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Effect of Gravity Modulation on Weakly Non-Linear Stability of Stationary Convection in a Dielectric Liquid

P. G. Siddheshwar and B. R. Revathi

Abstract—The effect of time-periodic oscillations of the Rayleigh-Bénard system on the heat transport in dielectric liquids is investigated by weakly nonlinear analysis. We focus on stationary convection using the slow time scale and arrive at the real Ginzburg-Landau equation. Classical fourth order Runge-kutta method is used to solve the Ginzburg-Landau equation which gives the amplitude of convection and this helps in quantifying the heat transfer in dielectric liquids in terms of the Nusselt number. The effect of electrical Rayleigh number and the amplitude of modulation on heat transport is studied.

Keywords—Dielectric liquid; Nusselt number; Amplitude equation.

NOMENCLATURE

Latin symbols

B	Amplitude of streamline perturbation
C_{VE}	effective heat capacity at constant volume and electric field
$\overrightarrow{D}$	electric displacement
$\overrightarrow{E}$	electric field
$\overrightarrow{E_0}$	root mean square value of the electric field at the lower surface
$\overrightarrow{g}$	gravitational acceleration (0,0,-g)
$\overrightarrow{g'}$	gravity modulation
h	depth of the fluid layer
$(\hat{i}, \hat{j}, \hat{k})$	unit vectors in the x,y and z directions respectively
k	dimensional wave number
k_1	thermal conductivity
Nu	Nusselt number
$\overrightarrow{P}$	dielectric polarisation
Pr	Prandtl number
$\overrightarrow{q}$	velocity vector(=(u,v,w))
R	thermal Rayleigh number
R_E	electric Rayleigh number
t	time
T	temperature
T_0	constant temperature of the upper boundary
ΔT	temperature differences between the lower and upper surfaces
∇^2	three dimensional laplacian operator $(= \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2})$

Greek symbols

α	thermal expansion coefficient
χ_e	electric suscptibility
δ_2	amplitude of gravity modulation
ε_0	electric permittivity of free space
ε_r	relative permittivity
μ_1	reference viscosity
Ω	scaledup frequency
Φ	electric scalar potential
ψ	Stream function
Ψ	Dimensionless stream function
ρ	fluid density
ρ_0	reference density
∇	Differential operator

Subscripts

b	basic value
c	Critical value

Superscripts

$\prime$	perturbed quantity
$*$	Dimensionless quantity

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I. INTRODUCTION

THE convective instability of a fluid in a horizontal layer of dielectric liquid due to a time dependent gravity is of practical importance. The regulation of convection is important from the applications point of view and thermogravitational vibration(called gravity modulation or g-jitter) is known to be an effective means of controlling instabilities. It is also of importance in the large-scale convection in atmosphere. Existence of adverse density variations within the fluid and a body force are the necessary conditions to initiate natural convection. The idea of using mechanical vibration as a tool to improve the heat transfer rate has received much attention. In the present paper the effect of time-periodic gravity modulation of the Rayleigh-Bénard convection problem on the heat transport in dielectric liquids is studied by weakly nonlinear analysis. Here the critical Rayleigh number is a function of the electrical Rayleigh number. We derive the real Ginzburg-Landau equation for slow time scale. The solution of the Ginzburg-Landau equation gives the amplitude of convection and we proceed to determine the Nusselt number. The effect of electrical Rayleigh number and the amplitude of modulation on Nusselt number is numerically studied and pointed out. We survey the literature pertaining to the problem.

