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Effect of Time-Periodic Boundary Temperature on the Onset of Nanofluid Convection in a Layer of a Saturated Porous Medium

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***Abstract*—The linear stability of nanofluid convection in a horizontal porous layer is examined theoretically when the walls of the porous layer are subjected to time-periodic temperature modulation. The model used for the nanofluid incorporates the effects of Brownian motion and thermopherosis, while the Darcy model is used for the porous medium. The analysis revels that for a typical nanofluid (with large Lewis number) the prime effect of the nanofluids is via a buoyancy effect coupled with the conservation of nanoparticles. The contribution of nanoparticles to the thermal energy equation being a second-order effect. It is found that the critical thermal Rayleigh number can be found reduced or decreased by a substantial amount, depending on whether the basic nanoparticle distribution is top-heavy or bottom-heavy. Oscillatory instability is possible in the case of a bottom-heavy nanoparticle distribution, phase angle and frequency of modulation.

***Keywords*—Brownian motion and thermophoresis, Porous medium, Nanofluid, Natural convection, Thermal modulation.

NOMENCLATURE

c	nanofluid specific heat at constant pressure
c_p	specific heat of the nanoparticle material
$(\rho c)_m$	effective heat capacity of the porous medium
g	gravitational accelaeration ($\mathrm{m/s^2}$)
D_B	Brownian diffusion coefficient ($\mathrm{m^2/s}$)
D_T	thermophoretic diffusion coefficient ($\mathrm{m^2/s}$)
H	dimensional layer depth (m)
k	thermal conductivity of the nanofluid
k_m	effective thermal conductivity of the porous medium
Le	Lewis number
N_A	modified diffusivity ratio
N_B	modified particle-density increment
p^*	pressure
p	dimensionless pressure, $p^* K/\mu\alpha_m$
Ra	thermal Rayleigh- Darcy number
Rm	basic-density Rayleigh number
Rn	concentration Rayleigh number
t^*	time (s)
t	dimensionless time, $t^*\alpha_m/\sigma H^2$
T^*	nanofluid temperature
T_r	reference temperature
(u,v,w)	dimensionless Darcy velocity components $\left(u^*,v^*,w^*\right)H/\alpha_m$
$\mathbf{v}$	nanofluid velocity
$\mathbf{v}_D$	Darcy velocity $\varepsilon\mathbf{v}$
$\mathbf{v}_D^*$	dimensionless Darcy velocity $\left(u^*,v^*,w^*\right)$
(x,y,z)	dimensionless Cartesian coordinate $\left(x^*,y^*,z^*\right)/H$; z is the vertically upward coordinate
$\left(x^*,y^*,z^*\right)$	Cartesian coordinates

Greek Symbols

α_m	thermal diffusivity of the porous medium, $k_m/\left(\rho c_p\right)_f$
ε	porosity of the medium
ε_t	amplitude of the modulation
μ	viscosity of the fluid
$\tilde{\mu}$	effective viscosity of the porous medium
ρ	fluid density
ρ_p	nanoparticle mass density
σ	parameter defined by (20)
ϕ^*	nanoparticle volume fraction
ϕ	relative nanoparticle volume fraction
Ω	dimensional frequency
ω	dimensionless frequency $\left(=\Omega H^2/k\right)$
ψ	phase angle

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I. INTRODUCTION

THE term "nanofluid" was first suggested by Choi [1] in his paper presented at the ASME Winter Annual Meeting. It refers to a liquid containing a dispersion of submicronic solid particles (nanoparticles) whose characteristic dimension is of the order of tens or hundreds of nanometers. The first SCI article on nanofluids was published by Choi's group in 1999. One of the most interesting features of nanofluids is the enhancement of thermal diffusivity that according to some data may exceed the limits predicted by conventional macroscopic theories of suspensions [2]-[6]. The enhancement of effective thermal conductivity was confirmed by experiments conducted by many researchers, including Masuda [7], although the level of enhancement is still a subject of a debate [8], [9]. The unique properties of nanofluids suggest the possibility of using nanofluids in a variety of engineering systems, from advanced nuclear systems [10]-[12] to drug delivery [13]. Heat transfer in nanofluids has been surveyed recently in a book by Das et al. [5].

