Abstract—Image coding based on clustering provides immediate access to targeted features of interest in a high quality decoded image. This approach is useful for intelligent devices, as well as for multimedia content-based description standards. The result of image clustering cannot be precise in some positions especially on pixels with edge information which produce ambiguity among the clusters.

Even with a good enhancement operator based on PDE, the quality of the decoded image will highly depend on the clustering process. In this paper, we introduce an ambiguity cluster in image coding to represent pixels with vagueness properties. The presence of such a cluster allows preserving some details inherent to edges as well for uncertain pixels. It will also be very useful during the decoding phase in which an anisotropic diffusion operator, such as Perona-Malik, enhances the quality of the restored image. This work also offers a comparative study to demonstrate the effectiveness of a fuzzy clustering technique in detecting the ambiguity cluster without losing lot of the essential image information. Several experiments have been carried out to demonstrate the usefulness of ambiguity concept in image compression. The coding results and the performance of the proposed algorithms are discussed in terms of the peak signal-to-noise ratio and the quantity of ambiguous pixels.

Keywords—Ambiguity Cluster, Anisotropic Diffusion, Fuzzy Clustering, Image Compression.

I. INTRODUCTION

After the last technological progress spreading worldwide, the information in all its forms becomes the most important product of contemporary society. Therefore, researchers are challenging to offer efficient algorithms which reduce the amount of data required to represent a given source of information. In fact, the need to save storage space and shorten transmission time has been the driving factors behind image compression methods.

Image compression deals with minimizing the amount of data needed to represent a digital image by removing redundant data. It involves encoding a 2-D array of pixels into a statistically uncorrelated data set. This transformation is applied before image storage or transmission. Afterward, the coded image is decompressed in order to reconstruct either the exact original image or an approximated version of the original one. For this reason, data compression system requires the definition of two functions: compression and decompression.

Most of the contemporary image coding techniques exploit the quantization principle, such as the JPEG standard based on the Discrete Cosine Transform (DCT) [1] or in the Log-exp transformation model [2]. The major limitation in JPEG is the blocky appearance of the decoded image. The Log-exp model suffers from its computation time which will be penalizing in real-time applications. The quantization stage is the core of every lossy image encoding algorithm: in the encoder part, quantization means partitioning of the input data into a smaller set of values [3].

Increasingly, many studies had proved the involvement of PDEs in the image processing field. They are largely the essential element of image compression and restoration [4]. The basic idea of applying PDEs is to encode some image points (landmarks) selected according to many criteria such as edges or corners whereas in the decompression phase, the decoded image with incomplete information will be fulfilled by applying PDE's anisotropic diffusion [5].

In this study, we suggest the exploitation of fuzzy clustering with ambiguity and PDEs in order to achieve good image quality after compression with a possibility to define image features by the mean of clustering or segmentation. This work is an extension of the proposed approach illustrated in [6].

Image segmentation is an essential and important phase in image analysis in order to produce meaningful features of interest. The purpose behind this step is to formulate the image into regions or clusters. In our work, we chose the fuzzy logic and more precisely fuzzy sets-based algorithms to offer a clustering tool [7, 8, 9]. The fuzzy sets are a generalization of the classical set theory but they offer greater flexibility to detect accurately the various aspects of vagueness or imperfection in image information.

The remainder of this paper is structured as follows: Section 2 presents our system architecture and the description of the related theories such as fuzzy clustering, entropy encoding and anisotropic diffusion process. The description of these theories is useful to better understand the core of our work. In section 3, the assessment of our proposed approach is illustrated with the help of experimental results and comparisons. The paper concludes with section 4 by providing an overview of the contributions and some potential perspectives.

II. SYSTEM WORKFLOW AND RELATED THEORY

In main purpose of this section is to explain the relevant theories which will be helpful for further understanding and discussion. Fig. 1 illustrates the system architecture and the principal steps performed in our proposed approach.