Gresho and Sani [1] and Greshuni et al. [2] were the first to study the effect of gravity modulation in a fluid layer. They used small amplitude approximation and found that the system may be stabilized in the same manner as an inverted pendulum is stabilized by vertical oscillation. In addition to that Gresho and Sani predicted that certain combinations of the flow parameters may stabilize or destabilize the development of convective flow for stable and unstable configurations respectively. Biringen and Peltier [3] investigated non-linear three dimensional Rayleigh-Bénard problem under gravity modulation by numerical solutions of full Navier-Stokes equations and confirmed the result of Gresho and Sani. Wheeler et al.[4] used the averaging method and the Floquet theory to analyze the stability of directional solidification problem under high frequency gravity modulation. Later Cleaver et al.[5] performed a detailed non-linear analysis of the problem and presented the stability limits to a much wider region of parameter space. Cleaver et al.[6], Rogers et al. [7], Aniss et al. [8], [9], Bhadauria et al. [10], [11] showed that gravitational modulation, acts on the entire volume of liquid and may have a stabilizing or destabilizing effect depending on the amplitude and frequency of the forcing. Here the onset of convection presents a competition between harmonic and subharmonic modes. Li[12], Pau and Li[13] show that gravity and magnetic fields represent different mechanisms of flow reduction and that they may be combined to further suppress the convection in a modulated gravity field. Malashetty and Padmavathy [14] studied the effect of small amplitude gravity modulation on the onset of convection in fluid and fluid-saturated porous layers. They found that low frequency oscillations have significant effect on the stability of the system. Skarda [15] studied the effect of gravity modulation in a Marangoni-Bénard problem. He observed that the instabilities are strongly influenced by the Prandtl number in Marangoni-Bénard problem while it is weakly affected by Prandtl number in the case of Rayleigh-Bénard problem. The linear stability theory of Govender[16], [17] showed that increasing the frequency of vibration stabilizes the convection in a gravity modulated homogeneous porous layer heated from below and same effect was seen in a gravity modulated cylindrical porous layer heated from below and in addition the aspect ratio was found to influence the stability of convection. Shu et al.[18] examined the effect of modulation of gravity and thermal gradients on natural convection in a cavity numerically and experimentally. They found that for low Prandtl number fluids, modulations in gravity and temperature produce the same flow field both in structure and in magnitude. The linear stability theory of Govender[19] showed that increasing the excitation frequency rapidly stabilizes the convection upto the transition point from synchronous to subharmonic convection beyond which slowly destabilizes the convection in a homogeneous gravity modulated porous layer heated from below. Siddavaram and Homsy [20], [21] analyzed the effect of harmonic gravity modulation on fluid mixing and later on they studied the effects of stochastic gravity modulation. They have compared the flow regimes and instabilities obtained with that of harmonic modulation. As the gravity modulation stabilized convection, the inverted pendulum with an oscillating pivot point also stabilizes the motion in a porous layer heated from below was studied by Saneshan Govender[22] Boulal et al.[23] focussed attention on the influence of gravitational modulation on the convective instability threshold and predicted that the threshold of convection corresponds precisely to quasi-periodic solutions. Siddheshwar and Annamma [24] investigated the thermal instability of dielectric liquid when the boundary of the layer is subjected to small amplitude time-periodic body force and this results in the delay of convection. The effect of gravity modulation on heat transport in the problem of magnetoconvection in a Newtonian fluid was analysed by Siddheshwar et al. [25].

A. *Mathematical Formulation*

Consider an infinite horizontal layer of a Boussinesquian dielectric liquid of depth ‘h’ that supports a temperature gradient and an *ac* electric field in the vertical direction. The upper and lower boundaries are maintained at constant temperatures T_0 and $T_0 + \Delta T$ ($\Delta T > 0$) respectively. The schematic of the physical configuration is shown in Figure 1. For mathematical tractability we confine ourselves to two-dimensional rolls so that all physical quantities are independent of y, a horizontal co-ordinate. Further, the boundaries are assumed to be free and perfect conductors of heat. In this paper we assume the effective viscosity μ to be constant and the reference viscosity μ_1 will be used to denote the constant viscosity.

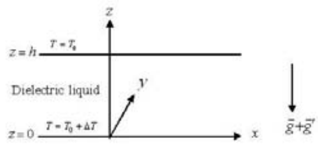

Fig. 1. Physical configuration of the Rayleigh-Bénard convection in a dielectric liquid with imposed time periodic gravity modulation.

