World Academy of Science, Engineering and Technology
International Journal of Bioengineering and Life Sciences
Vol:7, No:11, 2013

Examination of the Effect of Air Viscosity on Narrow Acoustic Tubes Using FEM Involving Complex Effective Density and Complex Bulk Modulus

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Abstract**—**Earphones and headphones, which are compact electro-acoustic transducers, tend to have a lot of acoustic absorption materials and porous materials known as dampers, which often have a large number of extremely small holes and narrow slits to inhibit the resonance of the vibrating system, because the air viscosity significantly affects the acoustic characteristics in such acoustic paths. In order to perform simulations using the finite element method (FEM), it is necessary to be aware of material characteristics such as the impedance and propagation constants of sound absorbing materials and porous materials. The transfer function is widely known as a measurement method for an acoustic tube with such physical properties, but literature describing the measurements at the upper limits of the audible range is yet to be found. The acoustic tube, which is a measurement instrument, must be made narrow, and the distance between the two sets of microphones must be shortened in order to take measurements of acoustic characteristics at higher frequencies. When such a tube is made narrow, however, the characteristic impedance has been observed to become lower than the impedance of air. This paper considers the cause of this phenomenon to be the effect of the air viscosity and describes an FEM analysis of an acoustic tube considering air viscosity to compare to the theoretical formula by including the effect of air viscosity in the theoretical formula for an acoustic tube.

Keywords**—**Acoustic tube, air viscosity, earphones, FEM, porous materials, sound absorbing materials, transfer function method.

I. Introduction

COMPACT acoustic transducers such as headphones, earphones, and speakers built into cellular phones use porous materials known as dampers to inhibit the resonance of the vibrating system to regulate the acoustic characteristics. These dampers often have a large number of extremely small holes and narrow slits which significantly affect the characteristics of the air viscosity in such acoustic paths. Computer-aided engineering (CAE) technology is currently used widely even in the acoustic field, as the processing speed of computers have accelerated and their capacities expanded.

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However, the subjects of analysis are mainly objects of larger dimensions such as building structures, and there are few cases that involve the analysis of acoustic propagation characteristics of compact spaces such as earphones or headphones at the upper limits of the audible range (20 kHz).

It is necessary to take into consideration the effects of porous materials when conducting simulations. In order to incorporate the effects of porous materials in simulations using methods such as the finite element method (FEM), the physical properties of the damping material, characteristic impedance, and propagation constant are required. The transfer function method [1] using an acoustic tube is available for taking measurements of characteristic impedance or propagation coefficients of porous materials, but the acoustic tubes available in the market have a measurable frequency range that only extends from approximately 200 Hz to 6 kHz. Therefore, such acoustic tubes cannot be used to take measurements further up the audible range, which extends to 20 kHz frequency. No previous publication examines the physical properties of damping materials up to 20 kHz.

It is necessary to narrow the diameter of the acoustic tube and to shorten the distance between the two microphones in order to take measurements of material properties at high frequencies. The effect of viscosity between air and the tube wall, along which the wave travels, can no longer be ignored as the tube is narrowed down. This paper reports on the effect of air viscosity inside acoustic tubes, analyzed through FEM using a complex effective density and complex bulk modulus. The analysis results are then examined by comparing them with the results of calculations using theoretical formula.

II. Three-Dimensional Closed Acoustic Field with Consideration of Complex Effective Density and Complex Bulk Modulus

The acoustic field of the narrow tube is discretized with three-dimensional finite elements.

The equation of motion for an ideal non-viscous compressed fluid subject to harmonic vibration of minute amplitudes is expressed as:

$$-\text{grad}\, p = -\rho\omega^2 \{U\} \tag{1}$$

Furthermore, the continuity equation is:

$$p = -E\ \text{div}\{U\} \tag{2}$$

where p is the pressure, $\{U\}$ is the particle displacement vector, ω is the angular frequency, ρ is the effective density, and E is the bulk modulus.

The relationship between the acoustic pressure p inside the element and the acoustic pressure $\{P_e\}$ of the nodal point can be approximated in the following manner using an appropriate interpolation function N_i, (i = 1,2,3…):

$$p = \left[N \right]^T \{ p_e \} \tag{3}$$

where $[N]^T = [N_1, N_2, N_3, \ldots]$, and the superscript T represents the transposition.