Nanofluids in porous media constitute an emerging topic; the review of recent literature points out to at least two possible applications. Porous foam and microchannel heat sinks (used for electronic cooling) are usually modeled and optimized utilizing the porous medium approach (e.g., see Kim et al. [14], Kim and Kuznetsov [15]). The utilization of nanofluids for cooling such microchannel heat sinks has been recently suggested by Abbassi and Aghanajafi [16], Tsai and Chein [17], Ghazvini et al. [18], Ghazvini and Shokouhmand [19]. Modeling of such heat sinks requires understanding of fundamentals of nanofluid convection in porous media.

Another area relevant to nanofluid convection in porous media is the utilization of nanoparticles hyperthermia for cancer treatment [20], [21]. The objective is to induce the maximum damage on the tumor (this requires elevating the temperature of at least 90% of the tumor above 43◦C) with the minimum damage to the normal tissue. Since a living tissue is a type of fluid-saturated porous medium (in fact, many medical studies use agarose gels with porous structures similar to human tissue for in vitro experiments; (see, for example, Salloum et al. [22]), the development of optimal protocols for this type of treatment again requires fundamental understanding of nanofluid convection in porous media.

A comprehensive survey of convective transport in nanofluids was made by Buongiorno [23] who, after considering alternative agencies, proposed a model incorporating the effects of Brownian diffusion and the thermophoresis. This model was applied to the Horton-Rogers-Lapwood problem (the onset of convection in a horizontal layer of a porous medium uniformly heated from below) by Nield and Kuznetsov [24] and Kuznetsov and Nield [25]-[27]. Both Brownian diffusion and thermophoresis give rise to cross-diffusion terms that are in some ways analogous to the familiar Soret and Dufour cross-diffusion terms that arise with a binary fluid.

There has been a growing interest in externally modulated hydrodynamic systems, both theoretically and experimentally. These systems may exhibit novel behavior in response to parametric forcing near a point of instability. Depending on the relative strength and rate of forcing, predictions exist for a variety of responses to the modulation. Among these are the upward or downward shifts of the convective threshold compared to the unmodulated problems. There are many works available in the literature, concerning how a time-periodic boundary temperature affects the onset of Rayleigh–Benard convection. Some of the findings related to this problem have been reviewed by Davis [28]. The studies related to the effect of thermal modulation on the onset of convection in a porous medium have also received equal importance (see, e.g. Nield and Bejan [29]).

The effect of time-dependent wall temperature on the onset of convection in a fluid-saturated porous medium has been studied by Caltagirone [30] using the Darcy model for the momentum equation. Chhuon and Caltagirone [31] have studied the stability of a fluid-saturated porous layer where the imposed temperature on the boundary is time-periodic, with a non-zero mean value. They performed experiments and compared their results with those obtained from Floquet theory. Rudraiah and Malashetty [32] investigated the stability of a fluid-saturated sparsely packed porous layer subject to time-periodic boundary temperature using the Brinkman model. They recovered the viscous flow results of Venezian [33], as a special case when the value of the porous parameter tends to zero. Linear stability analysis of the onset of convection induced by a non-uniform time-dependent volumetric heating in a fluid-saturated porous medium has been studied by Nield [34]. Analytical expression that gives upper bounds on an appropriate critical Rayleigh number is derived. The effect of thermal modulation on the convection in a porous medium is studied by Malashetty and Wadi [35] using the Brinkman model with effective viscosity larger than the fluid viscosity. Further, Malashetty and Basavaraja [36]-[38] have examined the single and double diffusive convections in a fluid saturated anisotropic porous layer subject to time-dependent wall temperature. Bhadauria [39] has studied the effect of thermal modulation on the onset of convection in a layer of sparsely packed porous medium bounded by rigid boundaries. Recently, Bhadauria [40] has included the effect of rotation, while Bhadauria and Aalam [41] have included the effect of magnetic field to study the onset of convection in a porous medium with temperature modulation. Recently Shivakumara et al. [42] analyzed the linear stability of Walters B viscoelastic fluid-saturated horizontal porous layer when the walls of the porous layer are subjected to time-periodic temperature modulation.

In the present study, the effect of thermal modulation on the onset of convection in a horizontal layer of porous medium saturated with nanofluid is investigated. In the absence of thermal modulation we get back the results of Nield and Kuznestove [24].

