Study on the Heat Transfer Performance of the Annular Fin under Condensing Conditions
Abdenour Bourabaa, Malika Fekih, Mohamed Saighi

Abstract—A numerical investigation of the fin efficiency and temperature distribution of an annular fin under dehumidification has been presented in this paper. The non-homogeneous second order differential equation that describes the temperature distribution from the fin base to the fin tip has been solved using the central finite difference method. The effects of variations in parameters including relative humidity, air temperature, air face velocity on temperature distribution and fin efficiency are investigated and compared with those under fully dry fin conditions. Also, the effect of fin pitch on the dimensionless temperature has been studied.

Keywords—Annular fin, Dehumidification, Fin efficiency, Heat and mass transfer, Wet fin.

I. INTRODUCTION

EXTENDED surfaces or fins are frequently designed to add additional surface area in order to enhance heat transfer between the surface and the ambient air. They can be used in variety applications such as in air conditioning systems, condensers and evaporators. In such application as evaporators, the surface temperature is generally below the dew point temperature of the incoming air. As a result, combined heat and mass transfer occurs over the cooled surface. In general, the fin performance is usually expressed in terms of heat transfer coefficient and fin efficiency. In the case when dehumidification of moist air occurs, the temperature over the fin surface changes simultaneously with the humidity ratio. This makes the wet fin performance study more complicated. In the other word, the study of the performance of the wet fin is considerably different from the study of the same fin under dry condition. Under dry fin conditions, Brown [1] derived an equation to determine the optimum dimensions of uniform annular fins. His second order differential equation was solved by the use of the Bessel functions. This approach has been later used by Ullmann and Kalman [2] in which the second order differential equation has been solved to show the efficiency and optimized fin dimensions. Under dehumidification, the results from Coney et al [3] show that condensation has a substantial influence on the fin performance. The importance of both heat and mass transfer on the fin performance of different geometries are presented in a large number of papers [4]-[13]. Kazeminejad et al. [4] carried out a numerical study to analyze the conjugate convection and conduction heat transfer characteristics of humid air flow over a vertical rectangular fin. The effect of various psychrometric and flow conditions on the fin efficiency has been discussed.

Theoretical analysis of the rectangular fin performance with combined heat and mass transfer has been carried out by Sunden [14]. Under dehumidification, a relation between the local changes in temperature with humidity ratio is necessary in order to determine fin efficiency. The temperature profile along the fin surface is governed by non-homogeneous second order differential equation. Several studies have been performed to solve this equation. For instance, Coney et al. [3], studied numerically the fin temperature distribution, the condensate film thickness and the fin effectiveness for a vertical fin in a laminar humid air cross flow. The value of the humidity ratio is related to temperature by a second-degree polynomial relationship. Chen [15] proposed a model to analyze the fin performance with simultaneous heat and mass transfer. He used the same humidity ratio/temperature relationship as Coney et al. [3]. Elmahdy and Biggs [16] presented an algorithm to determine the efficiency of longitudinal and circular fins with a uniform thickness. They proposed a linear relationship between the specific humidity of the saturated air on the fin surface and its corresponding temperature. Their results showed a decrease in fin efficiency and also temperature distribution with the increase of relative humidity. The temperature distribution over a circular partially wet or totally wet fin has been investigated by Liang et al. [17]. They used a third-degree polynomial correlation for the relationship between the specific humidity and the fin surface temperature. Sharqawy and Zubair [18] studied analytically a wet annular fin in which they proposed another linear relationship between the humidity ratio and its corresponding temperature. They suggest that the maximum temperature at the fin tip for wet surface condition is the dew point temperature of the air stream. Naphon [19] carried out numerical analysis to investigate the performance of an annular fin under totally dry, totally wet and partially wet fin conditions. He used a third degree polynomial developed by Liang et al. [17] to express the relationship between the saturated specific humidity on the fin surface and its corresponding temperature. Liang et al. [20] used also the correlation proposed by Liang et al. [17] to present their one-dimensional and two-dimensional fin efficiency of a plate fin-and-tube heat exchanger under condensing condition. The non-homogeneous second order differential equation that describes the temperature distribution from the fin tip to the

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fin base has been solved by Naphon [19] using the central finite difference scheme. Kazeminejad [21] studied a rectangular fin assembly under dehumidification. When only sensible heat transfer occurs Kazeminejad developed a solution to the differential equations that describe the temperature distribution. However, when dehumidification of moist air occurs, he solved these equations numerically by the shooting method which combines the Runge-Kutta method and the Newton-Raphson iteration. Salah El-Din [22] studied analytically a rectangular fin assembly under fully wet and partially wet fin surface conditions. He assumed also that the saturated humidity ratio varies linearly with the local fin surface temperature. Rosario and Rahman [23] studied the wet radial fin assembly. The effects of the cold fluid and air temperatures, relative humidity, fin thermal conductivity and also the effect of the fin assembly dimensions on the temperature distribution has been studied. His model can be used in prediction of partially wet fin assembly.

