model in Section III is a Mixed Integer Nonlinear Programming (MINLP). In this section we propose an effective method for overcoming the above difficulties in conventional methods. The advantages of the proposed approach are listed below:

1) The MINLP problem is converted into a Mixed 0-1 Linear Programming (M01LP) problem.
2) Guarantee to be a global optimum.
3) The set-up times and lot size are integer.

Consider the following propositions:

**Proposition 1.** Denote $[X/Q] = R$ be the smallest integer value larger than or equal to $X/Q$, then

$$[X/Q] = R \iff X/Q \leq R \leq X/Q + 1 - \epsilon/Q$$

where $\epsilon$ is a small positive value.

**Proof.** If $X/Q$ is an integer, then $[X/Q] = R$. Otherwise, $X/Q \leq [X/Q] = R \leq X/Q + 1$ which implies $X/Q \leq R \leq X/Q + 1 - \epsilon/Q$. □

For instance, given $X = 500$, $Q = 200$, $\epsilon = 0.001$, then $500/200 \leq R \leq 500/200 + 1 - 0.001/200$. Thus, $R = 3$.

**Proposition 2.** Denote $Q$ be a nonnegative integer value and $0 \leq Q \leq Q$, $0 \leq R \leq \overline{R}$, where $Q$, $\overline{Q}$, $\overline{R}$ are integer. $QR$ can then be expressed as $QR + \sum_{k=1}^{n} 2^{k-1}w_{k}$, where

1) $R - \overline{R}(1 - u_{k}) \leq w_{k} \leq R + \overline{R}(1 - u_{k})$, $k = 1, 2, \ldots, n$.
2) $0 \leq w_{k} \leq \overline{R}u_{k}$, $k = 1, 2, \ldots, n$.
3) $u_{k} \in \{0, 1\}, k = 1, 2, \ldots, n$.
4) $n$ is a smallest integer satisfying $n \geq \log_{2}(\overline{Q} - Q + 1)$.

**Proof.** A nonnegative integer value $Q$ can be expressed as $Q = QR + \sum_{k=1}^{n} 2^{k-1}w_{k}$, $u_{k} \in \{0, 1\}$, $k = 1, 2, \ldots, n$, where $\sum_{k=1}^{n} 2^{k-1} = 2^{n} - 1 \geq \overline{Q} - Q$. Let $w_{k} = u_{k}R$, then

$$QR = \overline{QR} + \sum_{k=1}^{n} 2^{k-1}w_{k}$$

Since $w_{k} = u_{k}R$, if $u_{k} = 0$, then $w_{k} = 0$, otherwise, $w_{k} = R$. □

By referring to Proposition 1 and Proposition 2, it is clear that:

1) $X/Q \leq R \leq X/Q + 1 - \epsilon/Q$.
2) $QR = \overline{QR} + \sum_{k=1}^{n} 2^{k-1}w_{k}$.

Then, the model in Section III can be converted into a Mixed 0-1 Linear Programming (M01LP) problem as below:
\[
\begin{align*}
\text{max} & \sum_t \left( \sum_c \sum_g \mu_{gct} \sum_w X_{gwct} - \text{Cost}_t \right) \\
\text{s.t.} & \sum_g (\alpha_g X_{gft} + \beta_g R_{gft}) \leq C_{ft}, \forall f, t \\
Q_{gft} R_{gft} - Q_{gft} + \epsilon & \leq X_{gft}, \forall g, f, t \\
X_{gft} & \leq Q_{gft} R_{gft}, \forall g, f, t \\
Q_{gft} = Q_{gft} + \sum_{k_{gft}=1}^{n_{gft}} 2^{k_{gft}-1} w_{k_{gft}} & \forall g, f, t \\
R_{gft} - \overline{R}_{gft}(1 - w_{k_{gft}}) & \leq w_{k_{gft}}, \forall k, g, f, t \\
w_{k_{gft}} & \leq R_{gft} + \overline{R}_{gft}(1 - w_{k_{gft}}), \forall k, g, f, t \\
0 & \leq w_{k_{gft}} \leq \overline{R}_{gft} \sum_{k_{gft}=1}^{n_{gft}} \forall k, g, f, t \\
0 & \leq Q_{gft} \leq \overline{Q}_{gft}, \forall g, f, t \\
0 & \leq R_{gft} \leq \overline{R}_{gft}, \forall g, f, t \\
w_{k_{gft}} & \in \{0, 1\}, \forall k, g, f, t \\
(8) - (13)
\end{align*}
\]

V. SYSTEM FRAMEWORK

This section illustrates the system framework of the Supply Chain Management System as shown in Fig. 2.

![System Framework Diagram]

The system is composed of several units:

1) User Interface: The user interface is developed with Java and NASA (National Aeronautics and Space Administration) World Wind SDK [8] as shown in Fig. 3. The NASA World Wind SDK provides 3D engine to zoom from satellite altitude into any place on Earth, leveraging high resolution LandSat imagery and SRTM (Shuttle Radar Topography Mission) elevation data to experience Earth in visually rich 3D. World Wind has a full copy of the Blue Marble, a spectacular true-color image of the entire Earth as seen on NASA’s Earth Observatory.

2) Model Solver: This is the kernel of the overall system. It provides the functionalities, such as “Parser” is responsible for verifying the syntax and format of the input model, “Pre-processor” performs range reduction to reduce the size of the feasible set, and “Solver” is devoted to searching for the solution with proposed method discussed in Section IV.

3) System Options: These handle the associated parameters in the problem-solving process. Users can decide the solving parameters according to the selected algorithms. Moreover, during the operation of the system, users can also keep a record of the associated information regarding their interactions with the system for future references.

4) External Solvers: The system offers a mechanism to provide external links with other popular solvers, such as LINGO, Mathematica, and Matlab.

5) System Outputs: The result can be generated after the problem-solving process is terminated. In addition to the text mode presentation, graphics and spreadsheet presentations of the final results have also been among the popular alternatives to general users.

VI. CONCLUSIONS

This study proposes an enhanced supply chain planning model with flexible planning capability and the algorithm which guarantees the global optimum in maximizing the objective function. The advantages of the proposed approach are (1) the MINLP problem is converted into a Mixed 0-1 Linear Programming (M01LP) problem; (2) guarantee to be a global optimum; (3) the set-up times and lot size are integer. In addition, a 3D supply chain planning system is illustrated.

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