The Governing equations describing the Rayleigh-Bénard instability situation in a constant viscosity dielectric liquid with gravity modulation are

$$\nabla.\vec{q} = 0, \tag{1}$$

$$\rho_0 \left[\frac{\partial \vec{q}}{\partial t} + (\vec{q}.\nabla)\vec{q} \right] = -\nabla p - \rho(g + g')\hat{k}$$
$$+(\vec{P}.\nabla)\vec{E} + \mu_1 \nabla^2 \vec{q}, \tag{2}$$

$$\rho_0 C_{VE} \left[\frac{\partial T}{\partial t} + (\vec{q}.\nabla)T \right] = k_1 \nabla^2 T. \tag{3}$$

The density equation of state is

$$\rho = \rho_0 \left[1 - \alpha(T - T_0)\right]. \tag{4}$$

The electrical equations are

$$\nabla.\vec{D} = 0, \quad \nabla \times \vec{E} = 0, \tag{5}$$

where

$$\vec{D} = \varepsilon_0 \vec{E} + \vec{P}, \quad \vec{P} = \varepsilon_0(\varepsilon_r - 1)\vec{E}, \tag{6}$$

The equation of state for dielectric constant ε_r is

$$\varepsilon_r = \varepsilon_r^0 - e(T - T_0), \tag{7}$$

where $\vec{E}$ is an *ac* electric field, which is assumed to oscillate sufficiently rapidly so as to make the body force on any free charges in the liquid inconsequential and the rest of the quantities have their usual meaning. We note here that the assumed strength of $\overrightarrow{E}$ is such that it does not induce any non-Newtonian characteristics in the dielectric liquid. It is expedient to write $\varepsilon_r^0 = (1+\chi_e)$, where χ_e is the electric susceptibility, for it enables us to arrive at the conventional definition $\overrightarrow{P} = \varepsilon_0\chi_e\overrightarrow{E}$ in the absence of the temperature dependence of ε_r, that is, when e = 0. We continue using Eq.(7) with ε_r^0 replaced by $(1+\chi_e)$. In writing Eq.(7) we have assumed that ε_r varies with the electric field strength quite insignificantly (Stiles et al., 1993).

We restrict ourselves to the two-dimensional analysis so that all physical quantities are independent of y, a horizontal co-ordinate. The electric boundary conditions are that the normal component of the electric displacement $\overrightarrow{D}$ and tangential component of the electric field $\overrightarrow{E}$ are continuous across the boundaries.

Taking the components of polarization and electric field in the basic state to be $[0, P_b(z)]$ and $[0, E_b(z)]$, we obtain the quiescent state solution

$$\left.\begin{array}{l} \overrightarrow{q}_b = (0,0), T_b = T_0 + \left(1-\frac{z}{h}\right)\triangle T, \\ \rho_b = \rho_0\left[1-\alpha\left(1-\frac{z}{h}\right)\triangle T\right], \\ \overrightarrow{E}_b = \left[\frac{(1+\chi_e)E_0}{(1+\chi_e)+e\left(1-\frac{z}{h}\right)\triangle T}\right]\hat{k}, \\ \overrightarrow{P}_b = \varepsilon_0 E_0(1+\chi_e)\left[1-\frac{1}{(1+\chi_e)+\left(1-\frac{z}{h}\right)\triangle T}\right]\hat{k}, \end{array}\right\} \quad (8)$$

where E_0 is the root mean square value of the electric field at the lower surface. On this basic state we superpose finite amplitude perturbations of the form:

$$\left.\begin{array}{l} \overrightarrow{q} = \overrightarrow{q_b} + (u', w'),\ T = T_b + T',\ P = P_b + P', \\ \rho = \rho_b + \rho',\ \overrightarrow{P} = \overrightarrow{P_b} + (P_1', P_3'), \\ \overrightarrow{E} = \overrightarrow{E_b} + (E_1', E_3'), \end{array}\right\} \quad (9)$$