In the present work both the temperature distribution and the fin efficiency over a circular fin surface are investigated. Following the same procedure presented by Bourabaa et al. [24] for rectangular wet fin, the second order differential formulation that describes the temperature distribution over circular fin has been simplified by avoiding the use of Bessel functions.

II. MATHEMATICAL PROCEDURE

A. Temperature Distribution-Analytical Method

When the surface is dry the driving potential for heat transfer is simply the surface to air temperature difference. However, when the surface is wet the heat rate transferred to the surface is governed by the dual driving potentials of temperature and water vapor concentration. It is shown from Fig. 1 that a heat balance on an element of a circular fin surface leads to the following expression:

\[ dQ = h_s dA \left( T_a - T_f \right) + \frac{h_s c_{pa}}{h_t} \left( w_a - w_f \right) \]

Equation (1), therefore, can be rewritten as

\[ dQ = h_s dA \left( 1 + \frac{i_{fg}}{c_{pa} Le} \right) \left( T_a - T_f \right) \]

The effect of the presence of moisture on the fin surface can be described by the second term on the right side of (3). The heat transfer equation cannot be expressed in term of the air to surface temperature difference alone. A humidity ratio difference also appears in this equation. Therefore, the humidity ratio of saturated air and its corresponding temperature at any location on the fin surface are required. To do this, it is necessary to solve the following second order differential equation that derived from a simple energy balance on a differential element of a circular fin of uniform thickness:

\[ \frac{d^2 T_r}{dr^2} + \frac{2h_t}{k_t} \left( T_a - T_r \right) + \frac{i_{fg}}{c_{pa} Le} \left( w_a - w_f \right) = 0 \]

The humidity ratio of saturated air at fin temperature can be calculated from the correlation by Sharqawy and Zubair [18]

\[ w_s = a_2 + b_2 T_r \]

where

\[ a_2 = w_{sb} - \frac{w_{sb} - w_{db}}{T_{dp} - T_b} \]
\[ b_2 = \frac{w_{sb} - w_{db}}{T_{dp} - T_b} \]

Here, \( T_b \) and \( T_{dp} \) are the base and the dew point temperatures; respectively, \( w_{sb} \) and \( w_{sdb} \) are the humidity ratios of saturated air evaluated at the base temperature and at the dew point temperature, respectively.

The one-dimensional temperature distribution described by (4) can be transformed to the following equation:

\[ \frac{d^2 \theta}{dr^2} + \frac{1}{r} \frac{d \theta}{dr} - m^2 \theta = m^2 BC_o \]

where

\[ \theta = \frac{T_b - T_r}{T_a - T_b} \]
\[ B = \frac{I_o}{c_w \rho L_e} \]  \quad \text{(10)}

\[ m_o^2 = \frac{b_o}{k_t} \]  \quad \text{(11)}

\[ m^2 = m_o^2 (1 + Bb_o) \]  \quad \text{(12)}

\[ C_o = \frac{w_o - a_o - b_o T_a}{T_o - T_b} \]  \quad \text{(13)}

In the above equations, \( k \) “W m\(^{-1}\) K\(^{-1}\)” and \( t \) “m” are the fin material conductivity and the fin thickness, respectively.

Assuming that the heat transfer through the fin tip is negligible, the following boundary conditions are used to solve the non-homogeneous second order differential equation.

at the fin base \( r = r_o \); \( \theta = \theta_b \) \quad \text{(14)}

at the Fin tip \( r = r_e \); \( \frac{d\theta}{dr} = 0 \) \quad \text{(15)}

Subject to the boundary conditions of (14) and (15), the associated analytical solution for circular fin gives us the temperature distribution along the fin surface, where \( I_n \) and \( K_n \) are the modified Bessel functions of first and second kind.

\[ \frac{\theta + \theta_p}{\theta + \theta_p} = \frac{I_n(m_r) K_{1}(m_r) + I_{1}(m_r) K_{1}(m_r)}{I_n(m_r) K_{1}(m_r) + I_{1}(m_r) K_{1}(m_r)} \]  \quad \text{(16)}

where \( \theta_p = \frac{BC_o}{1 + b_o B} \) \quad \text{(17)}

### B. Fin Efficiency

By definition, the fin efficiency is the ratio of the actual heat transferred to fin surface to the maximum possible heat transfer rate if the entire fin surface was at the base temperature.

