

A Joint Routing-Scheduling Approach for Throughput Optimization in WMNs

Hossein Nourkhiz Mahjoub, and Mohsen Shiva

Abstract—Wireless Mesh Networking is a promising proposal for broadband data transmission in a large area with low cost and acceptable QoS. These features' trade offs in WMNs is a hot research field nowadays. In this paper a mathematical optimization framework has been developed to maximize throughput according to upper bound delay constraints. IEEE 802.11 based infrastructure backhauling mode of WMNs has been considered to formulate the MINLP optimization problem. Proposed method gives the full routing and scheduling procedure in WMN in order to obtain mentioned goals.

Keywords—Mixed-Integer Non Linear Programming (MINLP), routing and scheduling, throughput, wireless mesh networks (WMNs)

I. INTRODUCTION

WIRELESS Mesh Networking concept has been proposed to provide a reliable data transmission strategy over wireless media.

Another major goal of wireless mesh networks is coverage range extension. Cost efficiency is the third important feature of this type of networks. Reliability achieved by mesh connectivity of WMNs which provides multiple routes over intermediate wireless links for transmission from a source node to its destination. Properly working of mesh structure requires nodes of the network to route data in a self organizing manner according to probable node failures. Multi-hop data handling provides a considerable range extension compared with single-hop networks, thanks to shadowing effect and path loss reduction. In WMNs infrastructure nodes are added one by one to improve network operation and balance demands and capabilities, without huge cabling engineering. This fact reduces the network setup cost considerably.

WMNs work in three different modes. The client mode of WMNs is similar to wireless ad-hoc networks, which nodes' positions are unknown according to their randomly distributed movements and data flows have arbitrary distribution pattern among network's nodes. But in infrastructure mode these two features are completely different. In this mode only network's backbone nodes which construct a stable topology are under consideration. Also traffic distribution has a particular pattern from intermediate routers to central access point (AP) or gate-

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way (GW) - which is connected directly to the backbone network with assumed infinite bandwidth- or vice versa. The third mode is a combination of two modes mentioned above which is called hybrid mode. Further general information has been presented in [1].

The main network's performance metrics are delay (mean or upper bound), coverage range and throughput. These factors' definitions are different in the literature according to different problem modeling strategies. Each of these features and also their tradeoffs in WMNs have been studied extensively in the literature. The main point is that these metrics move along contradictory directions. For example throughput increasing usually leads to coverage decreasing or delay increasing and so on. So balancing these factors in an appropriate way is a major issue of network performance improvement studies.

According to similarity of WMNs' client mode with ad-hoc networks, in almost all works WMNs have been considered in infrastructure mode. In [2] the important concept of *Bottleneck Collision Domain (BCD)* has been proposed to evaluate the maximum data generation rate of nodes which is limited by MAC throughput of links under a fair data routing procedure. References [3] and [4] offer a ring based WMN structure and a Phy-MAC cross layer throughput calculation framework to balance throughput, range and delay. Quality based routing metrics, like ETX, mETX and ENT, have been offered in [5] and [6] with the aim of data routing via the best paths in the network under stable and time varying channel conditions. A throughput-range trade off analysis has been done in [7]. A co-channel interference reduction solution by using WMNs structure for clustering has been introduced in [8] and its effect on normalized throughput of data flows on different channels has been shown.

In this paper an optimization framework for jointly routing and scheduling of data traffic in order to maximize uplink throughput in a single radio, multi channel WMN topology has been derived. The network topology is composed of a number of stable intermediate routers -which everyone is a data source too. These nodes forward their own and received traffic via other nodes to the central AP (which can be connected directly to the internet, for example). This model can be referred to as infrastructure, backhauling mode of WMNs.

In the first part, which scheduling is under consideration, the problem has been formulated as a mixed-integer nonlinear programming (MINLP) optimization problem with all sufficient and enough constraints. All constraints are linear,

but objective function is not a linear combination of variables. Upper bound delay requirement has been considered in this model too.