The kinetic energy, strain energy, and potential energy are obtained from (1), (2), and (3), and the following equations are obtained using the principle of minimum energy:

$$\left(\left[K \right]_e - \omega^2 \left[M \right]_e \right) \{ p_e \} = -\omega^2 \{ u_e \} \tag{4}$$

$$\left[K \right]_e = \left(1/\rho_e \right) \left[\tilde{K} \right]_e \tag{5}$$

$$\left[M \right]_e = \left(1/E_e \right) \left[\tilde{M} \right]_e \tag{6}$$

where $\{u_e\}$ is the nodal point displacement vector of the e^{th} element in (4), $[K]_e$ is the stiffness matrix of the element, and $[M]_e$ is the mass matrix of the element. $[K]_e$ and $[M]_e$ are, respectively, the interpolation function and the matrix comprised of its derivatives, while the matrix components of rows i and columns j, M_{eij} and K_{eij}, are expressed as:

$$\begin{aligned} \tilde{K}_{eij} = \iiint & \left(\partial N_i / \partial x \right) \left(\partial N_j / \partial x \right) + \left(\partial N_i / \partial y \right) \left(\partial N_j / \partial y \right) \\ & + \left(\partial N_i / \partial z \right) \left(\partial N_j / \partial z \right) dxdydz \end{aligned} \tag{7}$$

$$M_{eij} = \iiint N_i N_j dxdydz \tag{8}$$

The following model, along with the effective density and the bulk modulus expressed as complex numbers, has been proposed and examined for considering the acoustic field inside porous materials. However, for the cases of narrow tubes, the frequency-dependent effective density and bulk modulus are used. The complex effective density and the complex bulk modulus are given by:

$$\rho_e \Rightarrow \rho_e^* = \rho_{eR} + j\rho_{eI} \tag{9}$$

$$E_e \Rightarrow E_e^* = E_{eR} + jE_{eI} \tag{10}$$

Furthermore, the imaginary part of the effective density ρ_{eI} is a parameter for flow resistance. The imaginary part of the bulk modulus E_{eI} expresses the hysteresis of the relationship between the pressure p and the volumetric strain div$\{U\}$.

This is substituted into (9) and (5) to obtain the stiffness matrix of the element, $[K]_e$ as:

$$\left[K \right]_e = \left[K_R \right]_e \left(1 + j\eta_e \right) \tag{11}$$

However,

$$\left[K_R \right]_e = \left(\rho_{eR} / \left(\rho_{eR}^2 + \rho_{eI}^2 \right) \right) \left[\tilde{K} \right]_e ,$$

$$\eta_e = -\rho_{eI} / \rho_{eR} , \tag{12}$$

where $[K_R]_e$ is the real part of $[K]_e$.

This is substituted into (10) and (6) to obtain the mass matrix of the element, $[M]_e$.

$$\left[M \right]_e = \left[M_R \right]_e \left(1 + j\chi_e \right) \tag{13}$$

However,

$$\left[M_R \right]_e = \left(E_{eR} / \left(E_{eR}^2 + E_{eI}^2 \right) \right) \left[\tilde{M} \right]_e ,$$

$$\chi_e = -E_{eI} / E_{eR} \tag{14}$$

where $[M_R]_e$ is the real part of $[M]_e$.

The following discretization formula is obtained for the entire system by superposing all elements for the field that applies to (4)-(14) [2], [3]:

$$\sum_{e=1}^{e\max} \left(\left[K_R \right]_e \left(1 + j\eta_e \right) - \omega^2 \left[M_R \right]_e \left(1 + j\chi_e \right) \right) \{ p_e \} = \omega^2 \{ u \} \tag{15}$$

In (15), $[K_R]_e$, $[M_R]_e$, and $\{p_e\}$ have been rewritten to ensure that the matrix size is the same as the number of degrees of freedom for the entire system. The nodal particle displacement vector for the entire system is $\{u\}$. Equation (15) is a simultaneous linear equation with a complex coefficient, and it provides the nodal acoustic pressure $\{p_e\}$ on inserting ω and $\{u\}$ as known quantities. The frequency response of the narrow tube model was obtained from (15) in this analysis.