The heat transfer rate transferred to the entire fin surface is calculated from Fourier law of heat conduction:

\[ q_{\text{fin}} = kA_b \frac{dT}{dr}_{r=r_o} = -2 \pi t k \frac{m_o^2}{r_o^2} (T_o - T_b) \frac{d\theta}{dr} \]  \quad \text{(18)}

Using (16), the above equation becomes:

\[ q_{\text{fin}} = -2 \pi t k m_o^2 (T_o - T_b) \left( \theta + \theta_p \right) \times \frac{I_n(m_r) K_{1}(m_r) - I_{1}(m_r) K_{1}(m_r)}{I_n(m_r) K_{1}(m_r) + I_{1}(m_r) K_{1}(m_r)} \]  \quad \text{(19)}

Equation (1) is, now, used to calculate the maximum possible heat transfer rate in terms of radius \( r \) and integrating over the entire fin surface:

\[ q_{\text{max}} = \int_{t_o}^{t_e} 4 \pi b_t \left[ (T_o - T_a) + B (w_o - w_s) \right] r dr \]  \quad \text{(20)}

Combining (9), (10), (11), (12), (13), (17) and (20), the following equation that describes the maximum possible heat transfer rate is obtained:

\[ q_{\text{max}} = k \pi m^2 \left( \theta_b + \theta_p \right) (T_o - T_b) \left( r_e^2 - r_o^2 \right) \]  \quad \text{(21)}

Thus, the expression of the fin efficiency can be written as:

\[ \frac{q_{\text{fin}}}{q_{\text{max}}} = \frac{2 r_o}{m^2 (r_e^2 - r_o^2)} \times \frac{I_n(m_r) K_{1}(m_r) - I_{1}(m_r) K_{1}(m_r)}{I_n(m_r) K_{1}(m_r) + I_{1}(m_r) K_{1}(m_r)} \frac{1}{I_n(m_r) K_{1}(m_r) + I_{1}(m_r) K_{1}(m_r)} \]  \quad \text{(22)}

### C. Temperature Distribution-Numerical Method

In this section we performed a numerical method in order to simplify the solution to the non-homogeneous second order differential equation that describes the temperature distribution along the wet circular fin surface by avoiding the use of modified Bessel functions. Among all the approximations, the finite difference scheme is the simplest used one [24]. It is proposed to use a discrete spatial increment that can be computed by

\[ \Delta r = \frac{r_e - r_o}{N} \]  \quad \text{(23)}

Here, \( N \) is the number of the nodal points where the dimensionless temperature is to be determined, see Fig. 2.

![Finite difference scheme for 1-D circular fin](image)

In this approximation the temperature distribution at first
node and at the end node must be known as special cases. For $i = 1$ we have $\theta_i = \theta_0$ and (25) can be used directly, thus:

$$-\left(2 + m^2 \Delta r^2\right) \theta_i + \left(1 + \frac{\Delta r}{2 \theta_0}\right) \theta_2 = m^2 BC_0 \Delta r^2 - \left(1 + \frac{\Delta r}{2 \theta_0}\right) \theta_0.$$ (27)

For the end nod, at $i = N$, we can generate a backward approximation and, to do this, let’s write $\frac{d^2 \theta}{dr^2} = \theta''$, as follows:

$$\theta_i = \theta_N + \frac{d \theta}{dr} \bigg|_N = \theta_N - \frac{\theta_N - \theta_{N-1}}{\Delta r}.$$ (28)

The first derivative at point $N-1$ can be written as:

$$\frac{\theta_{N-1} - \theta_{N-2}}{2 \Delta r}.$$ (29)

Equations (29) and (28) can be rearranged and subject to boundary condition (15), the following equation can be derived:

$$\frac{\theta_{N-2} - \theta_{N-1}}{2 \Delta r^2} = \frac{\theta_{N-1} - \theta_N}{2 \Delta r^2}.$$ (30)

Together, (8), (15) and (30) give us for $i = N$ :

$$\theta_{N-2} - \left(1 + 2m^2 \Delta r^2\right) \theta_N = 2m^2 BC_0 \Delta r^2.$$ (31)

Equations (25), (27) and (31) constitute a system of $N$ equations, in which the temperature distribution along a wet circular fin can be easily found by solving this system.