In the second part, scheduling has not been mentioned and BCD concept is introduced. In this part a complete mathematical optimization problem with all of sufficient and enough constraints is derived in single channel mode, because the concept of collision domain is meaningless under multi-channel assumption. But, as will be shown, this model leads us to an expensive Mini-Max optimization problem which is not solvable with usual computational cost, according to its constraints' high degree of nonlinearity. For this part only mathematical framework is presented.

The rest of this paper is organized as follows. The network model details are explained in section II. The MINLP and Mini-Max problems setup are done in section III. Simulation results for MINLP problem are presented in section IV. Conclusion and future works is the last part of this paper.

II. NETWORK MODEL

A general perspective of network model is shown in Fig. 1. N intermediate routers have been considered, which every one is also a source of data and generates data in G_i bits/sec, for $i = 1, 2, \dots, N$. It has been supposed that time slot duration, in which one packet is transmitted is sufficiently small, and so bit streams could be considered rather than packets [7]. These N routers' coordination values have been assumed as known parameters: $(X_1, Y_1), \dots, (X_N, Y_N)$. AP which is the $N+1^{\text{th}}$ node is at the center of the plane ($X_{N+1} = Y_{N+1} = 0$). All links are directional. In Mini-Max formulation, timing is not mentioned. But in MINLP formulation every node, i , in each transmission interval (TI), relays data traffic which has received from other nodes during the time between the previous TI which i has been appeared as a transmitter up to now plus the amount of data which is generated by itself during this TI. It has been assumed that every node, i , only during its active TIs as a transmitter feeds its own data traffic by a constant bit rate G_i bits/sec. TIs could be defined as the time durations T_{S_0} ($S_0 = 1, 2, \dots, S, S = \text{Number of TIs}$), when a subset of nodes could be active as transmitter or receiver concurrently according to following constraints:

In every TI, every active node is only a transmitter (to only one node) or only a receiver (from only one node). In other words two or more links which are connected to one node can not be active simultaneously (in one TI). This constraint is forced by single radio assumption. This feature is referred to by the term *primary interference avoidance*.

IEEE 802.11[9] based WMNs have been considered with RTS/CTS exchange in the proposed model. So in single channel mode if a link between nodes i and j be active during one TI, then every link which its source or/and destination node be within the sensing range of i or/and j is silent in order to collision avoidance according to RTS/CTS mechanism. This region is BCD of link between i and j . This feature is ref-

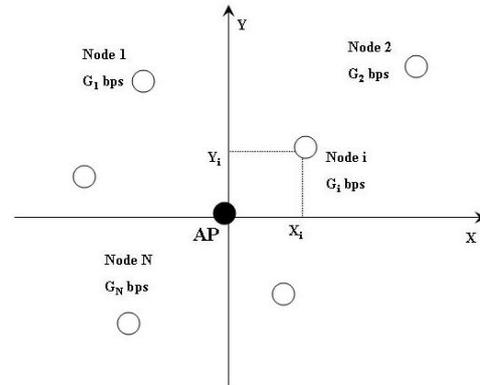


Fig. 1 General perspective of network model

erred to by term *secondary (co-channel) interference avoidance*.

Secondary interference avoidance constraint formula will be derived in section III. But, as will be shown, this constraint is a nonlinear combination of problem variables with a high degree of nonlinearity. This makes the optimization problem almost unsolvable with usual computational capabilities. So it is not efficient to consider single channel mode. According to enough non-overlapping frequency channels offered by IEEE 802.11, (for example 12 channels in IEEE 802.11a [10]), it has been assumed that separated links in one collision domain, can be active simultaneously by the means of an appropriate frequency planning mechanism based on graph edge coloring techniques [11]. As mentioned before, a fix number of TIs, S , has been supposed.

In Mini-Max problem the goal is finding the bottleneck (worst case) collision domain (or in other words finding the link that should relay the maximum amount of data and therefore limits the overall network's throughput) and minimizing its relayed traffic. In MINLP problem our goal is maximizing the amount of data which receives to the AP during a fixed number of TIs to total TIs' duration ratio. These TIs' durations could be fixed or variable. In later case the overall TIs' durations, which is referred to as upper bound delay, should be less than a maximum value.