III. Theoretical Equation for an Acoustic Tube with Consideration of Air Viscosity

It is possible to derive the acoustic pressure inside an acoustic tube using the wave motion equation. The following can be derived from the equation for a closed acoustic tube [4]:

$$P = j\rho_0 c_0 u_0 e^{j\omega t} \frac{\cos k \left(l - x \right)}{\sin kl} \tag{16}$$

where P is the sound pressure, ρ_0 is the density of air, c_0 is the sound speed in air, ω is the angular frequency, u_0 is the velocity amplitude of the plane of vibration, k is the wave number, l is the overall length of the acoustic tube, and x is the distance between the tip of the tube and a microphone. This equation is for an ideal condition and the viscosity of air is neglected in its derivation.

In order to take the air viscosity into consideration, it is necessary to derive the complex effective density, the complex

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sound speed, and the complex bulk modulus.

First, in order to derive the complex effective density, the particle velocity distribution inside the tube must be derived using the following equation for the complex effective density [5], [6]:

$$\rho^* = \rho_0 + \frac{2\mu}{j\omega a^2}\cdot\frac{\sigma T(\sigma)}{1-\frac{2}{j\sigma}T(\sigma)}, \tag{17}$$

where ρ^* is the complex effective density, μ is the air viscosity, and a is the radius of the tube. Furthermore, σ and $T(\sigma)$ are:

$$\sigma = a\left(\frac{\omega}{\nu}\right)^{\frac{1}{2}} \tag{18}$$

$$T(\sigma) = \frac{J_0\left(j\sqrt{j\sigma}\right)}{J_0'\left(j\sqrt{j\sigma}\right)}, \tag{19}$$

and J_0' is obtained by deriving J_0 with the Bessel function σ.

The bulk modulus is derived subsequently and expressed in the following manner [7]:

$$K^* = \frac{\gamma P_0}{1+(\gamma-1)\dfrac{2J_1\left(Bs\sqrt{-j}\right)}{Bs\sqrt{-j}J_0\left(Bs\sqrt{-j}\right)}}, \tag{20}$$

where K^* is the complex bulk modulus, P_0 is the atmospheric pressure, and γ is the specific heat ratio. Furthermore, B and s are given by:

$$B = \left(\frac{C_P\mu}{\kappa}\right)^{\frac{1}{2}}, \tag{21}$$

$$s = \left(\frac{\omega\rho_0 a^2}{\mu}\right)^{\frac{1}{2}}, \tag{22}$$

where C_P is the constant-pressure specific heat and κ is the coefficient of thermal conductivity.

The complex sound speed is expressed using the following equation based on the complex effective density from (17) and complex bulk modulus from (20):

$$c^* = \sqrt{\frac{\rho^*}{K^*}}. \tag{23}$$

Furthermore, the complex wave number k^* is:

$$k^* = \sqrt{\frac{\omega}{c^*}}. \tag{24}$$

The following equation is derived by substituting the complex sound speed c^*, complex effective density ρ^*, and complex wave number k^* for, respectively, the sound speed c_0, air density ρ_0, and wave number k from (16), which is a theoretical equation for the sound pressure inside the acoustic tube, by using (17), (23), and (24) :

$$P^* = j\rho^* c^* u_0 e^{j\omega t}\frac{\cos k^*(l-x)}{\sin k^* l}. \tag{25}$$

IV. FEM Simulation

The FEM simulation was performed using (15) for determining the physical properties of air inside the acoustic tube and to define the frequency-dependent complex effective density and complex bulk modulus with consideration of air viscosity.

In order to reduce the amount of calculations, a quarter-scale model of the circular tube was used, as shown in Fig. 1. In consideration of future experiments, the adopted diameters for the tubes were 14mm and 8mm. A thin straight tube of 1 mm, considered to show a significant air viscosity effect, was also analyzed. The length of the tubes was 250mm.

