Advanced ILQ Control for Buck-Converter via Two-Degrees of Freedom Servo-System

Sidshchadhaa Aumted, Shuhei Shiina, and Hiroshi Takami

Abstract—In this paper, we propose an advanced ILQ control for the buck-converter via two-degrees of freedom servo-system. Our presented strategy is based on Inverse Linear Quadratic (ILQ) servo-system controller without solving Riccati’s equation directly. The optimal controller of the current and voltage control system is designed. The stability and robust control are analyzed. A conscious and persistent effort has been made to improve ILQ control via two-degrees of freedom guarantees the optimal gains on the basis of polynomial pole assignment, which our results of the proposed strategy shows that the advanced ILQ control can be controlled independently the step response and the disturbance response by appending a feed-forward compensator.

Keywords—Optimal voltage control, inverse LQ design method, second order polynomial, two-degrees of freedom.

I. INTRODUCTION

Due to the buck DC-DC converter is used widely such as DC regulated power supply, photovoltaic system and DC motor regulated speed [1]. The sliding mode (SM) control technique is a possible option to control a DC-DC converters that must cope with their intrinsic nonlinearity, load variations, ensuring stability in any operating condition while providing fast transient response, and in addition to the traditional sliding mode control has the drawback of chattering phenomenon owing to its discrete control law [2], [3]. The optional of a practical servo-system on the basis of the optimal control theory is the inverse linear quadratic (ILQ) method, which finds the optimal gains to be based on the polynomial pole assignment without solving Riccati’s equation [4]. With this method, it has the advantages of the transfer functions between inputs and outputs can be asymptotically designated for the desired specification, the optimal solutions are analytically obtained and guaranteed, and the optimal gains can be easily adjusted to obtain the desired responses at the point of use [5]-[7]. The optimal voltage control by 2nd-order polynomial is achieved through the suppression of resonant output voltage by improving ILQ method [8]. Therefore, this paper proposes the improved ILQ control via two-degrees of freedom which enables to controlled the independently step response and the disturbance response by appending a feed-forward compensator. This section describes the advantages of our designed method and provides a summary of some of our current simulation results for using the optimal controller of the current and voltage control system. Section II of this paper, we describe the design of our modeling and propose the designed control strategy. The ILQ control via two-degrees of freedom using in this paper is defined as 2D-ILQ. The proposed control system consists of modeling for the buck-converter circuit, solving the 2D-ILQ servo-system, and finding the basic gains, the optimal conditions and the solutions of the feed-forward compensator [4]-[7]. In particular, we demonstrate how to get the basic equations from our modeling for the buck-converter circuit, to solve the 2D-ILQ servo-system, to find the basic gains, the optimal condition in the 2D-ILQ servo-system and the solution of the feed-forward compensator. Section III, we present the numerical simulations and describe the resulting important properties of this design. And finally, we discuss related work and present the conclusion.

II. MODEL AND PROPOSED CONTROL STRATEGY

A. Modeling for the Buck-Converter Circuit

In this step, we have designed the buck-converter circuit with a resonant LC low-pass-filter for the voltage control, and derived its system equations.

Fig. 1 The buck-converter circuit

Fig. 2 Equivalent circuit of the buck-converter circuit

Fig. 1 represents the proposed buck-converter circuit, and Fig. 2 is a corresponding equivalent circuit which leads to two basis voltage equations and a basis equation of current. In this
circuit, the load composed of two resistors with switch, which gives the step-functional load test for the buck-converter circuit.

Thus, the basis equations are as follows:

\[ v_1 = r_i + \frac{d}{dt}i_1 + v_2, \quad v_2 = \frac{1}{C_C} \int i_C dt = R_i, \quad i_2 = i_C + i_R + i_2 \]  

where \( L, C_C \) and \( R \) are inductance, capacitance and damping resistance of the passive low-pass filter. \( v_1 \) is a buck-converter output voltage, \( v_2 \) is a load voltage, \( i_2 \) is a buck-converter output current, \( i_R \) is a load current, \( i_C \) is a current of capacitor \( C_C \) and \( i_R \) is a damping resistance current, respectively.