III. OPTIMIZATION PROBLEM SETUP

A. MINLP Problem

In mentioned network model which was described above, distance of two arbitrary nodes i and j is referred to by notation $d(i, j)$. Now two binary parameters, $TR(i, j)$ and $CR(i, j)$, are defined which are related to distance between nodes i and j :

$$\text{For } 1 \leq i, j \leq N + 1 \quad \text{and} \quad 1 \leq S_0 \leq S$$

$$TR(i, j) = \begin{cases} 1 & \text{if } d(i, j) \leq R_t \\ 0 & \text{Otherwise} \end{cases} \quad (1)$$

$$CR(i, j) = \begin{cases} 1 & \text{if } d(i, j) \leq R_s \\ 0 & \text{Otherwise} \end{cases} \quad (2)$$

R_t and R_s are transmission (or reception) and sensing range of nodes, respectively. It is clear that for each pair of nodes, (i, j) , $TR(i, j) \leq CR(i, j)$ according to $R_t \leq R_s$ inequality.

For each pair of nodes, (i, j) , in each TI, S_0 , a binary variable $L_{ij}(S_0)$ is defined as follows:

$$\text{For } 1 \leq i, j \leq N+1 \text{ and } 1 \leq S_0 \leq S$$

$$L_{ij}(S_0) = \begin{cases} 1 & \text{If there is an active link from} \\ & \text{node } i \text{ to node } j \text{ in } S_0\text{th TI.} \\ 0 & \text{Otherwise} \end{cases} \quad (3)$$

If duration of S_0^{th} TI is denoted by T_{S_0} , the optimization problem variables set could be shown as follows:

$$\text{VariablesSet} = \{L_{ij}(S_0); 1 \leq i, j \leq N+1, 1 \leq S_0 \leq S\} \\ \cup \{T_{S_0}; 1 \leq S_0 \leq S\} \quad (4)$$

It is clear from above definitions that by finding the values of these variables, both routing and scheduling of the whole network is completely achieved. At this time the variables $X_{i,j}(S_0)$ can be defined as the total number of bits that are transmitted from node i to node j during S_0^{th} TI.

$$X_{ij}(S_0) = \left[\left[\sum_{h=1}^{S_0-1} \sum_{k=1}^{N+1} X_{ki}(h) - \sum_{h=1}^{S_0-1} \sum_{p=1}^{N+1} (X_{ip}(h) - Lip(h) \cdot G_i \cdot T_h) \right] + G_i \cdot T_{S_0} \right] \cdot L_{ij}(S_0) \quad (5)$$

As mentioned before, it has been assumed that G_i values are known. So it is clear that if the variable set is found completely, $X_{i,j}(S_0)$ s will be completely known according to above system of equations.

$X_{i,j}(S_0)$ s are nonlinear functions of the optimization problem variables, $L_{ij}(S_0)$ s. Throughput in proposed model has been defined as the ratio of total number of bits received by AP during all TIs to total TIs' duration. The goal in this problem is maximizing this throughput. So the objective function (throughput) can be defined as follows:

$$O.F. = \left(\sum_{S_0=1}^S \sum_{i=1}^N X_{i, N+1}(S_0) \right) \div \left(\sum_{S_0=1}^S T_{S_0} \right) \quad (6)$$

The nodes $1, 2, \dots, N$ (all nodes except AP) will be denoted by R in the rest of this paper. So the target is solving the following MINLP optimization problem:

MAX

$$L_{ij}(S_0), T_{S_0} \\ \left(\sum_{S_0=1}^S \sum_{i=1}^N X_{i, N+1}(S_0) \right) \div \left(\sum_{S_0=1}^S T_{S_0} \right)$$