where the prime denotes perturbation. The second of Eq.(6) now leads to

$$\left.\begin{array}{l} P_1' = \varepsilon_0\chi_e E_1' - e\varepsilon_0 T' E_1', \\ P_3' = \varepsilon_0\chi_e E_3' - e\varepsilon_0 T' E_0' - e\varepsilon_0 T' E_3', \end{array}\right\} \quad (10)$$

where it has been assumed that $e\triangle T \ll (1+\chi_e)$. Since we consider only two-dimensional disturbances, we introduce the stream function ψ'

$$u' = \frac{\partial\psi'}{\partial z}, \quad w' = -\frac{\partial\psi'}{\partial x}, \quad (11)$$

which satisfy the continuity equation (1) in the perturbed state. Introducing the perturbed electric potential Φ' through the relation $\overrightarrow{E'} = \nabla\Phi'$, eliminating the pressure p in Eq.(2), incorporating the quiescent state solution, and non-dimensionalizing the resulting equation, using the following scaling

$$(x^*, z^*) = \left(\frac{x}{h}, \frac{z}{h}\right), \quad t^* = \frac{\kappa}{h^2}, \quad \Psi^* = \frac{\psi'}{\kappa},$$

$$T^* = \frac{T'}{\triangle T}, \quad \Phi^* = \frac{(1+\chi_e)}{eE_0\triangle T h}\Phi',$$

we obtain the dimensionless form of the vorticity and heat transport equations as

$$\begin{aligned} &\tfrac{1}{Pr}\tfrac{\partial}{\partial t}(\nabla^2\Psi) = -(R+R_E)(1+gm)\tfrac{\partial T}{\partial x} + R_E\tfrac{\partial^2\Phi}{\partial x\partial z} \\ &+\nabla^4\Psi + R_E J(T, \tfrac{\partial\Phi}{\partial z}) + \tfrac{1}{Pr}J(\Psi, \nabla^2\Psi), \end{aligned} \quad (12)$$

$$\frac{\partial T}{\partial t} = -\frac{\partial\Psi}{\partial x} + \nabla^2 T + J(\Psi, T), \quad (13)$$

$$\nabla^2\Phi - \frac{\partial T}{\partial z} = 0, \quad (14)$$

where $gm = \frac{g'(t)}{g}$. g is the acceleration due to gravity, g' is the time-dependent gravity modulation due to the vibration of the Rayleigh-Bénard setup. ∇^2 is the Laplacian operator, the Prandtl number $Pr = \frac{\nu}{\kappa}$, the thermal Rayleigh number $R = \frac{\alpha g\Delta T h^3}{\nu\kappa}$ and the electric Rayleigh number $R_E = \frac{\varepsilon_0(eE_0\Delta Th)^2}{\mu_1\kappa(1+\chi_e)}$. In the above equations, the asterisks have been dropped for simplicity.

The boundary conditions to solve equation (12) - (14) are

$$\Psi = \frac{\partial^2\Psi}{\partial z^2} = T = \frac{\partial\Phi}{\partial z} = 0 \ \text{ at } \ z = 0, 1. \quad (15)$$

B. Non-Linear Theory

We now use the following asymptotic expansion in equations (12)-(14)

$$\left.\begin{array}{r} R = R_0 + \varepsilon^2 R_2 + \varepsilon^4 R_4 + ... \\ \Psi = \varepsilon\Psi_1 + \varepsilon^2\Psi_2 + \varepsilon^3\Psi_3 + ... \\ T = \varepsilon T_1 + \varepsilon^2 T_2 + \varepsilon^3 T_3 + ... \\ \Phi = \varepsilon\Phi_1 + \varepsilon^2\Phi_2 + \varepsilon^3\Phi_3 + ... \end{array}\right\}, \quad (16)$$

where R_0 is the critical value of the Rayleigh number at which stationary convection sets in when the gravity modulation is absent. We use the time variations only at the slow time scale $s = \varepsilon^2 t$ and gm(s) is taken as gm(s)=$\varepsilon^2\delta_2\cos(\Omega s)$.

