Construction Methods for Sign Patterns Allowing Nilpotence of Index $k$

Jun Luo

Abstract—In this paper, the smallest such integer $k$ is called by the index (of nilpotence) of $B$ such that $B^k = 0$. In this paper, we study sign patterns allowing nilpotence of index $k$ and obtain four methods to construct sign patterns allowing nilpotence of index at most $k$, which generalizes some recent results.

Keywords—Sign pattern, Nilpotence, Jordan block.

I. INTRODUCTION

The sign of a real number $a$, denoted by $\text{sgn}(a)$, is defined to be 1, −1 or 0, according to $a > 0$, $a < 0$, $a = 0$, respectively. A sign pattern matrix (or a sign pattern, for short) is a matrix whose entries are from the set $\{1, -1, 0\}$. The sign pattern of a real matrix $B$, denoted by $\text{sgn}(B)$, is the sign pattern matrix obtained from $B$ by replacing each entry by its sign.

Let $m$ be a positive integer. The integers $a$ and $b$ are congruent modulo $m$ if and only if there is an integer $t$ such that $a = b + tm$ (for short, written as $a \equiv b \pmod{m}$).

Let $Q_n$ be the set of all sign patterns of order $n$. For $A \in Q_n$, the set of all real matrices with the same sign pattern as $A$ is called the qualitative class of $A$, and is denoted by $Q(A)$ (12).

Suppose that a real matrix has the property $p$. Then a sign pattern $A$ is said to require $p$ if every real matrix in $Q(A)$ has property $p$, or to allow $p$ if some real matrix in $Q(A)$ has property $p$ (11).

In this paper, we investigate the property $Q$ of being nilpotent. Recall that a real matrix $B$ is said to be nilpotent if $B^k = 0$ for some positive integer $k$. The smallest such integer $k$ is called the index (of nilpotence) of $B$.

Let $k$ be a positive integer. We now consider sign patterns that allow nilpotence of index at most $k$. These sign patterns that allow nilpotence, are also referred to as the potentially nilpotent sign patterns (see [1], [4], [5], [6]). For convenience, we denote the class of all sign patterns that allow nilpotence of index $k \geq 4$, Eschenbacher and Li [4] studied $N_2$ and Gao, Li and Shao [1] studied $N_3$. In this paper, we mainly extend these results to any $N_k$.

II. PRELIMINARY

Lemma 1([4]). The set $N_k$ is closed under the following operations:

1) negation;

2) transposition;

3) permutational similarity, and

4) signature similarity.

As defined in [1], two sign patterns are equivalent if one can be obtained from the other by performing a sequence of operations listed in Lemma 1. This is indeed an equivalence relation.

Lemma 2 ([1]). A real matrix $B$ is nilpotent if and only if its eigenvalues are equal to zero.

Recall that a reducible (real or sign pattern) matrix is permutationally similar to a matrix in Frobenius normal form (see page 57 in [8]). Consequently, a reducible sign pattern $A$ allows nilpotence if and only if each irreducible component (see [8]) of $A$ allows nilpotence.

Lemma 3. Let $B$ be a nilpotent real matrix of index at most $k$, and $J$ the Jordan form of $B$. Then each Jordan block in $J$ is one of the following:

$$J_1 = [0], \quad J_i = \begin{bmatrix} 0 & 1 & 0 & \cdots & 0 \\ \vdots & \ddots & \cdots & \vdots \\ 0 & 0 & 0 & \cdots & 1 \\ 0 & 0 & 0 & \cdots & 0 \end{bmatrix} \quad \text{for } i = 2, 3, \cdots, k.$$

Let $A$ be a sign pattern matrix. The minimal rank of $A$, denoted by $\text{mr}(A)$, is defined as $\text{mr}(A) = \min\{\text{rank}B : B \in Q(A)\}$ (13).