Subject to:

$$\forall (i, j, S_0), L_{ij}(S_0) \leq TR(i, j) \quad (7)$$

$$\forall (i, S_0), L_{N+1, i}(S_0) = 0 \quad (8)$$

$$\forall (i, S_0), L_{i, i}(S_0) = 0 \quad (9)$$

$$\forall (i, S_0), 0 \leq \sum_{j=1}^{N+1} L_{ij}(S_0) + \sum_{k=1}^{N+1} L_{ki}(S_0) \leq 1 \quad (10)$$

$$\forall (i \in R), \sum_{S_0=1}^S \sum_{j=1}^{N+1} L_{ij}(S_0) \geq K_i \quad (11)$$

$$\sum_{S_0=1}^S T_{S_0} \leq \text{MaximumDelay} \quad (12)$$

$$\forall (i \in R), \sum_{S_0=1}^S \sum_{j=1}^N (L_{ij}(S_0) \cdot TR(i, N+1)) = 0 \quad (13)$$

$$\forall (i, j \in R, S_0), L_{ij}(S_0) \leq \sum_{h=S_0+1}^S \sum_{k=1}^{N+1} L_{jk}(h) \quad (14)$$

It is clear that if for two nodes i and j , $TR(i, j)$ is zero, then $L_{ij}(S_0)$ must be equal to zero for each $S_0=1, 2, \dots, S$. This is the first constraint, (7). According to uplink assumption, all of the $L_{i, N+1}(S_0)$ variables have been set equal to zero by (8). The third constraint, (9), is evident. The fourth constraint, (10), has been forced by single radio assumption. By fifth constraint, (11), it has been ensured that during all S TIs, each intermediate node i , appears at least in K_i number of TIs as a transmitter ($K_i \geq 1$). K_i is calculated as follows:

For every intermediate node, i , minimum traffic feeding demand has been assumed as a known parameter (f_i bits). It means that node i needs to feed at least f_i bits to the network during all S TIs. (f_i can be the total traffic of its clients which should be received by AP after maximum S TIs). This parameter with G_i and the lower bound of TIs' durations which is denoted by $\min(T_{S_0})$ makes K_i according to (15):

$$K_i \cdot \min(T_{S_0}) \geq f_i / G_i \quad \text{For each } i=1, 2, \dots, N \\ \Rightarrow K_i = \lceil f_i / (G_i \cdot \min(T_{S_0})) \rceil \quad (15)$$

So by (11) it has been ensured that after maximum S TIs, each node, i , feeds at least f_i bits to the AP. It should be mentioned that each node has been assumed to have enough buffer size and so no data loss occurs in intermediate nodes. The sixth constraint, (12), satisfies application's delay requirement. By (13) the algorithm's finalizing condition has been made. It means that if in a sequence of routing links in consecutive TIs, destination node of a link falls into AP's reception range, the next link is the last one in this sequence, by the previous destination as its source and AP as its destination. And at last, (14) is the necessary and sufficient condition for network connectivity. Connectivity is an important constraint and means that if data traffic in one TI

flows on a link with a destination except AP, this traffic flow must be fed to the AP during next TIs. In other words all traffic produced by nodes during S TIs must be received by the AP at the end of Sth TI. Equation (14) ensures that if a link be activated between two nodes i and j (j ≠ N+1), in the S₀th TI, then j must appear as a transmitter in at least one of the next TIs. An important point is that this constraint should be applied to all TIs (S₀ = 1, 2, ..., S) in order to work properly. This constraint is enough condition for network connectivity according to the meaning of connectivity, because if network is not connected, then there is at least one node except AP which receives data in a TI, but does not relay it during next TIs. The necessity of (14) for connectivity is inevitable. It should be mentioned that this constraint, guarantees network connectivity conditioned on every node sends data at least once and this problem has been solved by (11). Because for every positive values for f_i, G_i and min(T_{S0}), K_i ≥ 1 is guaranteed.

The MAC throughput, B, has been assumed sufficiently large compared with all X_{ij}(S₀)/T_{S0} values.