II. Mathematical Formulation

We consider a nano-fluid saturated porous layer, confined between two infinite horizontal plates situated at $z^*=0$ and $z^*=H$. We select a coordinate frame in which the $z-$axis is aligned vertically upward. Further, in addition to a fixed temperature difference between the walls, an additional perturbation is applied to the wall temperatures, varying sinusoidally in time. Thus, the wall temperatures are

$$T=T_0+\frac{1}{2}\Delta T\left[1+\varepsilon_t\cos\left(\Omega t\right)\right] \text{ at } z^*=0 \tag{1}$$

$$T=T_0-\frac{1}{2}\Delta T\left[1-\varepsilon_t\cos\left(\Omega t+\phi\right)\right] \text{ at } z^*=H \tag{2}$$

where ε_t represents a small amplitude of the thermal modulation, Ω the frequency of modulation and ϕ the phase angle.

The conservation equation takes the form

$$\nabla^*.\mathbf{v}_{\mathrm{D}}^* = 0 \tag{3}$$

Here $\mathbf{v}_{\mathrm{D}}^*$ is the nanofluid Darcy velocity. We write $\mathbf{v}_{\mathrm{D}}^* = \left(u^*, v^*, w^*\right)$.

In the presence of thermophoresis, the conservation equation for the nanoparticles, in the absence of chemical reactions, takes the form

$$\frac{\partial \phi^*}{\partial t^*} + \frac{1}{\varepsilon}\mathbf{v}_D^*.\nabla\phi^* = \nabla^*.\left[D_B\nabla^*\phi^* + D_T\frac{\nabla^* T^*}{T^*}\right] \tag{4}$$

where ϕ^* is the nanoparticle volume fraction, ε is the porosity, T^* is the temperature, D_B is the Brownian diffusion coefficient, and D_T is the thermophoretic diffusion coefficient.

If one introduces a buoyancy force and adopts the Boussinesq approximation, and uses the Darcy model for a porous medium, then the momentum equation can be written as

$$0 = -\nabla^* p^* - \frac{\mu}{K}\mathbf{v}_{\mathrm{D}}^* + \rho\,\mathrm{g} \tag{5}$$

Here ρ is the overall density of the nanofluid, which we now assume to be given by

$$\rho = \phi^*\rho_{\mathrm{p}} + \left(1-\phi^*\right)\rho_0\left[1-\beta_{\mathrm{T}}\left(T^* - T_r^*\right)\right] \tag{6}$$

where ρ_{p} is the particle density, ρ_0 is a reference density for the fluid, and β_{T} is the thermal volumetric expansion. The thermal energy equation for a nanofluid can be written as [24]

$$\begin{aligned}&(\rho c)_{\mathrm{m}}\frac{\partial T^*}{\partial t^*} + (\rho c)_{\mathrm{f}}\,\mathbf{v}_{\mathrm{D}}^*.\nabla^* T^* \\ &= k_{\mathrm{m}}\nabla^{*2}T^* + \varepsilon(\rho c)_{\mathrm{p}}\left[D_{\mathrm{B}}\nabla^*\phi^*.\nabla T^* + D_{\mathrm{T}}\frac{\nabla T^*.\nabla T^*}{T_r^*}\right]\end{aligned} \tag{7}$$

The conservation of nanoparticle mass requires that

$$\frac{\partial \phi^*}{\partial t^*} + \frac{1}{\varepsilon}\mathbf{v}_{\mathrm{D}}^*.\nabla^*\phi^* = D_{\mathrm{B}}\nabla^{*2}\phi^* + \frac{D_{\mathrm{T}}}{T_r^*}\nabla^{*2}T^* \tag{8}$$

Here c is the fluid specific heat (at constant pressure), k_{m} is the overall thermal conductivity of the porous medium saturated by the nanofluid, and c_{p} is the nanoparticle specific heat of the material constituting the nanoparticles.

We write $\mathbf{v}_{\mathrm{D}}^* = \left(u^*, v^*, w^*\right)$

We assume that the volumetric fraction of the nanoparticles is constant on the boundaries. Thus, the boundary conditions are

$$w^* = 0,\ \ \phi^* = \phi_0^* \ \text{at}\ z^* = 0 \tag{9}$$

$$w^* = 0,\ \ \phi^* = \phi_1^* \ \text{at}\ z^* = H \tag{10}$$

A. Basic State

The basic state is quiescent and the temperature T_b, density ρ_b, and the pressure p_b satisfy

$$\rho_b\vec{g} + \nabla p_b = 0 \tag{11}$$

$$(\rho c)_m\frac{\partial T^*}{\partial t^*} = k_{\mathrm{m}}\nabla^2 T^* \tag{12}$$

$$\frac{d^2\phi_b^*}{dz^2} = 0 \tag{13}$$

Following Nield and Kuznestove [24], (12) and (13) are considered from (5) and (6).