At the lowest order we have

$$\begin{bmatrix} \nabla^4 & (R_0+R_E)\frac{\partial}{\partial x} & -R_E\frac{\partial^2}{\partial x\partial z} \\ \frac{\partial}{\partial x} & -\nabla^2 & 0 \\ 0 & -\frac{\partial}{\partial z} & \nabla^2 \end{bmatrix} \begin{bmatrix} \Psi_1 \\ T_1 \\ \Phi_1 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}. \quad (17)$$

The solution of the first order system is given by

$$\left.\begin{array}{l} \Psi_1 = A(s) \ \sin k_c x \ \sin\pi z \\ T_1 = B(s) \ \cos k_c x \ \sin\pi z \\ \Phi_1 = C(s) \ \cos k_c x \ \cos\pi z \end{array}\right\}, \quad (18)$$

where

$$A(s) = -\frac{\delta^2}{k_c}B(s), \quad C(s) = -\frac{\pi}{\delta^2}B(s), \quad \delta^2 = k_c^2 + \pi^2.$$

The system (17) gives us the critical Rayleigh numbers for stationary onset and their expression is given as

$$R_0 = \frac{\delta^6}{k_c^2} - \frac{R_E k_c^2}{\delta^2}. \quad (19)$$

At the second order, we have

$$\begin{bmatrix} \nabla^4 & (R_0+R_E)\frac{\partial}{\partial x} & -R_E\frac{\partial^2}{\partial x\partial z} \\ \frac{\partial}{\partial x} & -\nabla^2 & 0 \\ 0 & -\frac{\partial}{\partial z} & \nabla^2 \end{bmatrix} \begin{bmatrix} \Psi_2 \\ T_2 \\ \Phi_2 \end{bmatrix} = \begin{bmatrix} R_{21} \\ R_{22} \\ R_{23} \end{bmatrix}, \tag{20}$$

where

$$\left.\begin{array}{l} R_{21} = R_E\left[\frac{\partial T_1}{\partial x}\frac{\partial^2\Phi_1}{\partial z^2} - \frac{\partial T_1}{\partial z}\frac{\partial^2\Phi_1}{\partial x\partial z}\right] \\ +\frac{1}{Pr}\left[\frac{\partial\Psi_1}{\partial x}\frac{\partial}{\partial z}(\nabla^2\Psi_1) - \frac{\partial\Psi_1}{\partial z}\frac{\partial}{\partial x}(\nabla^2\Psi_1)\right] \\ R_{22} = \frac{\partial\Psi_1}{\partial x}\frac{\partial T_1}{\partial z} - \frac{\partial\Psi_1}{\partial z}\frac{\partial T_1}{\partial x} \\ R_{23} = 0 \end{array}\right\}. \tag{21}$$

The second order solution can be obtained as

$$\left.\begin{array}{l} \Psi_2 = 0 \\ T_2 = -\frac{\delta^2}{8\pi} \quad \sin 2\pi z \quad B^2 \\ \Phi_2 = 0 \end{array}\right\}. \tag{22}$$

The horizontally-averaged Nusselt number, Nu, for the stationary mode of convection (the preferred mode of convection in this problem) is given by

$$Nu(s) = \frac{\left[\frac{k_c}{2\pi}\int_{x=0}^{x=\frac{2\pi}{k_c}}(1-z+T_2)_z dx\right]_{z=0}}{\left[\frac{k_c}{2\pi}\int_{x=0}^{x=\frac{2\pi}{k_c}}(1-z)_z dx\right]_{z=0}}. \tag{23}$$