Theorem 1. Let $A \in Q_n$. If $A \in N_k$, then

$$\text{mr}(A) \leq \frac{k-1}{n}.$$

Proof. Let $A \in Q_n$ and $A \in N_k$. Then there exists a real matrix $B \in Q(A)$ such that $B^k = 0$. By Lemma 3 we can assume that the Jordan form $J$ of $B$ is a direct sum of $k_i$ copies of $J_i$ ($i = 1, 2, \cdots, k$), where $\sum_{i=1}^{k} ik_i = n$. Then

$$\text{rank}(B) = \text{rank}(J) = \sum_{i=1}^{k} (i-1)k_i \leq \frac{k-1}{k} k_1 + \frac{k-1}{k} 2k_2 + \cdots + \frac{k-1}{k} kk_k = \frac{k-1}{k} n.$$

Hence $\text{mr}(A) \leq \text{rank}(B) \leq \frac{k-1}{k} n$. □
Remark 1. Note that the sign pattern

\[
A = \begin{bmatrix}
1 & 1 & 1 & 1 \\
1 & 1 & -1 & -1 \\
1 & 1 & 0 & 0 \\
1 & 1 & 0 & 0 \\
1 & 1 & 0 & 0 \\
\end{bmatrix}
\]

satisfies \( mr(A) = 3 \leq \frac{2}{3} \times 5 \). However, \( A^4 \neq 0, A \notin N_4 \). So the condition in Theorem 1 is not a sufficient one.

Theorem 2. Let \( B \) be a real matrix of order \( n \) with \( \text{rank}(B) = r \). Then \( B^4 = 0 \) if and only if there exist nonnegative integers \( l, m \) and nonzero real column vectors \( \alpha_1, \alpha_2, \cdots, \alpha_r \) and \( \beta_1, \beta_2, \cdots, \beta_r \) such that \( \alpha_i \leq m \), \( m \leq \frac{2}{r} \), \( 2r - 2l - m \leq n \) and

\[
\beta_j^T \alpha_i = \begin{cases} 
1 & j \equiv 1 \pmod{3}, 1 \leq j \leq 3l - 1, \text{ and } i = j + 1, \\
1 & j \equiv 2 \pmod{3}, 1 \leq j \leq 3l - 1, \text{ and } i = j + 1, \\
0 & \text{otherwise},
\end{cases}
\]

such that

\[
B = \sum_{1 \leq i \leq r} \alpha_i \beta_i^T.
\]

Proof. Sufficiency. Let \( B = \alpha_1 \beta_1^T + \alpha_2 \beta_2^T + \cdots + \alpha_r \beta_r^T \). By (1), we have

\[
B^2 = (\alpha_1 \beta_1^T + \alpha_2 \beta_2^T + \cdots + \alpha_r \beta_r^T) = \alpha_1 \beta_1^T + \alpha_2 \beta_2^T + \cdots + \alpha_r \beta_r^T = \alpha_1 \beta_1^T + \alpha_2 \beta_2^T + \cdots + \alpha_r \beta_r^T
\]

\[
B^3 = (\alpha_1 \beta_1^T + \alpha_2 \beta_2^T + \cdots + \alpha_r \beta_r^T) = \alpha_1 \beta_1^T + \alpha_2 \beta_2^T + \cdots + \alpha_r \beta_r^T = \alpha_1 \beta_1^T + \alpha_2 \beta_2^T + \cdots + \alpha_r \beta_r^T
\]

\[
B^4 = (\alpha_1 \beta_1^T + \alpha_2 \beta_2^T + \cdots + \alpha_r \beta_r^T) = \alpha_1 \beta_1^T + \alpha_2 \beta_2^T + \cdots + \alpha_r \beta_r^T = \alpha_1 \beta_1^T + \alpha_2 \beta_2^T + \cdots + \alpha_r \beta_r^T
\]

\[
B = \sum_{1 \leq i \leq r} \alpha_i \beta_i^T.
\]

The conclusion follows. \( \square \)

Next, we generalize the above result to any \( B^k = 0 \), that is, \( N_k \).

Theorem 3. Let \( B \) be a real matrix of order \( n \) with \( \text{rank}(B) = r \). Then \( B^k = 0 \) if and only if there exist nonnegative integers \( l_1, l_2, \cdots, l_k \) and nonzero real column vectors \( \alpha_1, \alpha_2, \cdots, \alpha_r \), \( \beta_1, \beta_2, \cdots, \beta_r \) of order \( n \) with \( \sum_{i=1}^{k} l_i = n \), \( l_k \), \( \beta_j^T \alpha_i = \begin{cases} 
1 & j \equiv i \pmod{k-1}, 1 \leq j \leq (k-1)l_i - 1, \text{ and } i = j + 1, \\
1 & j \equiv (k-1)l_i \pmod{k-1}, 1 \leq j \leq (k-1)l_i - 1, \text{ and } i = j + 1, \\
0 & \text{otherwise},
\end{cases}
\]

such that

\[
B = \sum_{1 \leq i \leq r} \alpha_i \beta_i^T.
\]