S. Al-Ali, A. Shahin, and F. Chakik are with the Laboratory of Mathematics and Applications, Doctoral School of Sciences and Technology, the Lebanese University, Mitein Street, face to Malaab High School, Tripoli, Lebanon (e-mails: Safaa_alali@hotmail.com, ashahin@ul.edu.lb, fchakik@ul.edu.lb).
In our model, the compression algorithm is performed in the following way:

1) Firstly, Unsupervised Fuzzy clustering algorithm is applied on the input image. The FcM proposed by Bezdeck [8] is often a good tool to segment an image. In our work, we also compare the FcM result to the AFcM [9] which is based on the FcM but converges faster. At the end of this step, well-defined members are assigned to the cluster C with the highest and certain membership. Pixels with uncertain memberships will be assigned to a special class that contains all rejected points.

2) Define the vector of ambiguous points indexed by the ambiguity cluster.

3) A second stage of entropy encoding is executed. It’s applied on both image clusters and the vector of ambiguous points separately. This will help reducing the large amount of redundancy. Multiple studies was performed and improved to find effective techniques in order to encode information without dropping their principal entities. Among several algorithms recently applied, we suggest the use of Huffman or Arithmetic coding algorithms.

Accordingly, the order of processing appears reversed in the corresponding decompression phase:

1) Apply entropy decoding.

2) De-quantization which assigns the well-defined members with the corresponding cluster center is carried out. For the ambiguity cluster, indexing based on the vector of ambiguous points is used.

3) Apply nonlinear anisotropic diffusion. We suggest the use of Perona–Malik filter [10]. It is an iterative process that continues until a sufficient degree of smoothing is obtained. The used equation encourages diffusion (hence smoothing) within regions and prohibits it across strong edges. The process convergences once the image quality is degraded from the previous iteration.

The quality of the decoded image is controlled by many parameters such as the choice of the number of clusters, the ambiguity threshold for uncertainty decision, the maximum number of iterations in the diffusion phase, etc.

In the remaining part of this section, we will describe the methods used in the proposed compression/decompression algorithms.

A. Fuzzy Clustering with Ambiguity Class

Clustering is the process of organizing objects in groups having similar properties. Clustering methods can be used to create groups of pixels that are similar in regard to a measure, often their color or gray level; therefore simplifying the image by reducing the number of discrete possible pixel values. Image clustering can be used to get a simple segmentation of the image.

The Alternative Fuzzy c-Means algorithm (often abbreviated to AFcM) [9] is an iterative algorithm inspired from FcM proposed by Bezdeck [8]. These algorithms find clusters in data and use the concept of fuzzy membership: instead of assigning a pixel to a single cluster, each pixel will have different membership values on each cluster.

The AFcM attempts to find clusters in the data by minimizing an objective function shown in the equation below:

$$J = \sum_{i=1}^{C} \sum_{j=1}^{N} \mu_{ij}^m d^2(x_j, c_i)$$  

(1)

Where:

- $J$ is the objective function, a kind of quality criterion to minimize
- $N$ is the number of pixels in the image
- $C$ is the number of clusters used in the algorithm, and which must be decided before execution
- $\mu$ is the membership matrix of $N \times C$ entries which contains the membership values of each pixel to each cluster
- $m$ is a fuzziness factor (a value larger than 1)
- $x_j$ is the $j$th pixel in the image
- $c_i$ is the $i$th cluster
- $d(x_j, c_i)$ is the distance between $x_j$ and $c_i$. In the FcM, the Euclidean distance is used while in the AFcM it’s defined as:

$$d^2(x, y) = 1 - e^{-\beta||x-y||^2}$$  

(2)

Where $\beta > 1$ and could be estimated from the image variance $\bar{x}$ as:

$$\beta = \left(\frac{\sum_{j=1}^{n}||x_j - \bar{x}||^2}{n}\right)^{-1}$$  

(3)

This metric is a robustness estimator because it is insensitive to small variations and robust against noise [9].