Substituting equation (22) in equation (23) and completing the integration, we get

$$Nu(s) = 1 + \frac{\delta^2}{4}B^2. \tag{24}$$

At the third order we have

$$\begin{bmatrix} \nabla^4 & (R_0+R_E)\frac{\partial}{\partial x} & -R_E\frac{\partial^2}{\partial x\partial z} \\ \frac{\partial}{\partial x} & -\nabla^2 & 0 \\ 0 & -\frac{\partial}{\partial z} & \nabla^2 \end{bmatrix} \begin{bmatrix} \Psi_3 \\ T_3 \\ \Phi_3 \end{bmatrix} = \begin{bmatrix} R_{31} \\ R_{32} \\ R_{33} \end{bmatrix}, \tag{25}$$

where

$$\left.\begin{array}{l} R_{31} = -\frac{1}{Pr}\frac{\partial}{\partial s}(\nabla^2\Psi_1) - R_0\left(\frac{R_2}{R_0} + \delta_2\cos(\Omega s)\right)\frac{\partial T_1}{\partial x} \\ +R_E\left[\frac{\partial T_1}{\partial x}\frac{\partial^2\Phi_2}{\partial z^2} - \frac{\partial T_1}{\partial z}\frac{\partial^2\Phi_2}{\partial x\partial z} - \frac{\partial T_2}{\partial z}\frac{\partial^2\Phi_1}{\partial x\partial z}\right] \\ R_{32} = \frac{\partial\Psi_1}{\partial x}\frac{\partial T_2}{\partial z} - \frac{\partial T_1}{\partial s} \\ R_{33} = 0 \end{array}\right\}. \tag{26}$$

Substituting Ψ_1, T_1, T_2, Φ_1 and Φ_2 from equations (18) and (22) into equations (26), we get

$$\left.\begin{array}{l} R_{31} = \frac{-\delta^4}{Prk_c}\frac{dB}{ds} - R_E\frac{k_c\pi^2}{4}\sin k_c x \ \sin\pi z \ \cos 2\pi z B^3 \\ +R_0\left(\frac{R_2}{R_0} + \delta_2\cos(\Omega s)\right)k_c\sin k_c x \ \sin\pi z B \\ R_{32} = \left[-\frac{dB}{ds} + \frac{\delta^4}{4}\cos 2\pi z B^3\right]\cos k_c x \ \sin\pi z \\ R_{33} = 0 \end{array}\right\}. \tag{27}$$

The adjoint system corresponding to the system (17) is

$$\left.\begin{array}{l} -(R_0+R_E)\frac{\partial\hat{T}_1}{\partial x} - R_E\frac{\partial^2\hat{\Phi}_1}{\partial x\partial z} - \nabla^4\hat{\Psi}_1 = 0 \\ -\frac{\partial\hat{\Psi}_1}{\partial x} - \nabla^2\hat{T}_1 = 0 \\ \nabla^2\hat{\Phi}_1 + \frac{\partial\hat{T}_1}{\partial z} = 0 \end{array}\right\}. \tag{28}$$

The eigenfunction of this adjoint system are given by

$$\left.\begin{array}{l} \hat{\Psi}_1 = -\frac{\delta^2}{k_c}\ B(s)\sin k_c x \ \sin\pi z \\ \hat{T}_1 = -B(s)\ \cos k_c x \ \sin\pi z \\ \hat{\Phi}_1 = -\frac{\pi}{\delta^2}B(s)\cos k_c x \ \cos\pi z \end{array}\right\}. \tag{29}$$

Substituting equations (27) and (29) in the solvability condition and completing the integration, we get on simplification the real Ginzburg-Landau equation for stationary instability in the form:

$$\left[\frac{\delta^6}{Prk_c^2} + (R_0+R_E)\right]\frac{dB}{ds} = -\left[(R_0+R_E)\frac{\delta^4}{8} - \frac{\delta^2\pi^2R_E}{8}\right]B^3 + R_0\left(\frac{R_2}{R_0} + \delta_2\cos(\Omega s)\right)\delta^2 B. \tag{30}$$

The solution of equation (30) subject to the initial condition $B(0) = a_0$, where a_0 is a chosen amplitude of convection is obtained by using classical fourth order Runge-Kutta method. In our calculations we have assumed $R_2 = R_0$ to keep the parameters to the minimum.