Proof. Sufficiency. Let \( B = \alpha_1 \beta_1^T + \alpha_2 \beta_2^T + \cdots + \alpha_r \beta_r^T \). By (4), we have

\[
B^2 = (\alpha_1 \beta_1^T + \alpha_2 \beta_2^T + \cdots + \alpha_r \beta_r^T) = \alpha_1 \beta_1^T + \alpha_2 \beta_2^T + \cdots + \alpha_r \beta_r^T
\]

\[
B^3 = (\alpha_1 \beta_1^T + \alpha_2 \beta_2^T + \cdots + \alpha_r \beta_r^T) = \alpha_1 \beta_1^T + \alpha_2 \beta_2^T + \cdots + \alpha_r \beta_r^T
\]

\[
B^{k-1} = (\alpha_1 \beta_1^T + \alpha_2 \beta_2^T + \cdots + \alpha_r \beta_r^T) = \alpha_1 \beta_1^T + \alpha_2 \beta_2^T + \cdots + \alpha_r \beta_r^T
\]

\[
B^k = 0.
\]
and

\[ B^k = BB^{k-1} = (\alpha_1 \beta_1^T + \alpha_2 \beta_2^T + \cdots + \alpha_r \beta_r^T)(\alpha_1 \beta_1^{T(k-1)} + \cdots + \alpha_{(k-2)} \beta_{k-1}^{T(k-1)}), \]

\[ = 0. \]

**Necessity.** Let \( B^k = 0 \) with \( \text{rank}(B) = r \). By Lemma 3, the Jordan form \( J \) of \( B \) is a direct sum of \( I_k \) copies of \( J_i \), where \( \sum_{i=1}^k n_i = n, \sum_{i=1}^k (i-1)n_i = n \) and it implies that there exists a nonsingular real matrix \( D \) of order \( n \) such that

\[ D^{-1}BD = J = \begin{bmatrix} J_{11} & J_{12} & \cdots & J_{1n-r,1}\cr J_{21} & J_{22} & \cdots & J_{2n-r,1}\cr \vdots & \vdots & \ddots & \vdots\cr J_{n-r,1,1} & J_{n-r,1,2} & \cdots & J_{n-r,1,n-r}\cr J_{n-r,n-r,1} & J_{n-r,n-r,2} & \cdots & J_{n-r,n-r,n-r}\cr \end{bmatrix}, \]

where

\[ J_{11} = \cdots = J_{1k} = \lambda_k, \]

\[ J_{1k+1,1k+1} = \cdots = J_{1k+1-k,1k+1-k} = \lambda_{k+1}, \]

\[ \cdots, \]

\[ J_{1n-r,1n-r} = \cdots = J_{n-r,n-r} = \lambda_1. \]

Write

\[ D = (u_1, u_2, \cdots, u_n) \text{ and } D^{-1} = (v_1, v_2, \cdots, v_n)^T, \]

where \( u_1, u_2, \cdots, u_n \) are column vectors of \( D \) and \( v_1, v_2, \cdots, v_n \) are column vectors of \( D^{-1} \). \( v_i^T u_i = 1 \) for \( i = 1, 2, \cdots, n \) and \( v_i^T u_i = 0 \) for \( i \neq j \). Let

\[ \alpha = (s-1)i+m + \sum_{j=i+1}^k \beta_j, \]

\[ \alpha = (s-1)i+m + \sum_{j=i+1}^k \beta_j, \]

for \( s = 1, 2, \cdots, k, i = 1, 2, \cdots, l_i - 1, \) and \( m = 1, 2, \cdots, s - 1, 0 \). It is easy to see that \( \lambda_i \) and \( \beta_i \) satisfy the condition (4). By (6), we have that

\[ B = DJD^{-1} = \alpha_1 \beta_1^T + \alpha_2 \beta_2^T + \cdots + \alpha_r \beta_r^T. \]

The proof is completed. \( \square \)

**III. MAIN RESULTS**

Based on the above analysis, one can obtain the following construction methods to find a sign pattern in \( N_k \).

