Acoustic Finite Element Analysis of a Slit Model with Consideration of Air Viscosity

M. Sasajima, M. Watanabe, T. Yamaguchi, Y. Kurosawa, Y. Koike

Abstract—In very narrow pathways, the speed of sound propagation and the phase of sound waves change due to the air viscosity. We have developed a new finite element method (FEM) that includes the effects of air viscosity for modeling a narrow sound pathway. This method is developed as an extension of the existing FEM for porous sound-absorbing materials. The numerical calculation results for several three-dimensional slit models using the proposed FEM are validated against existing calculation methods.

Keywords—Simulation, FEM, air viscosity, slit.

I. INTRODUCTION

Computer-aided engineering has been used extensively in recent years for acoustic analyses. However, the conventional analysis approach is used predominantly for relatively large structures or large equipment. For example, for a structure with a small volume like a portable electronic device, very few methods of sound propagation analysis are available. The viscosity of air in these narrow pathways results in damping. Consequently, the speed of sound propagation decreases, and a phase delay occurs. Therefore, to carry out accurate acoustic analysis for small electronic devices, we must consider the effect of the air viscosity. This effect is not considered in conventional acoustic analysis. In the present study, we developed a new finite element method (FEM) that includes the effects of air viscosity in narrow places in the sound pathway of small electronic devices. This has been developed as an extension of the acoustic FEM proposed by Yamaguchi [1], [2] for a porous sound-absorbing material. We attempted numerical analysis in the frequency domain with our acoustic solver that uses the proposed FEM [3]-[5].

II. NUMERICAL PROCEDURES

We have developed a new FEM that incorporates air viscosity at small amplitudes. Fig. 1 shows the direct Cartesian coordinate system and a constant strain element of a three-dimensional (3D) tetrahedron. Here, \( u_x, u_y, \) and \( u_z \) are the displacements in the \( x, y, \) and \( z \) directions at arbitrary points in the element. The strain energy \( U \) can be expressed as follows:

\[
U = \frac{1}{2} E \iiint \left( \frac{\partial u_x}{\partial x} + \frac{\partial u_y}{\partial y} + \frac{\partial u_z}{\partial z} \right)^2 dx dy dz \quad (1)
\]

where \( E \) is the bulk modulus of elasticity of the air. The time derivative of the particle displacement is expressed as \( \dot{u} \). Therefore, the kinetic energy \( T \) can be expressed as follows:

\[
T = \frac{1}{2} \iiint \rho \dot{u} \dot{u}^T dx dy dz \quad (2)
\]

where \( \rho \) is the effective density of the element, and \( \dot{u} \) represents a transposition. The viscosity energy \( D \) of a viscous fluid can be expressed as follows:

\[
D = \iiint \frac{1}{2} \{ \dot{\tau} \} \{ \dot{\tau} \}^T dx dy dz \quad (3)
\]

where \( \{ \dot{\tau} \} \) is the stress vector attributable to viscosity. The relationship between the particle velocity and the stress can be expressed as follows:
$$\mathbf{\epsilon} = \begin{bmatrix} \frac{4}{3} & 0 & -\frac{2}{3} & 0 & \frac{2}{3} \\ 0 & \frac{2}{3} & 0 & -\frac{2}{3} & 0 \\ -\frac{2}{3} & 0 & \frac{2}{3} & 0 & -\frac{2}{3} \\ 0 & -\frac{2}{3} & 0 & \frac{2}{3} & 0 \\ \frac{2}{3} & 0 & -\frac{2}{3} & 0 & \frac{2}{3} \end{bmatrix}$$

where \( u_x, u_y, \) and \( u_z \) are the particle velocities in the \( x, y, \) and \( z \) directions, respectively, at arbitrary points in the element, and \( \mu \) is the coefficient of viscosity of the air. In the above equation, \( \mathbf{\epsilon} \) is the strain vector. The relationship between the particle velocity and the strain can be expressed by the constant strain element of a 3D tetrahedron as shown in Fig. 2.

