Calculation of the Ceramics Weibull Parameters

V. Fuis and T. Navrat

Abstract—The paper deals with calculation of the parameters of ceramic material from a set of destruction tests of ceramic heads of total hip joint endoprosthesis. The standard way of calculation of the material parameters consists in carrying out a set of 3 or 4 point bending tests of specimens cut out from parts of the ceramic material to be analysed. In case of ceramic heads, it is not possible to cut out specimens of required dimensions because the heads are too small (if the cut out specimens were smaller than the normalised ones, the material parameters derived from them would exhibit higher strength values than those which the given ceramic material really has). On that score, a special testing jig was made, in which 40 heads were destructed. From the measured values of circumferential strains of the head’s external spherical surface under destruction, the state of stress in the head under destruction was established using the final elements method (FEM). From the values obtained, the sought for parameters of the ceramic material were calculated using Weibull’s weakest-link theory.

Keywords—Hip joint endoprosthesis, ceramic head, FEM analysis, Weibull’s weakest-link theory, failure probability, material parameters

I. INTRODUCTION

A problem that is being solved is the destruction of the ceramic heads of total hip joint endoprostheses in vivo, that had occurred in a series of Czech hospitals (Fig. 1). The ceramic heads are made of Al₂O₃ and put on conical stem made of austenitic steel. The implant’s failure of the „ceramic head destruction” type has always traumatic consequences for the patient, since a part of or even the whole endoprosthesis has to be re-operated, after which must again follow reconvalescence and rehabilitation. Hence, it is desired to reduce the number of implant re-operations to the minimum. The reliability of the ceramic component is based on Weibull weakest-link theory [1] and the failure probability depends on three Weibull’s parameters. These parameters are obtained from the statistical analysis of the set of the destructed specimens which are subjected to 3- or 4-point bending.

In case of ceramic heads of endoprosthesis, the snag is that the head’s dimensions are too small and do not allow to cut out from them a specimen that would comply with the standard for 3- or 4-point bending. One of the possible solutions to this problem is not to cut out the specimens from the heads, but to calculate the material parameters directly from the destruction of whole ceramic heads.

Fig. 1 In vivo destructed ceramic heads
II. GENERALLY ABOUT THE FAILURE PROBABILITY OF CERAMIC MATERIAL

For computational modelling of failure probability, two computation models are used which differ from each other in that which stresses are included into the computation. The first more simple model includes into the computation only the first principal stress \( \sigma_1 \) (see relation (1)), whereas the second model includes into the computation all three principal stresses. This model is suitable for cases where the second and the third principal stress take marked tensile values nearing to the values of the first principal stress \([2, 4]\). In case of a ceramic head of total hip joint endoprosthesis, the probability of its failure can be computed using a more simple model, since the maximum values of the second principal stress take at most 30% of the value of the first principal stress [3]. For computation of the ceramic head failure probability, the following relation is used, that is based on Weibull’s weakest-link theory [1]:

\[
P_f = 1 - e^{-\sum_{i=1}^{n} \left( \frac{\sigma_i - \sigma_u}{\sigma_o} \right)^m \Delta V_i}, \quad \sigma_i \geq \sigma_u,
\]

where: \( n \) – number of elements into which the analysed head is divided, using the finite elements method, \( \Delta V_i \) – volume of the i-th element, \( \sigma_i \) – first principal stress (\( \sigma_1 \)) acting in volume \( \Delta V_i \), \( \sigma_o \) – stress, beneath the value of which material failure does not occur, \( \sigma_o \) – normalised material strength of a volume unit of material, \( m \) - Weibull modulus (connected with the scatter of measured values). These parameters are derived from processing the destruction forces obtained from a set of 3- or 4-point bending tests. This way of assessing material parameters is universally applicable to any ceramic product, from which specimens are made for the mentioned bending test. The number of specimens must exceed 35, if the Weibull statistical method of the weakest link is at all to be applied. Small components present another problem which applies also to ceramic heads, namely that the cut out specimens are too small. Small specimens can be subjected only to 3-point bending, at which maximum tensile stress is concentrated only in a small area in the middle of the specimen, in contrast to 4-point bending at which maximum tensile stresses concentrate in two areas between two supports. The material characteristics obtained from 4-point bending tests show a greater plausibility than those from 3-point bending tests; therefore 4-point bending test is to be preferred.