The solution of (12) satisfying the thermal conditions given by (1) and (2) is

$$T_b = T_1(z) + \varepsilon_t T_2(z,t)$$

where

$$T_1(z) = \frac{\Delta T}{2}\left(1-\frac{2z}{H}\right) + T_r \tag{14}$$

$$T_2(z,t) = \mathrm{Re}\left[\left\{b(\lambda)e^{\lambda z/H} + b(-\lambda)e^{-\lambda z/H}\right\}e^{-i\omega t}\right] \tag{15}$$

with

$$\lambda = (1-i)\left(\frac{(\rho c)_m\,\omega H^2}{2k_m}\right)^{1/2} \tag{16}$$

$$b(\lambda) = \frac{\Delta T}{2}\left(\frac{e^{-i\phi} - e^{-\lambda}}{e^{\lambda} - e^{-\lambda}}\right) \tag{17}$$

and Re stands for real part. The expression for p_b and ρ_b is not given as they are not explicitly required in the subsequent analysis.

B. Perturbation Solution

We now superimpose perturbations on the basic solution. We write

$$\mathbf{v} = \mathbf{v}', \quad p = p_b + p', \quad T = T_b + T', \quad \phi = \phi_b + \phi' \tag{18}$$

where $\mathbf{v}'$, p', T' and ϕ' represents the perturbed quantities.

We introduce dimensionless variables as follows. We define

$$(x,y,z) = (x^*, y^*, z^*)/H, \quad t = t^*\alpha_m/\sigma H^2, \quad p = p^* K/\mu\alpha_m,$$
$$(u,v,w) = (u^*, v^*, w^*)H/\alpha_m, \ \phi = \frac{\phi^* - \phi_0^*}{\phi_1^* - \phi_0^*}, \ T = \frac{T^*}{\Delta T},$$
$$\omega = \frac{\sigma\Omega H^2}{\alpha_m} \tag{19}$$

$$\text{where } \alpha_m = \frac{k_m}{(\rho c_p)_f}, \ \sigma = \frac{(\rho c_p)_m}{(\rho c_p)_f}. \tag{20}$$

Substituting (19) and (20) in (3)–(8), and linearise by neglecting products of primed quantities. The following equations are obtained

$$0 = \nabla.\mathbf{v}' \tag{21}$$

$$0 = -\nabla p' - \mathbf{v}' - Rm\,\hat{\mathbf{e}}_z + RaT'\hat{\mathbf{e}}_z - Rn\phi\,\hat{\mathbf{e}}_z \tag{22}$$

$$\begin{aligned}\frac{\partial T'}{\partial t} + w'\frac{\partial T_b}{\partial z} = \frac{\partial^2 T'}{\partial z^2} + \frac{N_B}{Le}\left(\frac{\partial T'}{\partial z} + \frac{\partial T_b}{\partial z}\right)\left(1 + \frac{\partial \phi'}{\partial z}\right) \\ + \frac{2N_A N_B}{Le}\left(\frac{\partial T_b}{\partial z} + \frac{\partial T'}{\partial z}\right)\end{aligned} \tag{23}$$

$$\frac{1}{\sigma}\frac{\partial \phi'}{\partial t} + \frac{1}{\varepsilon}w' = \frac{1}{Le}\nabla^2\phi' + \frac{N_A}{Le}\nabla^2(T' + T_b) \tag{24}$$

$$w' = 0, \ T' = 0, \ \phi' = 0 \text{ at } z = 0,1 \tag{25}$$

where the parameter Le is a Lewis number and Ra is the familiar thermal Rayleigh-Darcy number. The parameter N_A is a modified diffusivity ratio and is somewhat similar to the Soret parameter that arises in cross-diffusion phenomena in solutions, while N_B is a modified particle density increment. In deriving (22), Oberbeck–Boussinesq approximation is used (neglecting a term proportional to the product of ϕ and T). This assumption is likely to be valid in the case of small temperature gradients in a dilute suspension of nanoparticles. It will be noted that the parameter R_m is not involved in these and subsequent equations. It is just a measure of the basic state pressure gradient.

For the case of regular fluid (not a nanofluid) the parameters R_n, N_A and N_B are zero. The remaining equations are reduced to the familiar equations for the Horton-Roger-Lapwood problem.