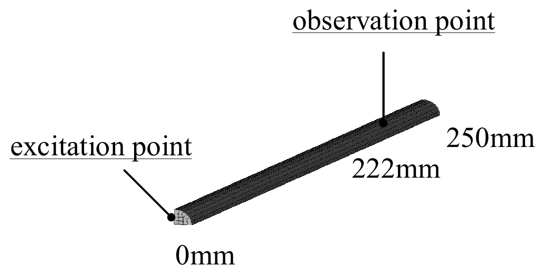
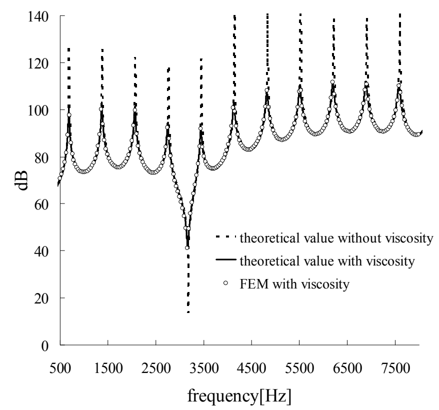
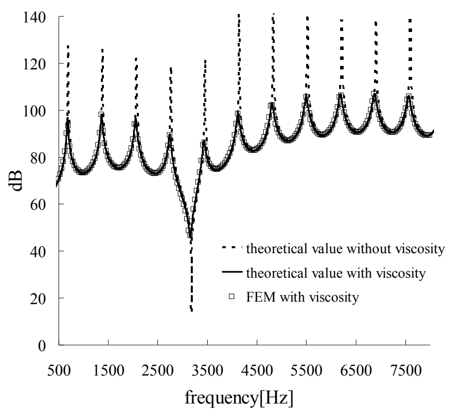


Fig. 1 The FEM model

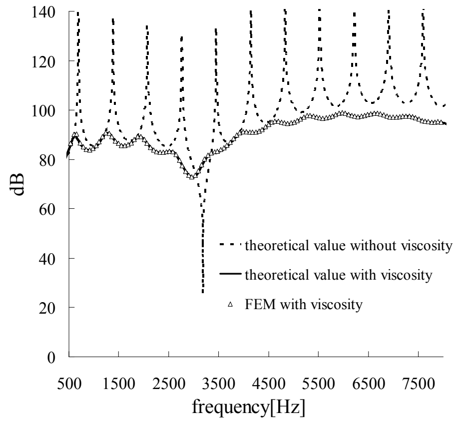
The comparison of the calculated values derived from the theoretical equation presented in (25) and the results of the FEM simulation is shown in Fig. 2. Furthermore, the graph also features a plot of the theoretical equation using the ordinary sound speed and density of air.



(a)



(b)



(c)

Fig. 2 Results of the theoretical calculation and FEM simulation for different tube diameters (a) 14mm diameter, (b) 8mm diameter, (c) 1mm diameter

The observations were performed at the position l = 222mm. First, a comparison of the theoretical equation with FEM reveals that the theoretical equation, which uses the ordinary sound speed without viscosity and density, has extremely acute resonance with the same plot, regardless of diameter. When the complex sound speed and the complex effective density are considered, the acuteness of the peak of the resonance point was inhibited with the narrowing of the diameter of the tube because of the air viscosity effect. The results of the FEM simulation revealed that the outcomes were similar to the theoretical values that consider the complex sound speed and complex effective density. Furthermore, a graph with a different diameter at one location of resonance is featured in Fig. 3 in order to observe the effect of viscosity clearly.

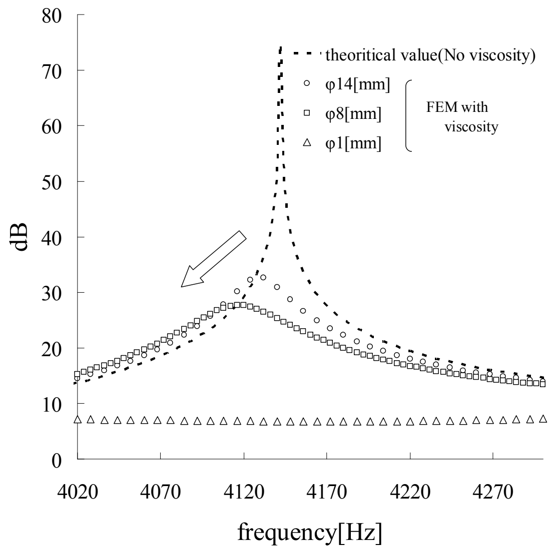
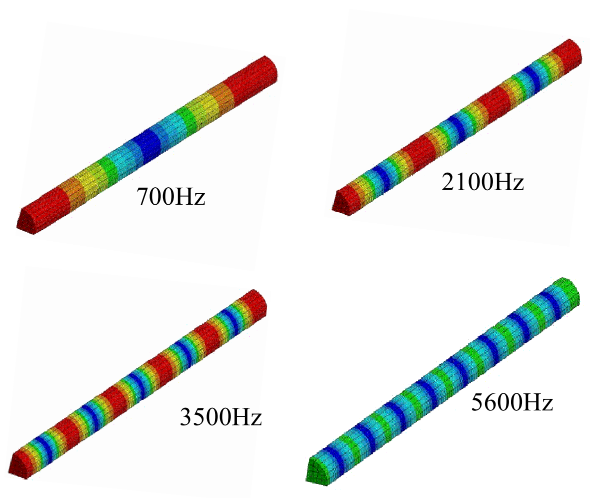