III. EXPERIMENTAL DESTRUCtIONS OF THE HEADS

For carrying out the destruction of a set of ceramic heads a special testing jig (Fig. 2) has been made in which the heads are subjected to compressive load that acts only on the surface of the head’s opening, as shown in Fig. 3. In the course of the test, circumferential strains are measured on the head’s external surface. On each head, two gauges were stuck (at 180°) which, in addition, provided information on whether the deformations in the head are distributed evenly on the circumference – the data of the gauges served for judging the reliability of the results of experiments. The compression load acting on the head was developed by the testing device ZWICK Z 020-TND (Fig. 4) whose head loaded the jig’s piston which, in turn, acted on rubber NBR 90 inside the jig, which, subsequently, acted on the ceramic head. The destructed head is shown in Fig. 5.

![Fig. 2 Dismounted jig for the head destructions](image)

![Fig. 4 Experimental jig fixed in the ZWICK testing machine](image)
The result of experiments was a set of the strain values established under the destruction of the individual heads – Fig. 6. The highest value of the difference of the measured values was 8.7 % (head No. 27 in Fig. 6 and Fig. 7). In two ceramic heads the gauges were destructed before the destruction of the head (heads No. 32 and 33 in Fig. 6). The values of the difference of the measured strains on the ceramic heads are show in Fig. 7 – only 6 measured stains have the large difference than 5 %, so the pressure loading of the heads were equable and the results are suitable for the calculation of the material parameters of the ceramic material.

IV. CALCULATION OF THE MATERIAL PARAMETERS

In the first phase, from the strains established under the heads’ destruction, the stress field produced by the destruction had to be defined. To this purpose, computational modelling (FEM method) was used. The circumferential strain field and the first principal stress field for compressive load $p = 100$ MPa (modelling the stress field in the sorted specimen No. 32 in the Table I) are shown in Fig. 8 and 9. The destruction strain $\varepsilon_{\text{dest}} = 332 \mu$m/m (Fig. 8) corresponds to the maximum value of the first principal stress $\sigma_{\text{1max-dest}} = 371$ MPa in the point of the conical opening’s transition into the bottom of the head’s opening (the transition radius gives rise to the concentration of stress), see Fig. 9. Since a linear task is the matter, even the remaining values $\varepsilon_{\text{dest}}$ can be recounted in a linear way to $\sigma_{\text{1max-dest}}$. Thus we obtain a set of values of destruction stresses that have to be arranged in descending order for further analysis. Each $j$-th destructed head is assigned the probability of its failure (see Table I and Fig. 10), e.g. from the relation $P_f(j) = j/(r+1)$, where $j$ is the serial number of the arranged head and $r$ is the total number of destructed heads, in our case $r = 40$. 

![Fig. 5 Destructed head](image)

![Fig. 6 Circumferential strain values under heads’ destruction](image)

![Fig. 7 Difference of the measured strains for destructed heads](image)

![Fig. 8 Circumferential strain in the head for load $p = 100$ MPa](image)

![Fig. 9 The first principal stress ($\sigma_1$) in the head for load $p = 100$ MPa](image)
TABLE I

<table>
<thead>
<tr>
<th>$j$</th>
<th>$\sigma_{\text{max-dec}}$ [MPa]</th>
<th>$P_j$</th>
<th>$j$</th>
<th>$\sigma_{\text{max-dec}}$ [MPa]</th>
<th>$P_j$</th>
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<tr>
<td>1</td>
<td>208.4</td>
<td>0.024</td>
<td>21</td>
<td>314.8</td>
<td>0.512</td>
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<td>2</td>
<td>216.4</td>
<td>0.049</td>
<td>22</td>
<td>322.8</td>
<td>0.537</td>
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<tr>
<td>3</td>
<td>217.1</td>
<td>0.073</td>
<td>23</td>
<td>324.5</td>
<td>0.561</td>
</tr>
<tr>
<td>4</td>
<td>221.8</td>
<td>0.098</td>
<td>24</td>
<td>330.5</td>
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</tr>
<tr>
<td>5</td>
<td>224.5</td>
<td>0.122</td>
<td>25</td>
<td>340.8</td>
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<tr>
<td>6</td>
<td>250.8</td>
<td>0.146</td>
<td>26</td>
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</tr>
<tr>
<td>7</td>
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<td>0.171</td>
<td>27</td>
<td>346.2</td>
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</tr>
<tr>
<td>8</td>
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<td>0.195</td>
<td>28</td>
<td>357.2</td>
<td>0.683</td>
</tr>
<tr>
<td>9</td>
<td>270.4</td>
<td>0.220</td>
<td>29</td>
<td>360.1</td>
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</tr>
<tr>
<td>10</td>
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<td>30</td>
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<td>0.732</td>
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<tr>
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<td>365.2</td>
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<tr>
<td>12</td>
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<td>0.293</td>
<td>32</td>
<td>371.2</td>
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<td>13</td>
<td>289.1</td>
<td>0.317</td>
<td>33</td>
<td>374.3</td>
<td>0.805</td>
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<td>14</td>
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</tr>
<tr>
<td>15</td>
<td>303.9</td>
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<td>379.3</td>
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<tr>
<td>16</td>
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<tr>
<td>18</td>
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<tr>
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<tr>
<td>20</td>
<td>314.4</td>
<td>0.488</td>
<td>40</td>
<td>501.7</td>
<td>0.976</td>
</tr>
</tbody>
</table>