Until now proposed MINLP problem has been formulated completely. At this time, as had been mentioned in section II, the secondary interference avoidance constraint is derived. This constraint has not been included in problem according to appropriate frequency planning used. This constraint has been shown in (16).

$$\forall (p, q, S_0, (i, j) \neq (p, q)), L_{pq}(S_0) \cdot \sum_{i=1}^{N+1} \sum_{j=1}^{N+1} [(CR(p, i) + CR(p, j) + CR(q, i) + CR(q, j)) \cdot L_{ij}(S_0)] = 0 \quad (16)$$

Obviously, this constraint is a nonlinear combination of variables with a high degree of nonlinearity according to large number of multiplications.

In next part the mathematical framework of Mini-Max problem will be presented.

B. Mini Max Problem

For Mini-Max formulation the notations and network model is same as previous section except that time variables and indexes (S₀s and T_{S0}s) are eliminated. Also it will be assumed that each node has only one outgoing link, but number of incoming links maybe zero, one or more than one link. It means that every node relays its traffic to only one node, but two or more nodes can relay data to the same node. This assumption does not have repugnance with single radio assumption, because these outgoing links will not be activated simultaneously. In this model the collision domain of each link, (the set of links that should be silent when this link is active in order to avoid primary and secondary interference), should be found, its carried traffic load should be calculated as the summation of the loads of all links in its collision domain, the bottleneck collision domain as the collision domain of the link which should transfer maximum load should be found,

and finally its relayed load should be minimized.

Carried traffic flow of directional link between the pair of nodes i and j which is denoted by X_{ij} is:

$$X_{ij} = \left(\sum_{k=1}^{N+1} (L_{ki} X_{ki}) + G_i \right) \cdot L_{ij} \quad (17)$$

L_{ij} has the same meaning of L_{ij}(S₀) in previous part, except that does not have timing characteristic.

The carried traffic load of link between (p,q) which is denoted by F(p,q) could be calculated as follows:

$$\begin{aligned} F(p, q) = & \sum_{i=1}^{N+1} \sum_{j=1}^{N+1} (X_{ij} (CR(p, i) + CR(p, j))) \\ & - \sum_{i=1}^{N+1} \sum_{j=1}^{N+1} (X_{ij} \cdot CR(p, i) \cdot CR(p, j)) \\ & + \sum_{i=1}^{N+1} \sum_{j=1}^{N+1} (X_{ij} (CR(q, i) + CR(q, j))) \\ & - \sum_{i=1}^{N+1} \sum_{j=1}^{N+1} (X_{ij} \cdot CR(q, i) \cdot CR(q, j)) \\ & - \sum_{i=1}^{N+1} \sum_{j=1}^{N+1} ((X_{ij} + X_{ji}) \cdot CR(p, i) \cdot CR(q, j)) \\ & + \sum_{i=1}^{N+1} \sum_{j=1}^{N+1} (CR(p, i) \cdot CR(p, j) \cdot CR(q, i) \cdot CR(q, j) \cdot X_{ij}) \end{aligned} \quad (18)$$

The first term accounts the load of links which are within the sensing range of node p. The second term corrects the value of F for links which their both ends are within the sensing range of p. the third and fourth terms are same as first and second ones respectively, for node q. The fifth term corrects F value for links that one of their ends is within the sensing range of p and other is within q's sensing range. And finally the last term deals with links which both their ends are within the sensing range of p and q concurrently.

So the optimization problem is as follows:

$$\text{Min Max } F(p, q)$$

$$F(p, q)$$

Subject to:

$$\forall (i, j), L_{ij} \leq TR(i, j) \quad (19)$$

$$\forall i, L_{N+1, i} = 0 \quad (20)$$

$$\forall i, L_{i, i} = 0 \quad (21)$$

$$\forall (i \in R), \sum_{j=1}^{N+1} L_{ij} = 1 \quad (22)$$

$$\forall (i \in R), L_{i, N+1} = TR(i, N+1) \quad (23)$$

$$\forall (i \in R), CCv(i) = 2 \quad (24)$$

Equations (19), (20) and (21) are same as (7), (8) and (9) in MINLP problem. Equation (22) shows that every node has one outgoing link. Algorithm finalizing is achieved by (23). This means that if a node be within the transmission range of the AP, its destination is AP. And finally (24) ensures the network connectivity. In (24) CCv, (Connectivity Check value), for each node i , except AP, is defined as follows:

$$\forall (i \in R), CCv(i) = \sum_{i1=1}^{N+1} [L_{i, i1} + (L_{i, i1} \equiv 1 \& i1 \neq N+1 \& i1 \equiv N+1)] \cdot \sum_{i2=1}^{N+1} [L_{i1, i2} + (L_{i1, i2} \equiv 1 \& i1 \neq N+1 \& i2 \equiv N+1)] \dots \sum_{iN+1=1}^{N+1} [L_{iN, iN+1} + (L_{iN, iN+1} \equiv 1 \& iN \neq N+1 \& iN+1 \equiv N+1)] \dots \quad (25)$$

For each node which has a path to the AP, CCv equals to 2. If a node does not have any path to the AP its CCv is 0 or 1.

In (25), the logical term of each summation (the term within parentheses) equals to 1 if all of its three parts be true. It is equal to 0 otherwise. These terms prevent from reporting nodes of loops as connected nodes. If these terms are omitted, then the second part of (24) should be replaced by 1. But this constraint does not work correctly in cases like Fig. 2. In Fig. 2 arrows could be Mini-Max problem solution offered links. These links satisfy all constraints of Mini-Max problem. Nodes A, B, and C are out of AP's reception range, so none of them is forced to have a link to AP by (23). In this case for all nodes A, B, and C the CCv (without logical terms), equals to 1, but it is clear that these nodes are not connected to the AP. So the logical terms of (25) are necessary.

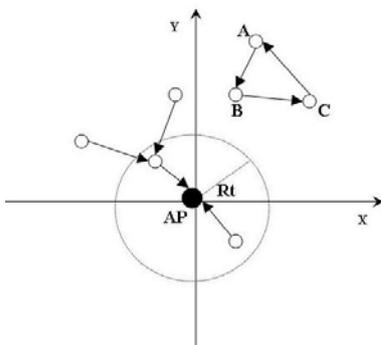


Fig. 2 Loops oblige logical terms of CCv formula

IV. SIMULATION RESULTS

In this section simulation results for MINLP problem are presented. For simulations, the MINLP problem has been implemented in MATLAB and solved by TOMLAB v6.1 [12] MINLP routines. TOMLAB is an open and integrated

MATLAB development environment in the field of optimization.

For simulations following numerical values have been assumed:

G_i and f_i values have been set equal to 1Mbps and 50Kb for all $i = 1, 2, \dots, N$, respectively. Min (T_{S0}) has been set to 50 ms. It should be mentioned that these assumptions has not limited problem's generality at all. According to above values, K_i equals to 1 for all intermediate nodes.

The network topology is depicted in Fig. 3. For each N , first N nodes of this topology have been selected and AP is always the $N+1^{th}$ node.

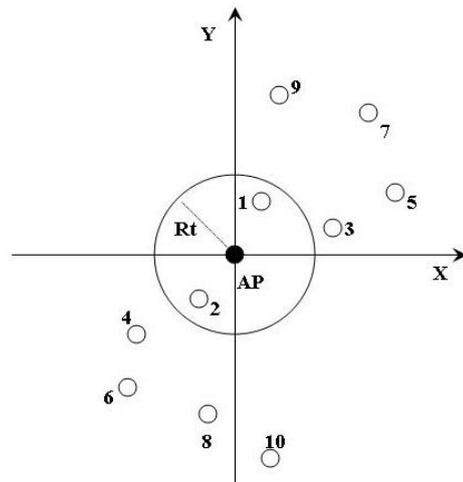


Fig. 3 Network topology for simulations

Table I shows a sample result of solved MINLP problem for $N = 8, S = 6, R_t = 300$ m, $R_s = 450$ m and Maximum Delay = 500ms. In each row active pairs of nodes in corresponding TI have been shown. T_{S0s} are in the first column.