**A. Construction Method 1—Jordan Method**

By Lemma 3, we may obtain the Jordan form method to construct a sign patterns in \( N_k \). For example, let

\[ J = \begin{bmatrix} J_1 & J_2\cr J_2 & J_1\cr \end{bmatrix}, \]

\[ D = \begin{bmatrix} 1 & 2 & 2 & 0 & 1 & 3\cr 0 & 1 & 0 & 0 & 1 & 0\cr 0 & 1 & 1 & 1 & 0 & 2\cr 0 & 0 & 1 & 0 & 0 & 0\cr 0 & 1 & 0 & 2 & 1 & 0\cr 0 & 1 & 0 & 0 & 0 & 1\cr \end{bmatrix}, \]

\[ D^{-1} = \begin{bmatrix} -1 & -1 & -2 & 2 & 2 & 3\cr 0 & 1 & 0 & 0 & -1 & 1\cr 1 & 1 & -1 & -2 & -2 & 3\cr 0 & 0 & 0 & 0 & 1 & 1\cr 0 & 0 & 0 & 0 & 1 & -1\cr 0 & -1 & 0 & 0 & 1 & 0\cr \end{bmatrix}, \]

\[ D^{-1} = \begin{bmatrix} -1 & -1 & -2 & 2 & 2 & 3\cr 0 & 1 & 0 & 0 & -1 & 1\cr 1 & 1 & -1 & -2 & -2 & 3\cr 0 & 0 & 0 & 0 & 1 & 1\cr 0 & 0 & 0 & 0 & 1 & -1\cr 0 & -1 & 0 & 0 & 1 & 0\cr \end{bmatrix}, \]

Note that

\[ B = DJD^{-1} = \begin{bmatrix} 0 & 2 & 2 & 0 & -2 & -1\cr 1 & 0 & 1 & -1 & -1 & -2\cr 0 & 1 & 1 & 0 & -1 & -1\cr -2 & 1 & 0 & 2 & 1 & 3\cr 1 & 0 & 1 & -1 & -1 & -2\cr 1 & 1 & -1 & -1 & -2 & 3\cr \end{bmatrix}, \]

\[ B^4 = 0, \]

Then

\[ A = \text{sgn}(B) = \begin{bmatrix} 0 & 1 & 1 & 0 & -1 & 1\cr 0 & 0 & 1 & -1 & -1 & 1\cr 0 & 0 & 1 & -1 & -1 & 1\cr 0 & 0 & 1 & -1 & -1 & 1\cr 0 & 0 & 1 & -1 & -1 & 1\cr 0 & 0 & 1 & -1 & -1 & 1\cr \end{bmatrix}. \]

**B. Construction Method 2—vectors spanning method**

Let \( l_1, l_2, \cdots, l_k \) be nonnegative integers with \( \sum_{i=1}^k n_i = n, \sum_{i=1}^k (i-1)n_i = n \). Let real column vectors \( \alpha_1, \alpha_2, \cdots, \alpha_r \) and \( \beta_1, \beta_2, \cdots, \beta_r \) of order \( n \) satisfy the condition

\[ \beta_j \alpha_i = \begin{cases} 1 & j \equiv s \pmod{k-1}, s = 1, 2, \cdots, k-2, \cr 1 \leq j \leq (k-1)l_i - 1, i = j + 1, \cr 0 & \text{otherwise.} \end{cases} \]

By Theorem 3, the real matrix

\[ B = \sum_{i=1}^{l_i} \alpha_i \beta_i^T \]

is nilpotent of index at most \( k \), and its sign pattern is in \( N_k \). For example, let \( n = 8, r = 6, l = m = 1, \)

\[ \alpha_1 = \begin{bmatrix} 2 & 3 & 0 & 1 \cr 1 & 1 & 2 & 1 \cr \end{bmatrix}, \]

\[ \alpha_2 = \begin{bmatrix} 1 & 3 & 0 & 1 \cr 1 & 1 & 2 & 1 \cr \end{bmatrix}, \]

\[ \alpha_3 = \begin{bmatrix} 1 & 1 & 1 & 1 \cr \end{bmatrix}. \]
\( \beta_1 = (1, 1, 0, -1, 0, 0, -1, -1), \quad \beta_2 = (-1, 1, 2, -1, -1, 1, -1, -1), \\
\beta_3 = (-1, 0, 0, 0, 0, 0, 1, 0), \quad \beta_4 = (2, 0, 0, -1, 1, 0, -1, -1), \\
\beta_5 = (1, -1, -1, 1, -1, -1, 1, -1), \quad \beta_6 = (-1, 0, 0, 0, 0, 0, 0, 1), \)