The first of Weibull parameters is stress $\sigma_u$, which must be lower than the minimum value $\sigma_{\text{max-dec}}$ ($\sigma_{\text{max-dec-min}} = 208.4$ MPa – see Table I or Fig. 10), so the stress $\sigma_u$ was assumed for the following values - 0, 50, 100, 150 and 200 MPa. Conservative approach considers $\sigma_u = 0$ MPa, then all tensile stresses influence the body's destruction (thus Weibull 3-parameter analysis changes to a 2-parameter analysis).

The second parameter (Weibull modulus $m$) is connected with the dispersion of experimentally established values and it is determined as gradient of a line interlaid with experimentally established data from the Table I. Weibull modulus for above assumed values of the stress $\sigma_u$ is show in the Fig. 11. The values of the correlation between the blue experimental data and the red line in Fig. 11 are shown in Fig. 12. The highest value of the correlation is for stress $\sigma_u = 160$ MPa – Fig. 13, so this is the first Weibull material parameter and the Weibull modulus $m = 2.4166$ (Figs. 12 and 13).

The last of the parameters (normalised volume strength $\sigma_v$) is calculated from the following equation

$$\sigma_v = \sqrt[n]{\sum_{i=1}^{n}(\sigma_i - \sigma_u)^{m}}\Delta V_i, \quad \sigma_i \geq \sigma_u,$$

(2)
which was derived by modification of the equation (1) for
\[ P_f = 1 - \frac{1}{e} = 0.63212 \] (failure probability for the experimental
data and for the material parameters will be the same –
0.63212 – see fitted point if Fig. 13). As values \( \sigma_i \) are used the
set of the values \( \sigma_i \) calculated by the finite elements method in
the head’s all elements, for which it holds that \( \sigma_i > \sigma_{cr} \).

The same way is used to calculation of the material
parameters for the conservative approach (\( \sigma_{fr} = 0 \) MPa) and
the comparison of experimental and computational data is show in Fig. 14. The material parameters for both models
(2-parameter and 3-parameter) are show in Table II.

From the comparison of the experimental and
computational data in Figs. 13 and 14 implies the following
results:
- 2-parameter Weibull material parameters better fit the
experimental data than 3-parameter parameters but
for the low values of the failure probability is large
difference (the bioimplants have to reach the very low
failure probability so this model have to be improved
– for example fitting point will be moved from
\( P_f = 0.63212 \) to value 0.024 (the first sample in the
Table I)),
- 3-parameter Weibull material parameters is not
conservative because the low values of the failure
probability is lower than experimental data (in the
reality is failure probability higher than calculated).

V. CONCLUSION

The material parameters of ceramic material used for
manufacture of heads of total hip joint endoprosthesis have
been determined on the basis of a set of head destruction tests.
The analysis of the calculated material parameters show that
both assumed models have the lacks but the 2-parameters
Weibull material parameters are usable than 3-parameters
ones.

The reliability of the parameters thus obtained is higher than
those obtained from specimens cut out from the heads,
because their dimensions are markedly smaller than those
required by the standard. The presented approach is general
and applicable to any brittle material whose strength
characteristics exhibit Weibull probability distribution.
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REFERENCES