Considering the time-invariant system, a state equation can be obtained from (1) as follows:

\[ \dot{x} = Ax + Bu + Dd, \quad y = Cx \]  

where \( \dot{x} = dx/dt, \quad x = \begin{bmatrix} i_1 \\ v_2 \end{bmatrix}, \quad u = v_1, \quad d = i_2 \)

\[ A = \begin{bmatrix} \frac{r}{L} & -\frac{1}{L} \\ \frac{1}{C_C} & -\frac{1}{RC_C} \end{bmatrix}, \quad B = \begin{bmatrix} 1 \\ 0 \end{bmatrix}, \quad C = \begin{bmatrix} 0 & 1 \end{bmatrix} \text{ and } D = \begin{bmatrix} 0 \\ -\frac{1}{C_C} \end{bmatrix} \]

Henceforth, the plant must satisfy the conditions of the controllable and observable system, the minimal-phase system, and the no zeros at the origin, which already has all of conditions. Damping resistance \( R \) is usually large, so that we can neglect it and then we can verify and proof the system via the robust control theory approach [4], [5], [9].

**B. How to Solve the 2D-ILQ Servo-System**

In order to design the 2D-ILQ optimal servo-system, it needs to use four following principles [4], [7], [10], [11] as follows:

1) Our proposed strategy extended the state feedback system.
2) We can find the analytical optimal solution based on the ILQ design method.
3) We can get the asymptotic feature of the ILQ optimal servo-system.
4) We can follow the procedure for the optimal solutions of the 2D-ILQ servo-system.

In this moment, we first derive the basic construction for the 2D-ILQ servo-system, and then we explain about a procedure for getting the optimal solutions of the 2D-ILQ servo-system.

**C. Basic Construction of the 2D-ILQ Servo-System**

Concerning the basic construction of the proposed 2D-ILQ servo-system, two additional feed-forward compensators \( G_F(s) \) and \( G_D(s) \) are improved the response of the reference input independently to the response of the disturbance, which are appended as shown in Fig.3, where \( y^* \) is a reference input, \( K_F \) is a state feedback gain, and \( K_i \) is an integral gain of the servo-controller.

Superposing the conventional ILQ servo-system of the feed-forward compensators \( G_F(s) \) and \( G_D(s) \) are an effective procedure for finding out an optimal relation and then we have achieved the optimal solutions of the 2D-ILQ servo-system.

**D. Finding the Optimal Gains and Conditions**

The basic gains are represented as

\[ \begin{bmatrix} K_F & K_i \end{bmatrix} = V^* \Sigma \begin{bmatrix} K_F^0 & K_i^0 \end{bmatrix} \]  

where \( K_F^0, K_i^0 \) are the basic optimal gains, \( \Sigma > 0 \) is a diagonal gain matrix as an adjusting parameter, and \( V \) is a suitable nonsingular matrix [6], [12], [13].

In other words, the first we apply the ILQ solution procedures for conventional ILQ servo-system with the condition of \( G_F(s) = 1 \) and \( G_D(s) = 0 \) in Fig. 3, and the second we determine the \( G_F(s) \) and \( G_D(s) \) to the optimal gains 2D-ILQ servo-system by the response of \( y^* \) to \( y \).

Considering the conventional ILQ servo-system with the gains \( K_F^0 \) and \( K_i^0 \), the features of the responses are given by assigned poles, and the transfer functions become independent as \( \Sigma \rightarrow \infty \) [4], [7] in Fig. 4.
where $D_r$, which must be nonsingular, i.e. necessary-and-sufficient condition, enables to decouple the system, $c_1$ is the 1$\times$1 row-vector of matrix $C$, and $d_1 = \min\{k | c_1 A^k B \neq 0, k = 1, 2, \ldots \}$ is the order difference between the denominator and the numerator of the plant.

The order difference, $d_1 = 2$ in the system was given by (2). Hence, the stable polynomial for $\phi_1(s)$ determines the response of the servo-system in the condition $\Sigma \rightarrow \infty$, can be defined as:

$$\phi_1(s) = a_1 + a_2 s + s^2$$

where $a_1$ and $a_2$ are the coefficients of the polynomial, thus the objective transfer function is given by

$$G_1^o(s) = \frac{\phi_1(0)}{\phi_1(s)} = \frac{a_1}{a_1 + a_2 s + s^2}.$$  (6)

A polynomial matrix can be defined as

$$N(d)(A) = c_1 \phi(A) = c_1 \left( a_1 + a_2 A + A^2 \right) = \left[ \frac{a_2}{C_C} - \omega_0^2 r \quad a_1 - \omega_0^2 \right]$$

where $\omega_0 = \sqrt{|L/C_C|}$ represents the resonant angular velocity of the passive second-order low-pass filter that composed of $L$ and $C_C$.