The steps of the algorithm are:

1. Initialize $\mu$ with random values between zero and one; but with the sum of all fuzzy membership elements for a particular pixel being equal to 1. In other words, the sum of the memberships of a pixel for all clusters must be one.
2. Estimate $\beta$ using (3)
3. Calculate an initial value for $J$ using (1).
4. Calculate the centroids of the clusters $c_i$ using

$$c_i = \frac{\sum_{j=1}^{n} h_{ij} |1- d^2(x_j,c_i)| x_j}{\sum_{j=1}^{n} h_{ij} |1- d^2(x_j,c_i)|}$$  \hspace{1cm} (4)

5. Calculate the fuzzy membership $\mu_{ij}$ using

$$\mu_{ij} = \frac{\sum_k |d^2(x_j,c_k)|^{-1/(m-1)}}{|d^2(x_j,c_i)|^{-1/(m-1)}}$$ \hspace{1cm} (5)

6. Go to step 4 until a stopping condition is reached.

Some possible stopping conditions are:

- A number of iterations were executed, and we can consider that the algorithm achieved a "good enough" clustering of the data.

- The difference between the values of $J$ in consecutive iterations is small (smaller than a user-specified parameter $\epsilon$), therefore the algorithm has converged.

Traditionally the algorithm defuzzify its results by choosing a "winning" cluster, i.e. the one which is closer to the pixel in the feature space, is the one for which the membership value is highest/certain and using that cluster center as the new values for the pixel. These membership values can be obtained for any kind of images (grayscale, RGB, etc...). The algorithm is adaptive and can be used with image of multiple channels.

The ambiguity cluster is built by grouping all pixels with uncertain memberships, i.e. they cannot belong to a precise class due to vagueness in properties. The criterion used to unclassify pixels is threshold based and is object to experimental evaluation.

B. Non-Linear Diffusion

The PDEs used in image restoration (smoothing, denoising, enhancing of image...) [11] are almost the same PDEs used in image compression by the diffusion (linear isotropic or nonlinear anisotropic).

Typical PDE techniques for image smoothing regard the original image as initial state of a parabolic (diffusion-like) process, and extract filtered versions from its temporal evolution.

Many evolution equations for restoring images can be derived as gradient descent methods for minimizing a suitable energy functional, and the restored image is given by the steady-state of this process.

This theory was proposed by Malik and Perona [10]. Their main idea is to introduce a part of the edge detection step in filtering itself, allowing distinguishing noise from edge. The principle is to spread strongly in areas with low gradients (homogenous areas), and low in areas with strong gradients (edges). These filters are difficult to analyze mathematically, as they may act locally like a backward diffusion process.

Perona and Malik proposed a nonlinear diffusion method for avoiding the blurring and localization problems of linear diffusion filtering. They applied an inhomogeneous process that reduces the diffusivity at those locations which have a larger likelihood to be edges. This likelihood is measured by $\|\nabla u\|^2$. The Perona-Malik filter is based on the equation:

$$\partial_t u = \text{div}(g(\|\nabla u\|^2)\nabla u)$$ \hspace{1cm} (6)

and uses the diffusivity:

$$g(s^2) = \frac{1}{1+s^2/\lambda^2} \hspace{0.5cm} (\lambda > 0)$$ \hspace{1cm} (7)

The experiments of Perona and Malik were visually very impressive: edges remained stable over a very long time. It was demonstrated that edge detection based on this process clearly outperforms the linear Canny edge detector. For more details, see [10].

III. RESULTS AND DISCUSSIONS

In order to evaluate the effectiveness of the proposed methods, we have tested our algorithm with the help of three benchmarks of grayscale images: Lena, Barbara and Baboon of size 512*512. In order to control the parameters on which the approach depends, we divided our study into many parts.