The six unknowns $u', v', w', p', T', \phi'$ can be reduced to three by operating on (22) with $\hat{\mathbf{e}}_z.curl\,curl$ and using the identity $curl\,curl \equiv grad\,div - \nabla^2$ together with (3).

The result is

$$\nabla^2 w' = -Ra\,\nabla_H^2 T' + Rn\nabla_H^2\phi', \tag{26}$$

Here ∇_H^2 is the two-dimensional Laplacian operator on the horizontal plane.

The dimensionless basic temperature gradient is given by

$$\frac{\partial T_b}{\partial z} = -1 + \varepsilon_t f \tag{27}$$

Here, f is the modulation temperature gradient and is given by

$$f = \mathrm{Re}\left[\left\{A(\lambda)e^{\lambda z} + A(-\lambda)e^{\lambda z}\right\}e^{-i\omega t}\right] \tag{28}$$

where

$$A(\lambda) = \frac{\lambda}{2}\left(\frac{e^{-i\varphi} - e^{-\lambda}}{e^{\lambda} - e^{-\lambda}}\right) \tag{29}$$

$$\lambda = (1-i)\left(\frac{\sigma\omega}{2}\right)^{1/2} \tag{30}$$

The differential equations (26), (23), (24) and the boundary conditions (25) constitute a linear boundary-value problem that can be solved using the method of normal modes.

We write

$$(w', T', \phi') = \left[W(z), \Theta(z), \Phi(z)\right]\exp(st + ilx + imy), \tag{31}$$

and substitute into the differential equations to obtain

$$(D^2 - \alpha^2)W + Ra\alpha^2\Theta - Rn\alpha^2\Phi = 0 \tag{32}$$

$$\begin{aligned}W(-1 + \varepsilon_t f) + \left(s - (D^2 - \alpha^2)\right)\Theta - \frac{2N_A N_B}{Le}(\varepsilon_t f - 1)D\Theta \\ - \frac{N_B}{Le}D\Theta - \frac{N_B}{Le}(-1 + \varepsilon_t f)D\Phi = 0\end{aligned} \tag{33}$$

$$\frac{1}{\varepsilon}W - \frac{N_A}{Le}(D^2 - \alpha^2)\Theta - \left(\frac{1}{Le}(D^2 - \alpha^2) - \frac{s}{\sigma}\right)\Phi = 0 \tag{34}$$

$$W = 0, \ \Theta = 0, \ \Phi = 0 \text{ at } z = 0 \text{ and at } z = 1 \tag{35}$$

where

$$D \equiv \frac{d}{dz} \text{ and } \alpha = (l^2 + m^2)^{1/2} \tag{36}$$

Thus α is a dimensionless horizontal wave number.

The vanishing the determinant of coefficients produces the eigenvalue equation for the system. One can regard Ra as the eigenvalue. This enables us to find Ra in terms of the other parameters.

III. Results and discussion

The eigenequation is

$$\det M = 0$$

where
$$M = \begin{bmatrix} M_{11} & M_{12} & M_{13} \\ M_{21} & M_{22} & M_{23} \\ M_{31} & M_{32} & M_{33} \end{bmatrix} \tag{37}$$

and for $i, j = 1, 2, 3............N$.

$$\left(M_{11}\right)_{ij} = \left\langle W_j D^2 W_i \right\rangle - \left\langle \alpha^2 W_j W_i \right\rangle, \tag{38}$$

$$\left(M_{12}\right)_{ij} = Ra\,\alpha^2 \left\langle W_j \Theta_i \right\rangle, \tag{39}$$

$$\left(M_{13}\right)_{ij} = -Rn\,\alpha^2 \left\langle W_j\ \Phi_i \right\rangle \tag{40}$$

$$\left(M_{21}\right)_{ij} = \left(-1 + \varepsilon_t f\right)\left\langle \Theta_j\ W_i \right\rangle \tag{41}$$

$$\left(M_{22}\right)_{ij} = -\Theta_j \left\langle D^2 \Theta_i \right\rangle + \alpha^2 \left\langle \Theta_j \Theta_i \right\rangle + s\left\langle \Theta_j\ \Theta_i \right\rangle - \left(\frac{2N_A N_B\left(-1+\varepsilon_t f\right)}{Le} + \frac{N_B}{Le} \right)\left\langle \Theta_j D\Theta_i \right\rangle \tag{42}$$