We derive the decoupling gain as follows:

$$K = D_r^{-1} N_{\phi}(A) = \begin{bmatrix} \frac{1}{C_C} \frac{a_2}{\omega_0^2} - r & \frac{a_1}{\omega_0^2} - 1 \end{bmatrix}$$

Henceforth, we can obtain the optimal basic gains as follows:

$$K_F^0 \begin{bmatrix} A \\ C \end{bmatrix} = K^0 \begin{bmatrix} B^0 \end{bmatrix} = \begin{bmatrix} L & \frac{a_2}{\omega_0^2} & \frac{a_1}{\omega_0^2} \end{bmatrix}.$$  (9)

In order to achieve a simple design of the responses, we give the objective transfer function of (6) as

$$a_1 := \omega_0^2, \quad a_2 := 2 \zeta \omega_0$$

where $\omega_0$ is a natural angular velocity, and $\zeta$ is a damping factor in its conventional ILQ servo-system.

According to reference [4], the optimal condition can be obtained as

$$\Sigma > 4 \zeta \omega_0 - 2/\tau$$

where $\tau = L/r$ is the time constant of the inductor.

Consequently, the transfer function from the reference $y^*$ to the output $y$ is given by

$$G_1(s) = \frac{\Sigma a_1}{s^3 + \Sigma s^2 + (\omega_0^2 - \omega_1^2) s + \Sigma a_1}.$$  (12)

### E. Finding the Solution of Feed-Forward Compensators

The 2D-ILQ servo-system can control independently the step response and the disturbance response by an additional feed-forward compensator that is concerned with the reference $y^*$ in Fig. 5, representing the block diagram that appended the feed-forward compensators, which we can get the nonsingular basic gain of (9).

![Fig. 5 Block diagram of the construction of the proposed 2D-ILQ servo-system](image-url)

Equation (9) is concerned an equivalent block diagram with additional pre-filter as shown in Fig. 6, and its definition of (13) as follows:

$$G_c(s) = G_c(s) + s K^0 V^{-1} \Sigma^{-1} V G_c(s).$$  (13)

We can redraw a simple block diagram of Fig. 6 from (12) and (13) as shown in Fig. 7. Thus, the total transfer function from $y^*$ to $y$ is derived from (12) and (13) as follows:

$$G_c^*(s) = \frac{\Sigma a_1 G_c(s) + s K^0 V^{-1} \Sigma^{-1} V G_c(s)}{s^3 + \Sigma s^2 + (\omega_0^2 + \Sigma a_1) s + \Sigma a_1}.$$  (14)

The order difference between the numerator and the denominator is second. Therefore, the objective transfer function is given by

$$G_c^*(s) = \frac{\omega_0^3}{s^3 + 2 \zeta \omega_0 s + \omega_0^2}.$$  (15)

where $\omega_0$ and $\zeta$ are the target natural angular velocity and the damping factor of the 2D-ILQ servo-system, respectively.

From (14) and (15), the feed-forward transfer function is determined as

$$G_c(s) = \frac{\omega_0^3 s^2 + \Sigma (\omega_0^2 - \omega_1^2) s + \omega_0^2 (\omega_0^2 + \Sigma a_1) s + \Sigma a_1}{\omega_0^3 (s^3 + 2 \zeta \omega_0 s + \omega_0^2)}.$$  (16)
III. Numerical Simulations and Results

All simulations, which were carried out at the condition of the DC input-voltage of 24V and the carrier frequency of 20kHz. The amplitudes of the reference voltage were varied from 9V to 12V at 10ms, and the step-functional load of 5Ω is connected to the output terminals at 15ms. The red line represents the reference voltage \( v_2^* \) and the actual voltage \( v_2 \) is represented by the blue line.

For the evaluating response by the reference input, we define a “rising time”, which is an interval from 10% to 90% of the output voltage. Moreover for the evaluating response by the disturbance of system, we define a “recovery time”, which is an interval from the impact of its disturbance to the recovering level of 90%.

\[ G_c(s) = \frac{1}{\alpha_1 s^2 + \alpha_2 s + \alpha_3} \quad \text{and} \quad G_F(s) = \frac{1}{s} \]

\[ G_F(s) = 0 \]
Fig. 11 Proposed 2D-ILQ control by 2\textsuperscript{nd}-order polynomial
\(a_1=7500\), \(a_2=2*17500\), \(c=40000\), 
\(G_r(s) = \frac{2500}{s+2500}\) and \(G_d(s) = 0\)

Fig. 8 and Fig. 9 show the waveforms of the conventional ILQ control when we changed the values of the factors \(a_1\) and \(a_2\) due to (10) with the 2\textsuperscript{nd}-order polynomial while the waveforms of the proposed 2D-ILQ control are shown in Fig. 10 and Fig. 11. The rising times and the recovery times of all are summarized in Table I.

As the simulation result in Fig. 10, the rising time of the proposed 2D-ILQ control is 0.60ms that is faster than the rising time of the conventional ILQ control is 1.15ms as shown in Fig. 8 while the recovery times of all are corresponded to 0.6ms. This case shows that the rising time is able to adjust via \(G_d(s)\) for changing the speed response, whereas the recovery time is unaffected by \(G_d(s)\).