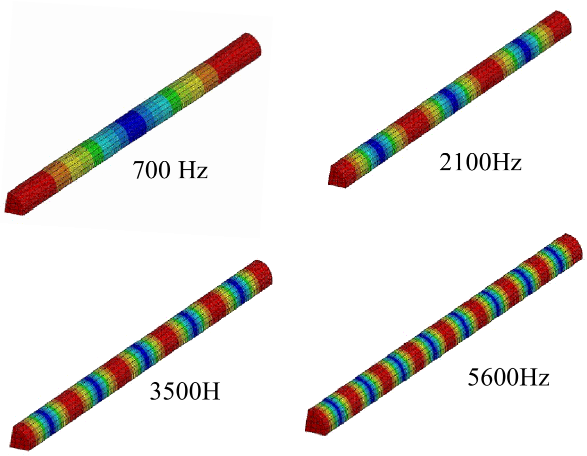
Fig. 3 Comparison of the results of the theoretical calculation and the FEM simulation for different tube diameters

The acuteness of the resonance point was inhibited as the diameter of the tube was narrowed, leading to even lower resonance frequencies, as shown in Fig. 3.

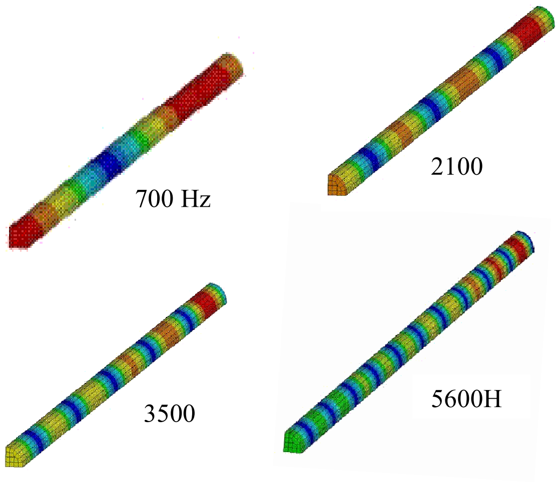
A number of contour diagrams for a number of pressure distributions |P| of resonance points are featured in Fig. 4. The loops and nodes of many modes are revealed inside the tube with the rise in the frequency, expressing the acoustic distribution inside the tube. In cases where the diameter of the tube was large, the contour is not significantly represented, but when the diameter of the tube is 1mm, the contour of the loops and nodes of sound pressure |P| take on a paler color as the distance from the sound source increases, indicating that damping occurs because of the air viscosity effect. Furthermore, in terms of pressure, the same pressure is distributed across the uniform sections, resulting in planar waves.



(a)



(b)



(c)

Fig. 4 Contour diagrams of pressure distributions from the FEM simulation results (a) 14mm diameter, (b) 8mm diameter, (c) 1mm diameter

V. Conclusion

The complex effective density and the complex bulk modulus were used to determine the effect of the air viscosity inside acoustic tubes. Results from the FEM and theoretical equation were compared and examined.

The FEM, used with the consideration of air viscosity and the use of the complex effective density and complex bulk modulus obtained by M. A. Biot (1992) [5], was compared with the corresponding theoretical solutions.

When the air viscosity was taken into consideration in the theoretical calculation, the acuteness of the resonance point was inhibited and the resonance frequency was decreased as the diameter of the tube became thinner. The FEM with consideration of air viscosity matched the theoretical values qualitatively as well as quantitatively. This made it possible to show the validity of the FEM simulation with consideration of the air viscosity.

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