TABLE I
 A SAMPLE ROUTING - SCHEDULING RESULT

TIs' Durations	Active Links
1st TI = 110 ms	(1,9) (4,2) (5,7) (8,6)
2nd TI = 130 ms	(2,9) (3,1) (6,4) (7,5)
3rd TI = 110 ms	(1,9) (4,2) (5,3) (8,6)
4th TI = 50 ms	(2,9) (3,1) (6,4)
5th TI = 50 ms	(1,9) (4,2)
6th TI = 50 ms	(2,9)

The optimum (maximum) throughput obtained in this example is 3.4 Mbps.

Fig. 4 shows the given optimized throughput values by MINLP problem versus maximum allowable delay. Each curve corresponds to a specific number of nodes. It is clear that maximum achievable throughput increases for weaker delay restrictions. Optimized throughput value has larger increasing rate when N increases. During simulations this fact has been found out that for each value of N there is a minimum value for number of TIs, S , which if S is set to a smaller value than this minimum, the problem is not solvable. In other words in order to satisfy all constraints (especially minimum traffic feeding demands of nodes), S should be

greater than a threshold. This threshold value depends on the network topology (number of nodes and their positions) and also K_i values. In mentioned topology this threshold equals to 3, 4, 5 and 6 for $N=4, 6, 8$ and 10 respectively. But the main point is that for a fixed maximum delay, if S be greater than this threshold then the maximum achievable throughput value is almost independent of S . This fact has been shown in Fig. 5 for a fixed maximum delay (600ms).

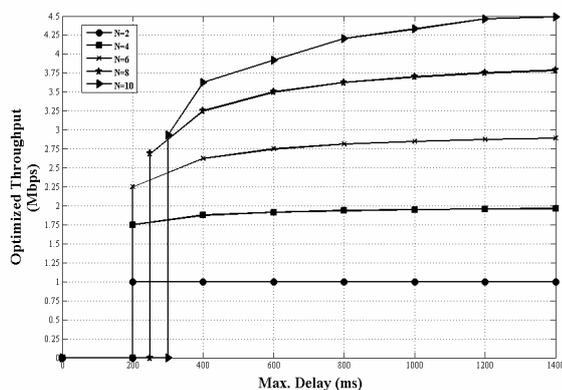


Fig. 4 Optimized throughput values vs. maximum delay

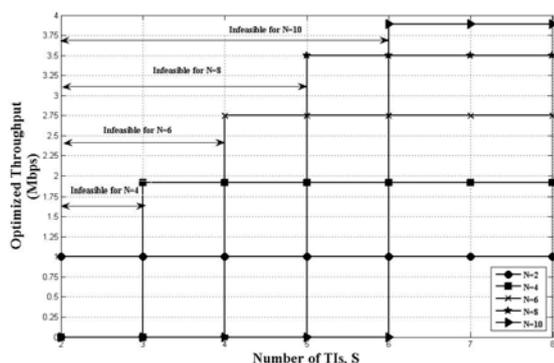


Fig. 5 Optimized throughput value is almost irrelevant to S for $S > S_{min}$

V. CONCLUSION

In this paper an optimization problem formulation for throughput maximizing in single radio, multi channel wireless mesh backhaul networks with delay constraints has been developed. Frequency planning has been assumed in order to avoid co-channel interference inside a collision domain. This mixed-integer nonlinear programming (MINLP) problem gives the best (upper bound) throughput which is achievable in a WMN according to application's delay requirements by trying all possible links between nodes and finding the best subset of links for each TI.

The main bottleneck of this approach is its large variables set which is equivalent to time consumption. This problem is acceptable according to infra structure mode which has a more or less stable topology. But if the same procedure is applied to a reduced number of candidate links in each TI, the results may be acceptable too, based on computational cost reduction. (Although they will not be the best possible values.) Finding

fine criteria to select the best subsets of links in order to balance the trade off between computational cost and results optimality degradation is under study.

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