Likewise, the rising time is 1.15ms as shown in Fig. 11, is slower than the rising time as shown in Fig. 9 is 0.61ms while the recovery times of all are corresponded to 0.85ms. This case shows that the rising time is also able to adjust via \(G_r(s)\) for changing the speed response, whereas the recovery time is also unaffected by \(G_r(s)\).

Consequently, the additional feed-forward compensators \(G_r(s)\) and \(G_d(s)\) are improved the response of the reference input independently to the response of the disturbance, which are appended as shown in Fig. 3 and concerned with (13) and (16).

Considering the simulation result in Fig. 10, the recovery time of the proposed 2D-ILQ control is 1.73ms that is longer than the recovery time of the conventional ILQ control is 0.85ms as shown in Fig. 9 while the rising times of all are corresponded to 0.6ms.

Likewise, the recovery time is 0.85ms as shown in Fig. 11, is shorter than the recovery time as shown in Fig. 8 is 1.73ms while the rising times of all are corresponded to 1.15ms.

This case shows that the recovery times is able to adjust via \(\alpha\) for changing the speed response, whereas the rising time is unaffected by \(\alpha\) under the condition of \(G_r(s)\) and \(G_d(s)\) are not equal to zero.

<table>
<thead>
<tr>
<th>Conditions</th>
<th>Rising Time</th>
<th>Recovery Time</th>
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<tr>
<td>Fig. 8 (a) Conventional ILQ control by 2\textsuperscript{nd}-order polynomial (\alpha=5000), (\sigma=40000), (G_r(s)=1), (G_d(s)=0)</td>
<td>1.15ms</td>
<td>1.73ms</td>
</tr>
<tr>
<td>Fig. 9 (a) Conventional ILQ control by 2\textsuperscript{nd}-order polynomial (\alpha=7500), (\sigma=40000), (G_r(s)=1), (G_d(s)=0)</td>
<td>0.61ms</td>
<td>0.85ms</td>
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<tr>
<td>Fig. 10 (a) Proposed 2D-ILQ control by 2\textsuperscript{nd}-order polynomial (\alpha=5000), (\sigma=7500), (c=40000), (G_r(s)=1), (G_d(s)=0)</td>
<td>0.60ms</td>
<td>1.73ms</td>
</tr>
<tr>
<td>Fig. 11 (a) Proposed 2D-ILQ control by 2\textsuperscript{nd}-order polynomial (\alpha=7500), (\sigma=40000), (G_r(s)=1), (G_d(s)=0)</td>
<td>1.15ms</td>
<td>0.85ms</td>
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\(\sigma\) is gain adjusting parameter
The results in Fig. 12 and Fig. 13 show the results of simulations, which were carried out at the condition with parameter errors in Fig. 10 and Fig. 11, respectively. The inductance, which is one of the key parameters and determines the performance of the buck-converter circuit, is changed at values two times as its nominal value. Compared Fig. 10 (a) with Fig. 12 (a), and then Fig. 11 (a) with Fig. 13 (a), we found that the rising times and the recovery times are almost the same as each other, so that we can verify that proposed control strategy makes the robust buck converter although the converter parameters are varied.

From Fig. 8 (b), Fig. 9 (b), Fig. 10 (b), Fig. 11 (b), Fig. 12 (b) and Fig. 13 (b), the buck converter output current in all systems can be controlled by the ILQ control method, and used to the current feedback part for our strategy. In the proposed 2D-ILQ control of Fig. 10 (c), Fig. 11 (c), Fig. 12 (c) and Fig. 13 (c), and the conventional ILQ control of Fig. 8 (c) and Fig. 9 (c), after the step-functional load of 5Ω is connected to the output terminals at 15ms, the peak value of the output current load is affected to correlate with the output voltage in the region 2 of each figure. That has the least significance for controlling overall system because the voltage converges to the reference voltage at a short amount of time. At last in Fig. 8 (d), Fig. 9 (d), Fig. 10 (d), Fig. 11 (d), Fig. 12 (d) and Fig. 13 (d), the duty factor of all cases fall in the region of (0,1) are under control.

IV. CONCLUSION
In this paper, we have proposed an advanced ILQ control (2D-ILQ control) for the buck-converter via two-degrees of freedom servo-system based on the conventional ILQ servo-system. The 2D-ILQ control can be controlled independently the step response and the disturbance response by appending a feed-forward compensators. However, the disturbance response must be suppressed in the practical servo-system.

REFERENCES

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