![Fig. 2 Relationship between particle velocity and strain](image)

$$\mathbf{\epsilon} = \begin{bmatrix} \frac{4}{3} \partial \xi/\partial x & 0 & -\frac{2}{3} \partial \xi/\partial y & 0 & \frac{2}{3} \partial \xi/\partial z \\ 0 & \frac{2}{3} \partial \xi/\partial x & 0 & -\frac{2}{3} \partial \xi/\partial y & 0 \\ -\frac{2}{3} \partial \xi/\partial x & 0 & \frac{2}{3} \partial \xi/\partial y & 0 & -\frac{2}{3} \partial \xi/\partial z \\ 0 & -\frac{2}{3} \partial \xi/\partial x & 0 & \frac{2}{3} \partial \xi/\partial y & 0 \\ \frac{2}{3} \partial \xi/\partial x & 0 & -\frac{2}{3} \partial \xi/\partial y & 0 & \frac{2}{3} \partial \xi/\partial z \end{bmatrix}$$

We can obtain the following discretized equation of an element by using Lagrange’s equations:

$$\frac{d}{dt} \{ \mathbf{\dot{u}}_e \} = -\mathbf{K}_e \{ \mathbf{u}_e \} + j \omega \mathbf{C}_e \{ \mathbf{u}_e \} + \mathbf{f}_e$$

where \( \mathbf{u}_e \) is the \( i^{th} \) component of the nodal displacement vector \( \{ \mathbf{u}_e \} \), and \( \mathbf{f}_e \) is the \( i^{th} \) component of the nodal particle velocity vector \( \{ \mathbf{\dot{u}}_e \} \). We can obtain the following discretized equation of an element by substituting (1)–(7) into (9):

$$-\omega^2 \{ \mathbf{M}_e \} \{ \mathbf{u}_e \} + \{ \mathbf{K}_e \} \{ \mathbf{u}_e \} + j \omega \{ \mathbf{C}_e \} \{ \mathbf{u}_e \} = \{ \mathbf{f}_e \}$$

We use \( \{ \mathbf{\dot{u}}_e \} = j \omega \{ \mathbf{u}_e \} \) in this equation because a periodic motion having angular frequency \( \omega \) is assumed. \( \{ \mathbf{M}_e \}, \{ \mathbf{K}_e \}, \{ \mathbf{C}_e \}, \) and \( \{ \mathbf{f}_e \} \) are the element mass matrix, element stiffness matrix, element viscosity matrix, and nodal force vector, respectively.

### III. Calculation

#### A. Damping Analysis by the Three-Dimensional Finite Element Method

To verify our method, we carried out an acoustic damping analysis for slits using 3D FEM. As shown in Fig. 3, this model is a 1/4 solid model symmetrical about the \( x-z \) plane and the \( x-y \) plane. The width of the model was 2.0mm, the height was 0.5mm, and the length was 16.6mm.
This model used 3D tetrahedral elements having four nodes. There were 39 divided elements in the length (x) direction, 10 layers in the height (y) direction and 5 layers in the width (z) direction. Both ends of this model were closed. We selected the effective density \( \rho_R = 1.2 \text{ kg/m}^3 \), the coefficient of viscosity \( \mu = 1.82 \times 10^{-5} \text{ N s/m}^2 \), the real part of the complex volume elasticity \( E_R = 1.4 \times 10^9 \text{ Pa} \), and the sound propagation speed \( c = 340 \text{ m/s} \) in air. As the boundary conditions, the particle displacements of all nodes on the outside in contact with surfaces were fixed, except for the plane of symmetry and the side walls which were not defined in the theoretical analysis. Fig. 4 shows the contours of the calculated particle displacements and the isosurface view of the model for the proposed finite element method, near the resonance conditions (10,200 Hz and 20,400 Hz). As can be seen, the magnitude of the displacement of the particles changes significantly near the contact surface. However, the displacement becomes flatter with distance from the contact surface.