\[
B = \sum_{1 \leq i \leq 6} \alpha_i \beta_i = \begin{bmatrix} 1 & 2 & 1 & -3 & 1 & 0 & -4 & 1 \\
2 & 5 & 5 & -7 & 4 & -2 & -8 & 1 \\
-2 & 0 & 0 & 0 & 0 & 0 & 1 & 1 \\
2 & 2 & 2 & -3 & 2 & -1 & -3 & 1 \\
3 & 1 & 1 & -3 & 1 & 0 & -4 & 0 \\
-1 & 2 & 3 & -3 & 2 & -1 & -4 & 3 \\
1 & 2 & 1 & -3 & 1 & 0 & -4 & 1 \\
1 & 2 & 1 & -3 & 1 & 0 & -4 & 1 
\end{bmatrix}
\]

\( D^1 = 0. \)

Then

\[ A = \text{sgn}(B) \in N_4. \]

**C. Construction Method 3—block method**

**Theorem 4.** Suppose \( A = \begin{bmatrix} A_1 & A_2 \\ A_3 & A_4 \end{bmatrix} \in N_k, \) where \( A_1 \) and \( A_4 \) are square, then for any positive integer \( m, \) we have

\( \tilde{A} = \begin{bmatrix} A_1 & A_2 & \cdots & A_2 \\ A_3 & A_4 & \cdots & A_4 \end{bmatrix} \in N_k, \)

where \( \tilde{A} \) has \((m + 1)^2\) blocks.

**Proof.** Note that, if \( A \in N_k, \) then there is a real matrix

\[ B = \begin{bmatrix} B_{11} & B_{12} \\ B_{21} & B_{22} \end{bmatrix} \in Q(A), \]

where \( B_i \in Q(A_i)(i = 1, 2, 3, 4), \) such that \( B^k = 0. \)

Let

\[ B_j = \begin{bmatrix} f_{1j}(B_1, B_2, B_3, B_4) & f_{2j}(B_1, B_2, B_3, B_4) & f_{3j}(B_1, B_2, B_3, B_4) & f_{4j}(B_1, B_2, B_3, B_4) \end{bmatrix} \]

for \( j = 1, 2, \cdots, k - 1. \)

For short, we denote

\[ B^j = \begin{bmatrix} f_{1j} & f_{2j} & f_{3j} & f_{4j} \end{bmatrix} \]

for \( j = 1, 2, \cdots, k - 1. \) And

\[ \tilde{B} = \begin{bmatrix} f_{11} & f_{12} & \frac{1}{m} B_2 & \frac{1}{m} B_2 \\ f_{21} & f_{22} & \frac{1}{m} B_2 & \frac{1}{m} B_2 \\ \vdots & \vdots & \vdots & \vdots \\ f_{41} & f_{42} & \frac{1}{m} B_2 & \frac{1}{m} B_2 \end{bmatrix} \]

When \( k = 2, \) it follows that

\[ B^2 = \begin{bmatrix} B_{11}^2 + B_{21} B_{32} & B_{11} B_{22} + B_{21} B_{32} \\ B_{11} B_{22} + B_{31} B_{42} & B_{31} B_{42} + B_{21} B_{32} \end{bmatrix}. \]

So \( \tilde{B}^2 = 0. \) Then \( A \in N_k. \)

Suppose that we have

\[ \tilde{B}^s = \begin{bmatrix} f_{1s} & \frac{1}{m} f_{2s} & \cdots & \frac{1}{m} f_{2s} \\ f_{3s} & \frac{1}{m} f_{4s} & \cdots & \frac{1}{m} f_{4s} \\ \vdots & \vdots & \ddots & \vdots \\ f_{3s} & \frac{1}{m} f_{4s} & \cdots & \frac{1}{m} f_{4s} \end{bmatrix} \]

for \( 2 \leq s \leq k, \)