A. Compression without Ambiguity

We will start our experiments using fuzzy clustering without ambiguity class. It means that pixels are assigned to the cluster of the highest membership. The results are summarized in Table I. AFcM and FcM produce similar image quality in most of the cases. AFcM converges faster. In some situation, FcM produces better quality but with PDE operator, AFcM also achieves good image quality. The quality of the image is enhanced when we choose higher number of clusters and the PDE operator enhances slightly the image quality.

<table>
<thead>
<tr>
<th>Benchmark</th>
<th>Number of classes</th>
<th>FcM without PDE</th>
<th>FcM with PDE</th>
<th>AFcM without PDE</th>
<th>AFcM with PDE</th>
</tr>
</thead>
<tbody>
<tr>
<td>Lena</td>
<td>4</td>
<td>22.9033</td>
<td>22.9037</td>
<td>22.9085</td>
<td>22.9069</td>
</tr>
<tr>
<td></td>
<td>8</td>
<td>30.9650</td>
<td>30.9650</td>
<td>30.9664</td>
<td>30.9661</td>
</tr>
<tr>
<td></td>
<td>16</td>
<td>37.0802</td>
<td>37.0683</td>
<td>37.0505</td>
<td>37.0874</td>
</tr>
<tr>
<td></td>
<td>8</td>
<td>30.6522</td>
<td>30.6532</td>
<td>30.6531</td>
<td>30.6530</td>
</tr>
<tr>
<td></td>
<td>16</td>
<td>36.9588</td>
<td>36.9214</td>
<td>36.9612</td>
<td>36.9543</td>
</tr>
<tr>
<td>Baboon</td>
<td>4</td>
<td>23.9814</td>
<td>23.3814</td>
<td>23.3824</td>
<td>23.3822</td>
</tr>
<tr>
<td></td>
<td>8</td>
<td>29.9185</td>
<td>29.9222</td>
<td>29.9173</td>
<td>29.9225</td>
</tr>
<tr>
<td></td>
<td>16</td>
<td>32.2582</td>
<td>36.2508</td>
<td>36.0599</td>
<td>36.2513</td>
</tr>
</tbody>
</table>

AFcM produces similar results compared to FcM

B. Compression with Ambiguity

In Fig. 2, we compared the quality of the tested images by using FcM and AFcM in the clustering phase. The threshold value controls the quantity of ambiguous points. We have concluded that AFcM converges faster and gives similar quality compared to FcM.
Fig. 2 The horizontal axis represents the ambiguity threshold. The vertical axis indicates the PSNR for the clustering algorithm for C=8 classes.

In Table II, we have the results of applying FcM and AFcM on the three test images. These experiments were controlled by the ambiguity threshold which is highly related to the size of the ambiguity cluster. Again AFcM can give good image quality with smaller ambiguity cluster.

<table>
<thead>
<tr>
<th>Number of classes</th>
<th>Benchmark</th>
<th>Ambiguity threshold via FcM</th>
<th>PSNR via FcM</th>
<th>Number of Ambiguous Points</th>
<th>Ambiguity threshold via AFcM</th>
<th>PSNR via AFcM</th>
<th>Number of Ambiguous Points</th>
</tr>
</thead>
<tbody>
<tr>
<td>Lena 4</td>
<td></td>
<td>0.575</td>
<td>24.161</td>
<td>9.661 0.55</td>
<td>23.841</td>
<td>7.315</td>
<td></td>
</tr>
<tr>
<td>Lena 16</td>
<td></td>
<td>0.5</td>
<td>38.122</td>
<td>8.136 0.5</td>
<td>38.086</td>
<td>7.665</td>
<td></td>
</tr>
<tr>
<td>Barbara</td>
<td></td>
<td>0.6</td>
<td>24.306</td>
<td>9.384 0.6</td>
<td>24.305</td>
<td>7.384</td>
<td></td>
</tr>
<tr>
<td>Baboon</td>
<td></td>
<td>0.6</td>
<td>24.542</td>
<td>9.032 0.6</td>
<td>24.621</td>
<td>9.690</td>
<td></td>
</tr>
<tr>
<td>Lena 16</td>
<td></td>
<td>0.5</td>
<td>38.148</td>
<td>9.088 0.525</td>
<td>38.241</td>
<td>9.593</td>
<td></td>
</tr>
<tr>
<td>Baboon</td>
<td></td>
<td>0.525</td>
<td>37.885</td>
<td>9.288 0.525</td>
<td>37.560</td>
<td>7.744</td>
<td></td>
</tr>
</tbody>
</table>