II. Results and discussion

Here we deal with the effect of time-periodic vertical oscillations on the heat transport in dielectric liquids. The effect of the time-periodic vertical oscillations comes through the amplitude of modulation (δ_2) and the effect of the applied electric field comes through the electrical Rayleigh number(R_E). It is well known from the linear theory that the effect of time-periodic vertical oscillations on convection is to delay the onset (see Siddheshwar and Abraham, 2007). From the linear theory we have obtained the values for critical Rayleigh number, R_c and critical wave number, k_c for different values of R_E and are documented in Table I. Further, since stationary convection is the preferred mode of onset in dielectric liquids, the Prandtl number has no effect on onset.

The weakly nonlinear theory gives us the Nusselt number. The variation of Nu with time 't' for various parameters is

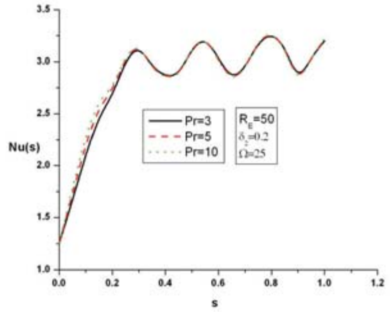

Fig. 2. Nusselt number Nu(s) vs slow time s for different values of Prandtl number and fixed value of electrical Rayleigh number R_E, amplitude of modulation δ_2.

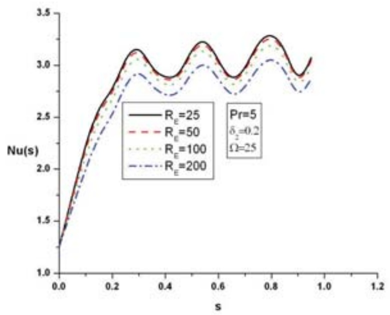

Fig. 3. Nusselt number Nu(s) vs s for different values of R_E.

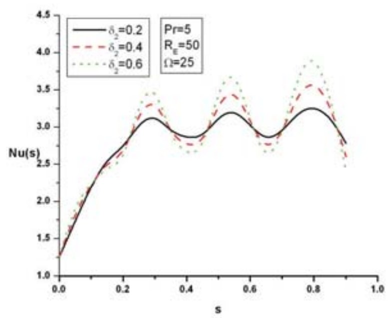

Fig. 4. Nusselt number Nu(s) vs s for different values of δ_2.

shown in figures 2-4. From these figures it is clear that the Nusselt number oscillate with time.

The solution of the Ginzburg-Landau equation gives us the amplitude of convection which helps in quantifying the Nusselt number. From the results of nonlinear stability analyses, we may conclude the following:

1) $Nu(Pr = 3) < Nu(Pr = 5)$.
   The Nusselt number vs s curves levels off after sometime. This result is seen when the amplitude of modulation is quite small.

2) $Nu(R_E = 0) > Nu(R_E \neq 0)$

3) $Nu|_{\delta_2=0.2} < Nu|_{\delta_2=0.4} < Nu|_{\delta_2=0.6}$

III. Conclusion

The effect of increasing electric Rayleigh number is to reduce the amount of heat transfer whereas increasing the amplitude of modulation results in increase in the amount of heat transport. Thus it is possible to regulate heat transfer with the help of time-periodic vertical oscillations and applied electric field.

TABLE I
Critical values of Rayleigh number and wave number for different values of R_E.

R_E	k_c	R_c
0	2.22	657.511
25	2.24	649.143
50	2.25	640.703
100	2.28	623.609
200	2.34	588.540

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