then

\[ B^{s+1} = B^s B = \begin{bmatrix} f_{1s} & f_{2s} & \cdots & f_{2s} \\ f_{3s} & f_{4s} & \cdots & f_{4s} \\ f_{3s} & f_{4s} & \cdots & f_{4s} \\ f_{3s} & f_{4s} & \cdots & f_{4s} \end{bmatrix} \begin{bmatrix} B_1 & B_2 \\ B_3 & B_4 \end{bmatrix} \]

and

\[ \tilde{B}^{s+1} = \begin{bmatrix} f_{1s} & \frac{1}{m} f_{2s} & \cdots & \frac{1}{m} f_{2s} \\ f_{3s} & \frac{1}{m} f_{4s} & \cdots & \frac{1}{m} f_{4s} \\ \vdots & \vdots & \ddots & \vdots \\ f_{3s} & \frac{1}{m} f_{4s} & \cdots & \frac{1}{m} f_{4s} \end{bmatrix} \begin{bmatrix} B_1 & \frac{1}{m} B_2 & \cdots & \frac{1}{m} B_2 \\ B_3 & \frac{1}{m} B_4 & \cdots & \frac{1}{m} B_4 \\ \vdots & \vdots & \ddots & \vdots \\ B_3 & \frac{1}{m} B_4 & \cdots & \frac{1}{m} B_4 \end{bmatrix} \]

So

\[ B^k = \begin{bmatrix} f_{1(k-1)} B_1 + f_{2(k-1)} B_3 & f_{1(k-1)} B_2 + f_{2(k-1)} B_4 \\ f_{3(k-1)} B_1 + f_{4(k-1)} B_3 & f_{3(k-1)} B_2 + f_{4(k-1)} B_4 \end{bmatrix} \in N_k, \]

\( \tilde{B}^k = 0. \)

By the principle of mathematical induction, we have \( \tilde{A} \in N_k. \)

**D. Construction Method 4—null space method**

**Theorem 5.** Let \( B \) and \( C \) be nilpotent real matrices of indices of \( k \) with order \( v_1 \) and \( v_2, \) respectively. Let \( p \) be a positive integer. The kernel of a matrix \( B, \) denoted by \( \text{Ker}(B), \) also called the null space, is the kernel of the linear map defined by the matrix \( B. \) Suppose that the following conditions hold:

\[ u_1, u_2, \cdots, u_p \in \text{Ker}(B^i), \quad v_1, v_2, \cdots, v_p \in \text{Ker}((C^j)^T), \]

\( (9) \)
where $1 \leq i < k$, $1 \leq j < k$, and $i + j \leq k$. Then the following partitioned block real matrix of order $n_1 + n_2$

$$D = \begin{bmatrix} B & X \\ 0 & C \end{bmatrix}$$

is nilpotent of index at most $k$ and $A = \text{sgn}(D) \in \mathbb{N}_k$, where $X = u_1v_1^T + u_2v_2^T + \cdots + u_pv_p^T$.

**Proof.** In fact, let $D = \begin{bmatrix} B & X \\ 0 & C \end{bmatrix}$, where $B$ and $C$ are square. Then

$$D^k = \begin{bmatrix} B^k & B^{k-1}X + B^{k-2}XC + \cdots + XC^{k-1} \\ 0 & C^k \end{bmatrix}.$$

Thus $D^k = 0$ if and only if $B^k = 0, C^k = 0$ and

$$B^{k-1}X + B^{k-2}XC + \cdots + XC^{k-1} = 0.$$

It is obvious that $B^k = 0$ and $C^k = 0$. In addition, we observe that

$$B^{k-1}X + B^{k-2}XC + \cdots + XC^{k-1} = B^{k-2}(Bu_1v_1^T + Bu_2v_2^T + \cdots + Bu_pv_p^T) + B^{k-3}(u_1, \cdots, u_p) \begin{bmatrix} v_1^T \\ \vdots \\ v_p^T \end{bmatrix} C + \cdots + (u_1, \cdots, u_p) \begin{bmatrix} v_1^T \\ \vdots \\ v_p^T \end{bmatrix} C^{k-1}.$$

Therefore, we get the desired result with the above condition (9). $\square$

**IV. Conclusion**

In this paper, sign patterns allowing nilpotence of index at most $k$ are researched and four methods to construct sign patterns under the condition that allows nilpotence of index at most $k$ are obtained, which generalizes some recent results in [1], [4] and has a certain theoretical and practical value.

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**References**


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