Results of ambiguity tests varying the number of segmented regions.

In Fig. 3, we show how the size of the ambiguity cluster is related to the ambiguity threshold value. They increase linearly.

Fig. 3 The quantity of ambiguity pixels depends on the threshold value using AFcM fuzzy clustering with C=8 classes.

From Table III, we can clearly affirm that the Perona-Malik diffusion does not have a remarkable influence on enhancing the quality of the decoded sparse images.

<table>
<thead>
<tr>
<th>Number of classes</th>
<th>Benchmark</th>
<th>Ambiguity threshold with PDE</th>
<th>PSNR with PDE</th>
<th>% of ambiguous points</th>
<th>PSNR without PDE</th>
</tr>
</thead>
<tbody>
<tr>
<td>Lena 4</td>
<td></td>
<td>0.55</td>
<td>23.945</td>
<td>8.028</td>
<td>23.841</td>
</tr>
<tr>
<td>Lena 16</td>
<td></td>
<td>0.5</td>
<td>38.196</td>
<td>8.520</td>
<td>38.086</td>
</tr>
<tr>
<td>Barbara</td>
<td></td>
<td>0.6</td>
<td>24.305</td>
<td>9.384</td>
<td>24.305</td>
</tr>
<tr>
<td>Baboon</td>
<td></td>
<td>0.6</td>
<td>24.621</td>
<td>9.690</td>
<td>24.621</td>
</tr>
<tr>
<td>Lena 16</td>
<td></td>
<td>0.5</td>
<td>38.120</td>
<td>8.307</td>
<td>38.241</td>
</tr>
<tr>
<td>Baboon</td>
<td></td>
<td>0.525</td>
<td>37.758</td>
<td>9.183</td>
<td>37.560</td>
</tr>
</tbody>
</table>

Results of PDE influence with AFcM and ambiguity cluster.

IV. CONCLUSION

We investigated an efficient image compression technique based on fuzzy clustering and ambiguity concept.
We studied the quality of compressed images without extracting ambiguous points. The AFcM gave better results compared to FcM using anisotropic diffusion.

We also applied fuzzy concept provided with its ambiguity part. A comparative study had shown the effectiveness of the AFcM algorithm in the clustering stage in our compression path. The AFcM algorithm has the advantage of reducing the convergence time and the ability to produce a good quality for the coded image with the minimal size of ambiguity consideration.

We have also conducted different experiments to evaluate the performance of the Perona-Malik diffusion on the test images by varying the threshold value and the number of clusters in AFcM. The PDE operator increases very slightly the image quality and may not be beneficial in some conditions. More attention needs to be carried out with the use of that filter. We should conduct tests on better anisotropic filters such as tensors [5], CED or EED [4] to enhance the diffusion quality.

We believe that introducing ambiguity factor in the clustering phase was beneficial for the decoding phase. We’d like to work on segmentation algorithms that integrate this ambiguity notation such as the Fuzzy c+2 Means introduced by [11].

This work could be extended in many ways. We’d like to test the efficiency of our approach on the compression of big pictures in order to work on these pictures with normal PCs, notebooks, iPad etc. We shall also try to evaluate our algorithms on color images and videos.

**REFERENCES**