**B. Damping Analysis by Theoretical Analysis**

We have carried out theoretical analysis of the resonant response of the slit to verify the proposed FEM. The frequency response of the pressure can be expressed by the following general expression [6]:

\[
P = -j \rho c v_0 e^{jw} \frac{\cos(k(x-l))}{\sin kl} \tag{11}
\]

where \( \rho \) is density of the air, \( c \) is the speed of sound, \( l \) is the length of the slit, \( x \) is the position of a reference point, \( k = \omega/c \), \( v_0 \) is an excitation velocity, and \( t \) is time.

In this equation, we introduce the complex sound speed \( c^* \) and the complex effective density \( \rho_c^* \) to include the attenuation due to the viscosity of the air. We replace the speed of sound and the density with the complex sound speed and the complex effective density as shown below.

\[
\rho \Rightarrow \rho_c^* \tag{12}
\]

\[
c \Rightarrow c^* \tag{13}
\]

Using the two substitutions above in (11) and the slit model is assumed infinite width, as shown in Fig. 5 the effective density can be expressed as follow [7]:

\[
\rho_c^* = \frac{\rho_0}{1 - \tanh(s'j/2)} \tag{14}
\]

where \( \rho_0 \) is mass density, \( s' \) and \( c^* \) are expressed as follow.
\[
    s' = \sqrt{\frac{\omega \rho a^2}{\mu}} \\
    c' = \sqrt{\frac{\kappa}{\rho v}} = \sqrt{\frac{\rho v}{\rho v}} 
\]

where \(a\) is the distance between the contact surfaces, \(\kappa\) is the bulk modulus, \(p_0\) is atmospheric pressure, and \(\gamma\) is the specific heat at constant volume. For the case of \(a = 0.5\)mm, the complex density and complex sound velocity are shown in Figs. 6 and 7, respectively. By calculating the frequency response with (11) using the values of these parameters, the theoretical solution that includes the viscosity is obtained.

![Fig. 6 Effective density \(\rho_c'\) in the slit model (\(a = 0.5\)mm)](image)

![Fig. 7 Sound speed \(c'\) in the slit model (\(a = 0.5\)mm)](image)

**C. Verification and Comparison of the Proposed Method**

We have analyzed frequency responses using the proposed FEM, and compared them with the above-described theoretical method that includes the viscosity and with the conventional FEM that does not include the attenuation.

![Fig. 8 Pressure versus frequency response for a slit model](image)

Fig. 8 shows the comparison of the analysis results for models of \(a = 0.8, 0.5, 0.25\)mm. The condition of excitation
was constant displacement excitation. From Fig. 8, we
determined the effect of damping on the calculated results by
using the proposed FEM and the theoretical method. The
conventional FEM does not show attenuation of the resonance
peaks. As can be seen from Fig. 8, when the slit is narrow, the
resonance peak decreases. That is, the attenuation increases
when the slit width is narrowed. In addition, it can be seen that
over the entire frequency domain, the calculated results of the
proposed FEM and the theoretical method are approximately
equal.

IV. CONCLUSION

We developed a new acoustic FEM that considers the effects
of damping by the viscosity of air. We compared calculation
results for sound pressure versus frequency characteristics
using the proposed method with that of the theoretical method,
and the conventional acoustic FEM without viscosity of air for
slit models. The comparison showed that the calculated results
are very close. Therefore, proposed new acoustic FEM was
confirmed that have good analytic accuracy. We plan to
consider the possibility of applying this analysis to various
shapes in the future.

REFERENCES

Sound Absorbing Materials in 3D Rooms Using Damping Contributions
with Eigenvectors as Weight Coefficients,” Journal of System Design and
Analysis Using Finite Element Method with Approximated Modal
Damping for Automotive Double Walls with a Porous Material,” Journal
Element Method Considering Effects of Damping Caused by Air
Normal Sound Absorption Coefficient for Multi-Layer Sound Absorbing
Materials by Using the Boundary Element Method,” Transactions of
Japan Society of Mechanical Engineers, Vol. 56-532C, pp. 3248–3252,
1990.
Acoustic Analysis with Consideration of Damping Effects of Air
Viscosity in Sound Pathway,” World Academy of science, Engineering