$$\left(M_{23}\right)_{ij} = -\frac{N_B\left(-1+\varepsilon_t f\right)}{Le}\left\langle \Theta_j D\Phi_i \right\rangle \tag{43}$$

$$\left(M_{31}\right)_{ij} = \frac{1}{\varepsilon}\left\langle \Phi_j\ \ W_i \right\rangle \tag{44}$$

$$\left(M_{32}\right)_{ij} = \frac{N_A}{Le}\left[-\Phi_j \left\langle D^2 \Theta_i \right\rangle + \alpha^2 \left\langle \Phi_j \Theta_i \right\rangle \right] \tag{45}$$

$$\left(M_{33}\right)_{ij} = \frac{1}{Le}\left[-\Phi_j \left\langle D^2 \Phi_i \right\rangle + \alpha^2 \left\langle \Phi_j \Phi_i \right\rangle + \frac{s}{\sigma}\left\langle \Phi_j \Phi_i \right\rangle \right] \tag{46}$$

Here
$$\left\langle f\left(z\right)\right\rangle = \int_0^1 f\left(z\right) dz \tag{47}$$

A. Non-Oscillatory Stability

We consider the case of non-oscillatory instability, when $\omega = 0$. For a first approximation we take $N = 1$, this produces the result

$$Ra\left(1-\varepsilon_t f\right) + Rn\left(N_A\left(1-\varepsilon_t f\right) + \frac{Le}{\varepsilon} \right) = 4\pi^2 . \tag{48}$$

If $\varepsilon_t = 0$ then (48) become

$$Ra + Rn\left(N_A + \frac{Le}{\varepsilon} \right) = 4\pi^2 \tag{49}$$

It is clear from (49) that the critical Rayleigh number for the onset of convection for an unmodulation case is the similar result obtained by Nield and Kuznestove [24].

B. Oscillatory Convection

$$Ra\alpha^2\left(\left(1-\varepsilon_t f\right)\left(\frac{J}{Le} + \frac{i\omega}{\sigma} \right) \right) + Rn\,\alpha^2\left(\frac{N_A J}{Le}\left(1-\varepsilon_t f\right) + \frac{J+i\omega}{\varepsilon} \right) = J\left(J+i\omega\right)\left(\frac{J}{Le} + \frac{i\omega}{\sigma} \right) \tag{50}$$

The real and imaginary parts of (50) yield

$$\frac{Ra\alpha^2}{Le}\left(1-\varepsilon_t f\right) + Rn\,\alpha^2\left(\frac{N_A}{Le}\left(1-\varepsilon_t f\right) + \frac{1}{\varepsilon} \right) = \frac{J^2}{Le} - \frac{\omega^2}{\sigma} \tag{51}$$

$$\omega\left[\frac{Ra\alpha^2}{\sigma}\left(1-\varepsilon_t f\right) + \frac{Rn\,\alpha^2}{\varepsilon} - J^2\left(\frac{1}{Le} + \frac{1}{\sigma} \right) \right] = 0 \tag{52}$$

Again the critical value of α is found to be π. Hence one obtains the results

$$\frac{Ra}{\sigma}\left(1-\varepsilon_t f\right) + \frac{Rn}{\varepsilon} = 4\pi^2\left(\frac{1}{Le} + \frac{1}{\sigma} \right) \tag{53}$$

where the frequency is found

$$\frac{Le\omega^2}{\pi^2\sigma} = 4\pi^2 - \left[Ra\left(1-\varepsilon_t f\right) + Rn\left(N_A\left(1-\varepsilon_t f\right)\right) + \frac{Le}{\varepsilon} \right] \tag{54}$$

In order for ω to be real it is necessary that

$$Ra\left(1-\varepsilon_t f\right) + Rn\left(N_A\left(1-\varepsilon_t f\right) + \frac{Le}{\varepsilon} \right) \le 4\pi^2 \tag{55}$$

In the absence amplitude of modulations $\left(\varepsilon_t = 0\right)$ (50)-(54) reduces to Nield and Kuznestove [24].

IV. Conclusion

The linear stability analysis of Nield and Kuznestove [24], for the onset of convection in a horizontal layer of a porous medium saturated by a nonofluid has been modified when the walls of the porous layer are subjected to time-periodic temperature modulation. The new effects such as frequency and phase angle enter the expressions for both non-oscillatory and oscillatory convection. The consequence of these factors is to advance or delay the onset of convection by controlling the frequency of modulation.

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