III. RESULTS AND DISCUSSION

The dimensionless fin surface temperature is plotted against the non-dimensional fin radius measured from the fin base. The present finite difference method for temperature distribution over wet circular fin has been compared with the commonly used Bessel equation. The results for 50% and 100% relative humidity (RH) values are shown in Fig. 3.

The variation of the fin efficiency with relative humidity is shown in Fig. 4 for two values of frontal velocity ($u_{fr} = 0.5$ and 3 m/s). As can be seen, there are two main regions. In the dry-surface fin no condensation of moisture is occurring. Thus, no variation of the fin efficiency with the relative humidity is seen. The reasonable explanation of the effect of the relative humidity on the fin efficiency is as follow.

The fin efficiency is defined as the ratio between the actual fin heat transfer rate to maximum possible heat transfer rate if the entire fin is at the base temperature. In the dry fin region, however, the actual fin heat transfer and the maximum possible heat transfer increase by the same amount. In the wet-surface fin, the fin efficiency decreases rapidly with the increase in relative humidity.

This situation corresponds to the partially-wet fin in which the actual heat transfer rate through the fin surface and the maximum possible heat transfer does not increase by the same amount. On the other word, the maximum possible heat transfer increases more rapidly than the actual fin heat transfer. However, the variation of the fin efficiency with the relative humidity diminishes as relative humidity increases.

By increasing relative humidity the wet portion on the fin surface increases until the fin becomes in fully wet situation. There, the ratio of the actual heat transfer rate to the maximum possible heat transfer rate is comparatively constant for fully wet fin surface.

![Fig. 3 Fin surface temperature comparison](image)

![Fig. 4 Effect of frontal velocity on fin efficiency](image)
by the presence of moisture on the fin surface. On the other hand, the presence of moisture reduces considerably the fin efficiency. As a result, lower fin efficiency can be seen for higher air velocity.

The influence of relative humidity on the fin surface temperature is illustrated in Fig. 5. The values of fin temperature gradient are plotted against the dimensionless radius for three values of relative humidity (RH=60%, 80% and 100%) and compared with a dry fin. As can be seen, the curves under dehumidification lie below that of dry fin. The wet portion on the fin surface increased with the increase of relative humidity. As reported by Elmahdy and Biggs [16] and Kazeminejad [21], when the inlet relative humidity increases the departure of the temperature profile from the dry surface curve becomes greater. Thus, the rise in relative humidity involves water vapor condensing on the fin surface. This results in higher amount of mass transfer and larger amount of latent energy and, by consequence, higher fin surface temperature.

For a given set of conditions, relative humidity, base temperature, fin pitch, outside diameter and fin thickness, Fig. 6 depicts the effect of dry bulb temperature on the dimensionless fin surface temperature as a function of the dimensionless radius. Lower dimensionless temperature curve is seen for higher dry air temperature. For a constant relative humidity, an increase in the surrounding temperature increases the heat transfer whatever the situation dry/or wet of the fin surface. It is noteworthy that the dew point temperature increases as the dry bulb temperature increases under the same relative humidity. Hence, higher dry air temperature can cause a larger mass of moisture, which consequently leads to a higher amount of latent energy.

Fig. 7 presents the fin efficiency as a function of relative humidity with two values of dry air temperature, 24 and 28°C. The fin efficiency decreases with the increase of dry air temperature in partially wet and fully wet regions. The amount of dehumidification increases with increasing dry temperature. It is important to note that the wet portion on the fin surface increases with dry bulb temperature.

![Fig. 6 Effect of air temperature on fin surface temperature](image6)

![Fig. 7 Effect of air temperature on fin efficiency](image7)

Fig. 8 shows the effect of fin pitch on the annular fin surface temperature for 0.5 m/s and 4 m/s air frontal velocities and 60% relative humidity. The dimensionless temperature decreases as the fin pitch decreases. When the fin pitch is small enough the condensate water is easier to adhere between the fins. The presence of the condensate water on the fin surface enhances the latent and sensible heat transfer rates and hence lower fin surface temperature curves are for lower fin pitch values. This effect tends to diminish as air face velocity decreases. This is in connection with the condensate blow off phenomenon. When air velocity and fin pitch are increased just a smaller amount of condensate flows alongside the fin whereas a larger amount of condensate may suspend between fins.
in the foregoing analysis, the non-homogeneous second order differential equation, derived from the heat balance on a circumferential control volume of an annular fin, has been solved numerically. The results presented here show the effect of dehumidification on both fin surface temperature and fin efficiency. The results here show the importance of the latent energy portion on the wet circular fin performance when dehumidification of moist air occurs.

**REFERENCES**


